Mass of $D_{sJ}^*(2317)$ and Coupled Channel Effect

Dae Sung Hwang
(Sejong Univ.)
in collaboration with
Do-Won Kim
(Kangnung Univ.)

- Coupled Channel Effect
  Basics
  Mechanism of Light Quark Pair Creation
- Mass shift of $D_{sJ}^*(2317)$ from Coupled Channel Effect PLB '04
- Decay Constants of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ hep-ph/0410301

First GHP Meeting

Fermilab, Oct 24-26, 2004

Coupled Channel Effect

Basics

$$H = H_0 + H_{pc}$$

$$H_0 = \begin{pmatrix} H_Q & 0 \\ 0 & H_c \end{pmatrix} \qquad H_{pc} = \xi \begin{pmatrix} 0 & H_{Qc} \\ H_{cQ} & 0 \end{pmatrix}$$

$$H_0 |i\rangle = M_i^{bare} |i\rangle, \quad H_c |\vec{P}, \lambda\rangle = E_c(P) |\vec{P}, \lambda\rangle$$

$$\langle i|j\rangle = \delta_{ij}, \quad \langle \vec{P}, \lambda | \vec{P}', \lambda' \rangle = (2\pi)^3 \delta^3(\vec{P} - \vec{P}') \delta_{\lambda\lambda'}, \quad \langle i | \vec{P}, \lambda \rangle = 0$$

physical mass eigenstates are eigenstates of the total hamiltonian H. We denote the physical mass eigenstate by $|N\rangle$:

$$H |N\rangle = M_N |N\rangle$$

$$|N\rangle = \sum_i a_i |i\rangle + \sum_\lambda \int \frac{d^3P}{(2\pi)^3} b_\lambda(P) |\vec{P}, \lambda\rangle$$

$$b_\lambda(P) = \sum_i \frac{\langle \vec{P}, \lambda | H_{pc} | i \rangle}{M_N - E_c(P)} a_i$$

$$H_{eff}\, a^N = M_N\, a^N$$

with $H_{eff} = M^{bare} + \Omega(M_N)$

where M^{bare} is a diagonal matrix with M_i^{bare} as diagonal elements, a^N is a column vector with probability amplitudes a_i^N as its elements, and $\Omega_{ij}(W)$ are given by

$$\Omega_{ij}(W) = \sum_\lambda \int \frac{d^3P}{(2\pi)^3} \frac{\langle i | H_{pc} | \vec{P}, \lambda \rangle \langle \vec{P}, \lambda | H_{pc} | j \rangle}{W - E_c(P) + i\epsilon}$$

The mass eigenvalues M_N are found from

$$\mathrm{Det}\left(M_N I - [M^{bare} + \Omega(M_N)] \right) = 0 .$$

Normalization constant $\mathcal{N}$ is calculated from

$$\mathcal{N}^2 \left(\sum_i |a_i|^2 + \sum_\lambda \int \frac{d^3P}{(2\pi)^3} |b_\lambda(P)|^2 \right) = 1 .$$

$$\mathcal{N}^2 = \left(\sum_i |a_i|^2 + \sum_{i,j} a_i^* \omega_{ij}(M_N) a_j \right)^{-1}$$

where $\omega_{ij}(M_N) = -\frac{d}{dW} \Omega_{ij}(W) \Big|_{W = M_N}$

The probability that $|N\rangle$ be in the quark-antiquark bound state sector is $Z = \mathcal{N}^2 \sum_i |a_i|^2$, while that in the two-meson state sector is $1 - Z$.

