7. Simple Quantum Problems

Present your solutions to the following problems using latex, if you have figures make sure they are publication quality, include your code in the solutions. Print your pdf files and bring them to class.

1. Scales

   Determine the length scales associated with each of the following Hamiltonians:
   (i) $p^2/2\mu + 1/2\mu\omega^2 r^2$
   (ii) $p^2/2\mu + \alpha/r$
   (iii) $p^2/2\mu + br$.
   One way to do this is to write the Schroedinger equation in dimensionless form by changing variables.

2. Bound State Errors

   Plot the relative error for first, fifth, and tenth S-wave eigenvalues of the SHO (set $\mu = \omega = 1$) vs. the number of grid points for $x_{max} = 2x_0, 5x_0, 10x_0$ where x_0 is the length scale for the SHO problem.

3. The Charmonium Spectrum

   Investigate a model of charmonium mesons. These are considered to be bound states of charm and anticharm quarks described by a Schroedinger equation. The Hamiltonian is taken to be

$$H = \frac{p^2}{2\mu} - \frac{4}{3}\frac{\alpha_S}{r} + br + \alpha_S \frac{32\sigma^3}{9m^2\sqrt{\pi}} \mathrm{e}^{-\sigma^2 r^2} \vec{S}_1 \cdot \vec{S}_2.$$

   The last term is the analogue of the hydrogenic hyperfine interaction. Parameters are $(\alpha_S, b, m, \sigma) = $ (0.5461, 0.1425 GeV^2, 1.4794 GeV, 1.0946 GeV). Work in 'high energy units' where $\hbar = c = 1$.

   Obtain the lowest five energies and RMS radii for the quantum numbers $^{(2S+1)}L_J =$, $^1S_0$, $^3S_1$, $^1P_1$, and ^3P_J. Get energies to 1 MeV.

4. Separable Scattering

(i) Obtain the analytic expression for the N-channel T-matrix assuming a separable potential: $V^{\alpha\beta}(k,k') = \lambda^{\alpha\beta} g(k) g(k')$. *Hint: assume that T is proportional to V.*

(ii) Specialise your answer to N=1 and perform the required integral to get an explicit form for T assuming that $g(k) = a^3/(k^2+a^2)^2$, where a is a parameter. *Hint: write $(k^2+a^2)^{-4}$ as the appropriate derivative of $(k^2+a^2)^{-1}$ with respect to a^2 and use Cauchy's theorem.*

(iii) Compare your solution to part (ii) to one obtained numerically using the momentum space method discussed in class (plot the real and imaginary parts of T up to k=2. Take $\mu = \lambda = a = 1$.)