2. Precision, Stability, Integrals

Present your solutions to the following problems using latex, if you have figures make sure they are publication quality, include your code in the solutions in an appendix. Bring a print out to class.

1. Determine your machine precision to within a factor of two in double and single precision.

2. Within a factor of two, determine the minimum and maximum single and double precision real numbers that your computer supports.

3. Implement a double precision subroutine for the spherical Bessel functions, $j_n(x)$, using the recursion relation

$$j_{n+1}(x) + j_{n-1}(x) = \frac{2n+1}{x} j_n(x)$$

and the initial conditions

$$j_0(x) = \frac{\sin x}{x} \qquad j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}.$$

Compare the results for $x \in (0, 1)$ for j_4, j_5, j_6 to the asymptotic expression given by

$$j_n(x) \to \frac{x^n}{(2n+1)!!}\left(1 - \frac{x^2}{2(2n+3)}\right).$$

Plot the relative error in each case (include the plots in your solution). Explain what you see.

4. Analytically compute the probability of a dart landing in a D-dimensional hypershere that is embedded in a D-dimensional cube. What is the probability for D=1, 10, 20? *Hint: evaluate* $\int \exp(-\vec{x}^2) d^D\vec{x}$ *in two different ways.*

5. Establish a viable numerical prescription for the principal value integral,

$$\mathrm{P}\int_0^\infty \frac{f(k)}{g(k) - g(k_0)} dk.$$

6. Compute the volume of a sphere using simple Monte Carlo techniques. Estimate the error in your answer by running your code many times, and making a histogram of the results. Call the number of throws N and the number of times you evaluate the volume M. Plot the distribution for $N = 1000$ and $N = 100,000$. Choose M large enough to get nice plots. What happens to the distribution as you increase N? Explain. Compute the estimated error in your estimate of the volume by evaluating the standard deviation of your histograms.

7. Use the VEGAS Monte Carlo routine (available on the class web site, along with its documentation) to evaluate

$$\int d^6x \exp(-\vec{x}^2).$$

Compare your result to the analytic result as a function of the number of Monte Carlo calls (a graph of relative error would be nice).