Mechanism of Light Quark Pair Creation

The Cornell group studied the effect of OZI allowed decay channels. They proposed that the following interaction hamiltonian is responsible for the decay as well as the binding of quark–antiquark bound states [Eichten et al. 1978, 1980]:

$$H_I = \frac{1}{2} \sum_{a=1}^{8} \int d^3x \, d^3y : \rho_a(\vec{x}) V(\vec{x}-\vec{y}) \rho_a(\vec{y}) :$$

where $V(r) = -\frac{\kappa}{r} + \frac{r}{a^2}$

$$\rho_a(\vec{x}) = \psi^\dagger(\vec{x}) \frac{1}{2} \lambda_a \psi(\vec{x})$$

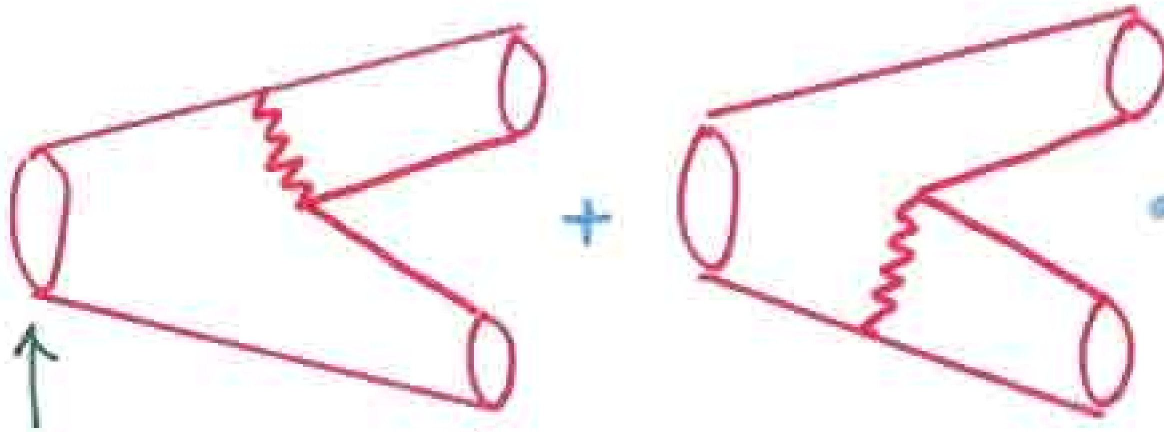
$$H_{eff} = M^{bare} + \Omega(M_N)$$

$$\Omega_{nL,m'L'}(W) = \sum_I \int P^2 dP \, \frac{H^I_{nL,mL'}(P)}{W - E_1(P) - E_2(P) + i\epsilon}$$

where

$$H^I_{nL,mL'}(P) = f^2 \sum_\ell C(JS, LL', J, J_2, \ell) \times I^\ell_{nL}(P) \, I^\ell_{mL'}(P)$$

