8. Anisotropic Heisenberg Antiferromagnet.

Present your solutions to the following problems using latex, if you have figures make sure they are publication quality, include your code in the solutions. Print your pdf files and bring them to class.

1. Solve the 1-d anisotropic Heisenberg antiferromagnet using the Lanczos algorithm and the techniques discussed in class.

$$H = \frac{J}{4}\sum_{(ij)}\left[\sigma_i^z\sigma_j^z + g(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y)\right]$$

Use periodic boundary conditions, map configurations to integers, store Hilbert space vectors as double precision arrays, use $S_z^{\rm tot} = 0$ basis states, use as little memory as possible and see how large a system you can solve.

(i) Prove analytically that $|\varphi_n\rangle$ and $|\varphi_{n+1}\rangle$ are orthogonal.

(ii) Obtain the analytic state code for the two Neel states.

(iii) Confirm that your code gives the correct answer for $L = 4$, $g = 1$. If you are ambitious, confirm it for $L = 4$ and general g.

(iv) Plot $E_0/(JL)$ vs. g for L=12.

(v) Obtain the ground state and first excited state energies (expressed as $E/(JL)$) for $g = 1$ to 5 or 6 digits and extrapolate to the infinite chain length limit. If you plot the energies against the right variable, this is an easy extrapolation.

(vi) Obtain the gap defined as $(E_1 - E_0)/J$ for $g = 1$. Extrapolate this to the bulk limit as well.