4. Simple Dynamical Systems

Present your solutions to the following problems using latex, if you have figures make sure they are publication quality, include your code in the solutions. Include your code as an appendix. Do **not** include library code or code I give you. Print your pdf files and bring them to class.

1. Verify explicitly that the Euler method does not conserve energy by finding the expression for

$$\delta E = \frac{E_{i+1} - E_i}{E_i}$$

for the case of the simple harmonic oscillator.

2. Write an Euler routine to compute the motion of a golf ball. Assume a constant gravitational field, a constant air density, a drag force, and a Magnus force (so neglect Coriolis and centrifugal forces). For your benefit I quote relevant parameters and formulae:

$$\vec{F}_{\rm mag} = S_0 \, \vec{\omega} \times \vec{v}$$

$$\vec{F}_{\rm drag} = -C(v) \rho A v^2 \hat{v}$$

$$C(v) = \frac{1}{2} \, \theta(v < v_c) + \frac{v_c}{2v} \, \theta(v > v_c)$$

and $A = 0.00143$ m^2, $\rho = 1.2$ kg/m^3, $S_0 \omega / m = 0.25$ 1/s, $v_0 = 70$ m/s (for Tiger Woods it is more like 100 m/s), $v_c = 14$ m/s, and $m = 0.046$ kg.

Obtain the distance travelled for $\theta_0 = 7$, 9, and 11 degrees. Plot the trajectories for a golf ball with the parameters specified (use $\theta_0 = 9$ deg), and for a smooth ball (C=1/2), and a ball with no backspin. Comment on your results.

3. Consider the driven, damped oscillator with the following parameters: $b = 1/2$, $g/\ell = 1$, $\Omega_D = 2/3$. Start with $\theta_0 = 0.2$ rad and $\omega_0 = 0$; try $dt = 0.04$ or less and use rk4.

Obtain the Poincaré map at $F_D = 1.2$ (plot ω vs. θ at stroboscope times $2\pi n = \Omega_D t$.

4. Consider the logistics map

$$x_{n+1} = f(x_n) \qquad\qquad f(x) = \mu x(1-x).$$

(i) Construct a bifurcation plot for $\mu \in (0, 4)$.
(ii) Plot the invariant density for $\mu = 4$ and $\mu = 3.8$.