with $f^2 = \frac{2}{3\pi^2 a^4 m_q^2} \frac{1}{\beta^3}$



$+$ $\propto I^{\ell}_{nL}(P)$

$R_{nL}(r)$

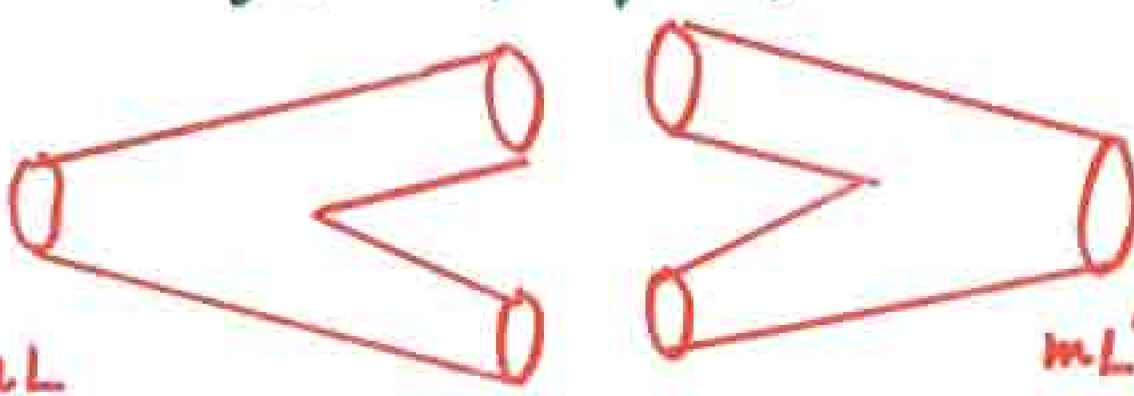
$\phi(r) = N e^{-\beta r^2}$

$$I^{\ell}_{nL}(P) = \int_0^\infty dt\, \Theta(t)\, R_{nL}\left(\frac{t}{\sqrt{\beta}}\right) j_\ell\left(\frac{\rho_Q P t}{\sqrt{\beta}}\right)$$

with

$$\Theta(t) = \left[t e^{-t^2} + (t^2-1) e^{-t^2/2} \sqrt{\frac{\pi}{2}}\, \mathrm{erf}\left(\frac{t}{\sqrt{2}}\right)\right]$$
$$+ 4\beta a^2 \kappa \left[-t e^{-t^2} + e^{-t^2/2} \sqrt{\frac{\pi}{2}}\, \mathrm{erf}\left(\frac{t}{\sqrt{2}}\right)\right]$$

where $\rho_Q = m_Q/(m_q + m_Q)$.



nL mL'

$= \Omega_{nL,mL'}(W)$

Mass Shift of $D_{sJ}^*(2317)$ from Coupled Channel Effect

Godfrey and Isgur obtained the mass of $1\,^3P_0\,(c\bar{s})$ as 2.48 GeV, and we take this value as M^{bare} and calculate the mass shift by coupled channel effect.

we use two parameterizations of potential:

(A) Eichten et al. $\kappa = 0.517$, $a = 2.12\ \mathrm{GeV}^{-1}$, $m_c = 1.84\ \mathrm{GeV}$.

(B) Hagiwara et al. $\kappa = 0.47$, $a = 1/\sqrt{0.19}\ \mathrm{GeV}^{-1}$, $m_c = 1.32\ \mathrm{GeV}$.

(we use $m_s = 0.55$ GeV, $m_{u,d} = 0.33$ GeV.)

$N r e^{-\beta_b r^2}$ $\left(\beta = \frac{\mu^2}{2}\right)$ $N e^{-\beta_a r^2}$

we use two values of μ_b:

0.46 GeV [Barnes et al.] (04)

0.57 GeV [Godfrey et al.] (91)

We use $\mu_a = 0.61$ GeV which is the average of $\mu_a(K) = 0.61$ and $\mu_a(D) = 0.60$ given in [Barnes et al.] (04)

The formula given in page 5 was derived for the case where two mesons in the coupled channel have the same heavy quark such as DD [Eichten et al. 78, 80]. However, in our present study two mesons are D and K, so our calculation is not exact and approximate since we use the formula given in page 5. We approximate our system by using this formula, μ values mentioned in the previous page, $\rho_Q = m_Q/(m_q+m_Q)$ with $m_Q = (m_c+m_s)/2$. In order to see the sensitivity of the results to the value of ρ_Q, we also performed the same calculation with $\rho_c = m_c/(m_q+m_c)$ and $\rho_s = m_s/(m_q+m_s)$.

In our calculation we also adopt the approximation of taking only one $c\bar{s}$ bound state of $1\,^3P_0$ for the quark-antiquark bound state sectors. We expect that the coupled channel effect to this state is dominant since the measured $1\,^3P_0$ state is very close to the threshold.

The statistical coupling coefficient $C(JS, LL', J, J_z, l)$ is

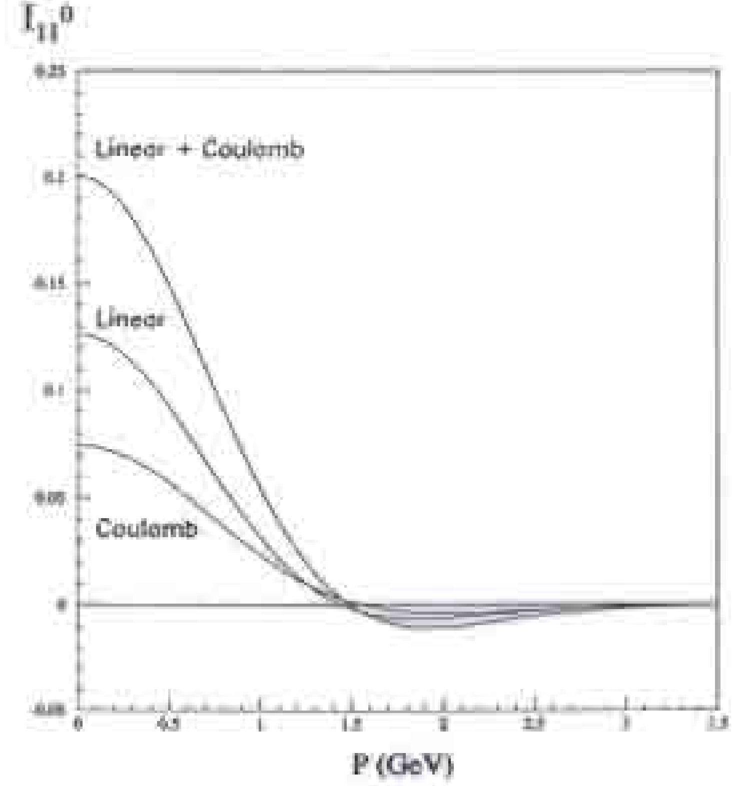
in DK channel 1 for $l=0$

in D^*K^* channel $\begin{cases} \frac{1}{3} & \text{for } l=0 \\ \frac{8}{3} & \text{for } l=2 \end{cases}$

and other coefficients are zero.

Figs. 1 and 2 show that the contribution of the Coulombic part is about 60 % of that from the linear part to $I_{nL}^{l}(P)$. Consequently, Ω becomes about 2.6 times when the Coulombic part is included compared to the case where it is not included.

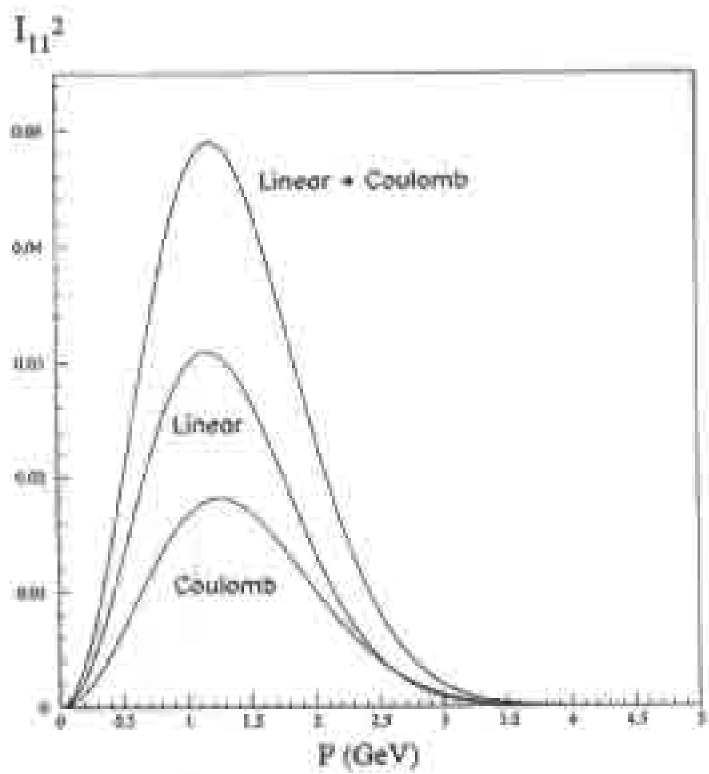
Fig. 3 shows that coupled channel effect shifts $M^{bare} = 2.48$ GeV to M_N near the mass of $D_{sJ}^*(2317)$.



$I_{nL}^{\ell}(P)$

$I_{11}^{0}(P)$

Figure 1: $I_{11}^0(P)$ for the potential A with μ_b=0.46 GeV.



$I_{11}^{2}(P)$

Figure 2: $I_{11}^2(P)$ for the potential A with μ_b=0.46 GeV.

10

$$\mathrm{Det}\left(M_N I - [M^{bare} + \Omega(M_N)]\right) = 0$$

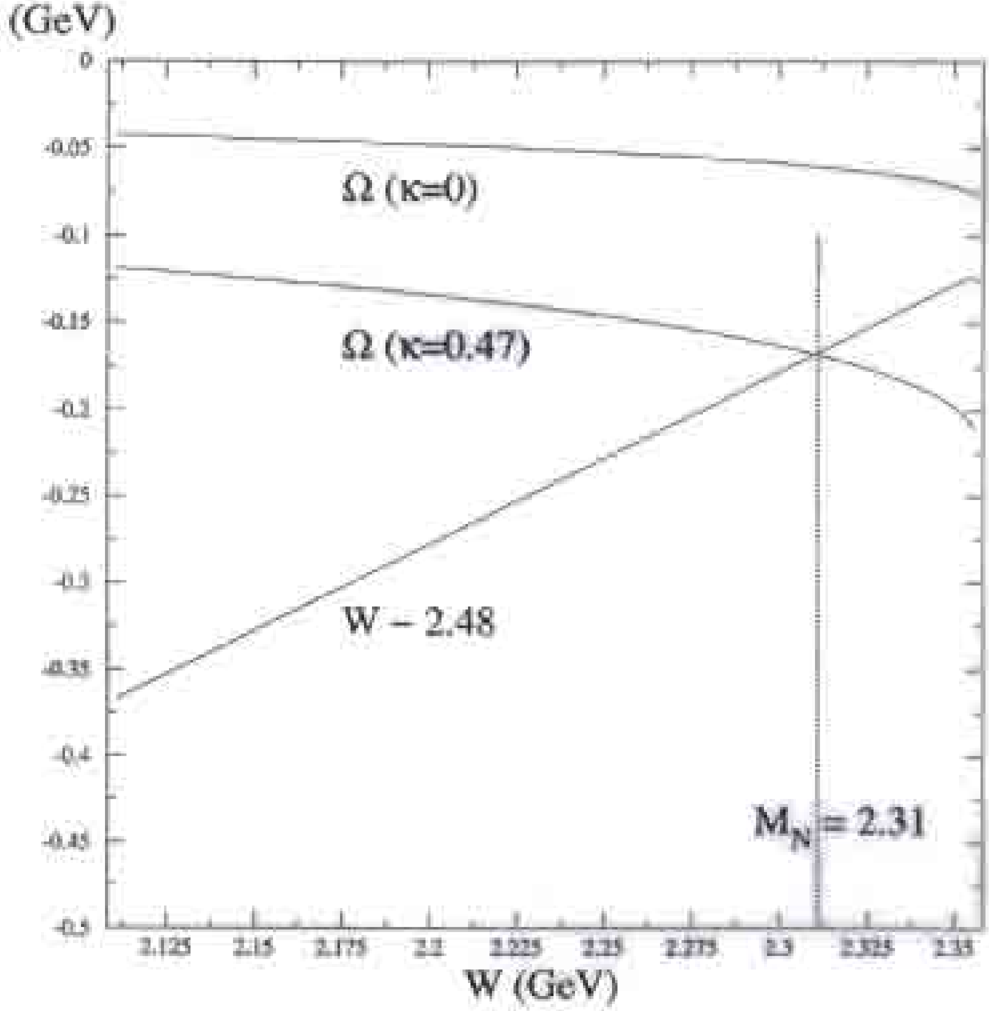


Figure 3: The result when the potential B and μ_b= 0.46 GeV were used. The upper first and second lines are Ω when the Coulombic part is not included and when it is include, respectively, and the lower diagonal line is $W - M^{bare}$ where M^{bare}=2.48 GeV given in [5]. As we see here, Ω without the Coulombic part does not meet with $W - M^{bare}$, which means that there does not exist a solution of the eigenvalue equation (9) below thresholds. On the other hand, Ω with the Coulombic part meets with $W - M^{bare}$ below thresholds and the value of W=2.31 GeV at the meeting point is the eigenvalue M_N of (9).

Parameterizations		M_N (GeV)	ΔM_N (GeV)	Z
Potential	μ_b (GeV)			
A	0.46	2.29	−0.19	0.67
	0.57	2.27	−0.21	0.70
B	0.46	2.31	−0.17	0.67
	0.57	2.29	−0.19	0.71

Table 2: The results of M_N, ΔM_N and Z for each parameterization of the potential and the μ_b value, when $\rho_Q (\equiv m_Q/(m_Q + m_{u,d}))$ with $m_Q = (m_c + m_s)/2$ was used.

Parameterizations			M_N (GeV)	ΔM_N (GeV)	Z
ρ_Q	Potential	μ_b (GeV)			
ρ_c	A	0.46	2.31	−0.17	0.63
		0.57	2.29	−0.19	0.68
	B	0.46	2.32	−0.16	0.62
		0.57	2.31	−0.17	0.67
ρ_s	A	0.46	2.23	−0.25	0.73
		0.57	2.19	−0.29	0.75
	B	0.46	2.27	−0.21	0.73
		0.57	2.25	−0.23	0.75

Table 3: The results of M_N, ΔM_N and Z for each parameterization of the potential and the μ_b value, when $\rho_Q (\equiv m_Q/(m_Q + m_{u,d}))$ with $m_Q = m_c$ or m_s was used in order to see the sensitivity of the results to the parameter values.

Decay Constants of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$

$(0^+, 1^+)$

we calculate the lower bounds of $f_{D_{sJ}^*(2317)}$ and $f_{D_{sJ}(2460)}$ from the measured branching ratios of

$B \to D\, D_{sJ}^*(2317)\, [D_s^+ \pi^0]$,

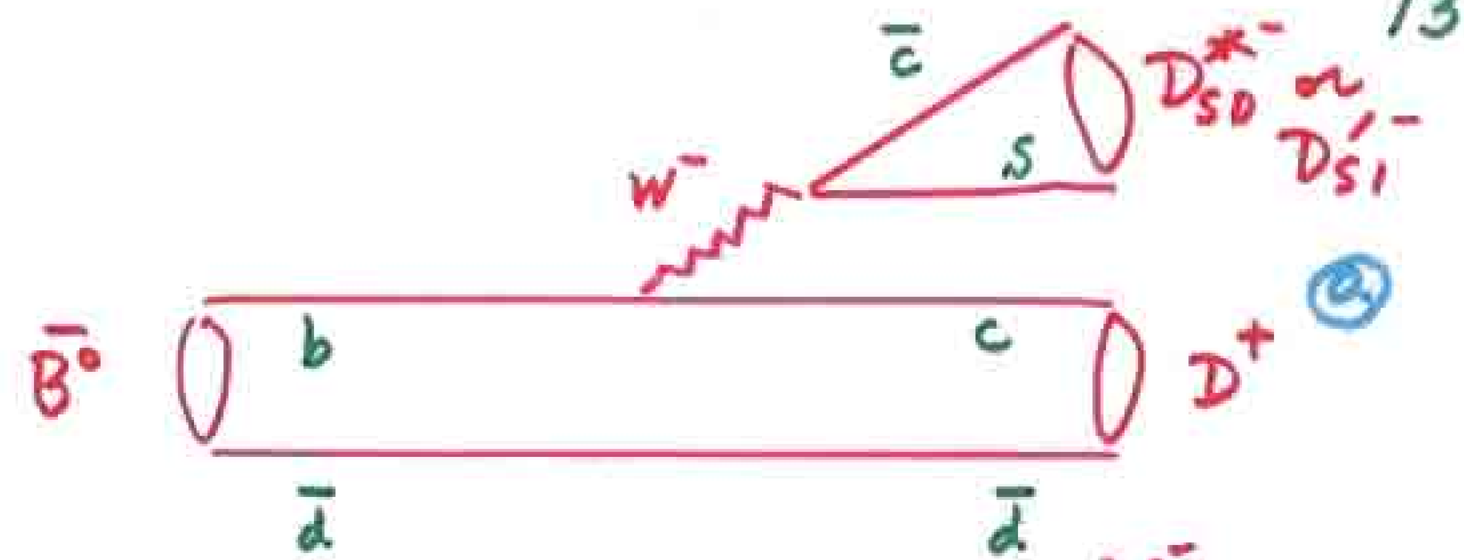
$B \to D\, D_{sJ}(2460)\, [D_s^{*+} \pi^0]$ and

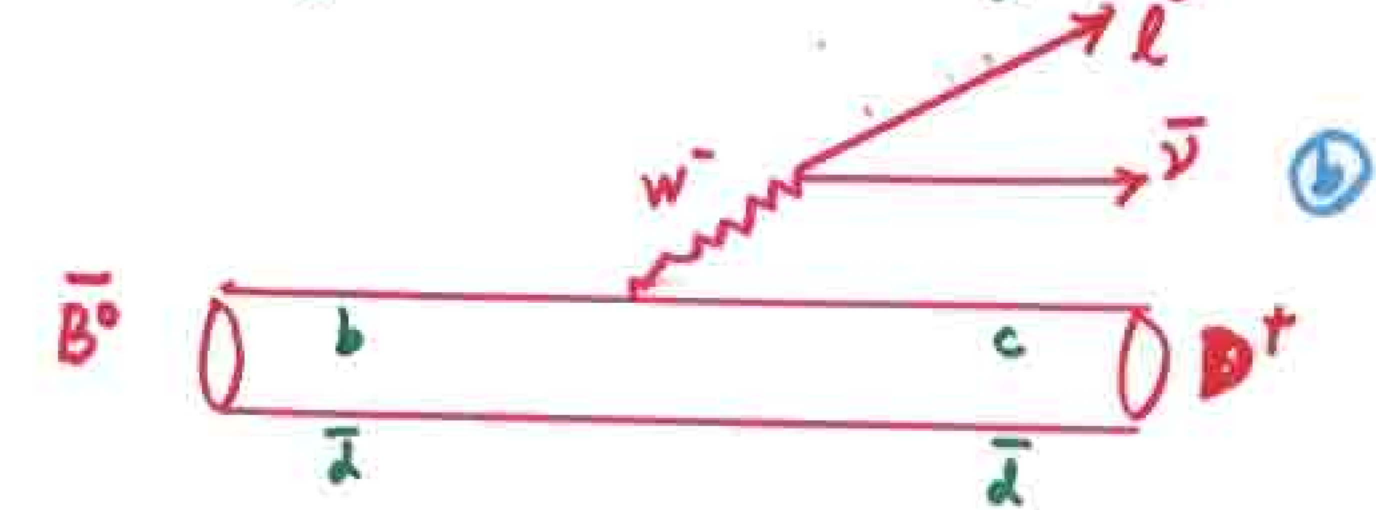
$B \to D\, D_{sJ}(2460)\, [D_s^+ \gamma]$

measured by Belle (03) and Babar (04), using the method of Rosner which is based on the factorization hyposis.

$$\langle 0 | \bar{Q} \gamma^\mu q | \underset{(P)}{D_{s0}^*} \rangle = i P^\mu f_{D_{s0}^*}$$

$$\langle 0 | \bar{Q} \gamma^\mu \gamma_5 q | \underset{(P,\varepsilon)}{D_{s1}'} \rangle = \varepsilon^\mu(P)\, m_{D_{s1}'} f_{D_{s1}'}$$





$$\underline{\mathcal{B}(\bar{B}^0 \to D^+ D_{s0}^{*-})} = [\text{kin. factor}] \times \underline{|a_1|^2 f_{D_{s0}^*}^2} \times \underline{|V_{cb} F_0^{BD}(m_{D_{s0}^*}^2)|^2}$$

$$\underline{\mathcal{B}(\bar{B}^0 \to D^+ D_{s1}^{\prime -})} = [\text{kin. factor}] \times \underline{|a_1|^2 f_{D_{s1}^\prime}^2} \times \underline{|V_{cb} F_1^{BD}(m_{D_{s1}^\prime}^2)|^2}$$

Belle (03) and Babar (04) results.

use the values given from ⓑ expt's.

Group	Decay Mode	$\|a_1\| f_{D'_{s1}}$ (MeV)	$\|a_1\| f_{D^*_{s0}}$ (MeV)	$f_{D'_{s1}}/f_{D^*_{s0}}$
Belle	$B^0 \to D^- D'^+_{s1}$	175 ± 39		2.61 ± 0.89
	$B^0 \to D^- D^{*+}_{s0}$		67 ± 20	
	$B^+ \to \overline{D}^0 D'^+_{s1}$	126 ± 33		2.00 ± 0.72
	$B^+ \to \overline{D}^0 D^{*+}_{s0}$		63 ± 19	
Babar	$B^0 \to D^- D'^+_{s1}$	189 ± 47		1.95 ± 0.64
	$B^0 \to D^- D^{*+}_{s0}$		97 ± 27	
	$B^+ \to \overline{D}^0 D'^+_{s1}$	173 ± 43		2.47 ± 0.91
	$B^+ \to \overline{D}^0 D^{*+}_{s0}$		70 ± 22	
Average		166 ± 20	74 ± 11	2.26 ± 0.41

Table 1: The results for the lower bounds of the decay constants of $D'^+_{s1}(2460)$ and $D^{*+}_{s0}(2317)$ and their ratio. $(|a_1| \sim 1)$

$$|a_1|\, f_{D^*_{s0}(2317)} > 74 \pm 11 \text{ MeV}$$

$$|a_1|\, f_{D'_{s1}(2460)} > 166 \pm 20 \text{ MeV}$$

$$f_{D'_{s1}} / f_{D^*_{s0}} \sim 2.26 \pm 0.41$$

The results are consistent with the results of Veseli and Dunietz (96) obtained from the relativistic quark model calculation given by

$$f_{0^+_{1/2}} = 110 \text{ MeV}, \; f_{1^+_{1/2}} = 233 \text{ MeV}$$

$$f_{1^+_{1/2}} \,/\, f_{0^+_{1/2}} = 2.12$$

- We obtained the ratio as about 2, which is contrary to the naive expectation of the heavy quark symmetry which gives the ratio as 1. This big deviation originates from the large internal motion of quarks inside these P-wave states.
- The obtained lower bounds provide a good evidence that those states are with $j = 1/2$, not with $j = 3/2$, since decay constants of states with $j = 3/2$ are much smaller.

Summary

We showed that coupled channel effect shifts $M^{bare} = 2.48$ MeV to M_N near the mass of $D^*_{sJ}(2317)$.

$\Delta M \sim -160$ MeV.

We showed that the decay constant of $D_{sJ}(2460)$ is about twice of that of $D^*_{sJ}(2317)$.