PY3726 Assignment 5

Due: Apr 27

1. Attractive Gravity.
Ignoring issues with interpretation, determine the range of the equation of state parameter w over which gravity is attractive when considering graviton exchange between "perfect fluids".

2. Simple Faddeev-Popov
Consider the integral

$$Z = \int dx\, dy\, \mathrm{e}^{-1/2(x+y)^2} \equiv \int dx\, dy\, \mathrm{e}^{-1/2\vec{x}^T K \vec{x}}.$$

a. Find the "gauge transformation" that leaves the integrand invariant (in the form $\vec{x} \to \vec{x} + \vec{\delta}$).
b. Prove that δ is an eigenvector of K with eigenvalue zero (work generally).
c. Implement the Faddeev-Popov procedure in the form

$$Z = \int dx\, dy\, \delta(F)\, \frac{\delta F}{\delta \epsilon}\, \mathrm{e}^{-1/2\vec{x}^T K \vec{x}}.$$

Set $F = x^2 y - const$ and verify that the procedure works as intended.

3. Graviton Gauge Constraints
Write the quadratic graviton action in the form

$$S = \int d^4x\, (a\partial_\lambda h^{\mu\nu}\partial^\lambda h_{\mu\nu} + b\partial_\lambda h \partial^\lambda h + c\partial_\lambda h^{\lambda\nu}\partial^\mu h_{\mu\nu} + dh\partial^\mu\partial^\nu h_{\mu\nu}$$

a. Argue that this is the most general form. What conditions must be observed for this to be the case?
b. Impose gauge invariance for $h_{\mu\nu}$ and determine the values of the constants.

4. Photon-photon Scattering via Graviton Exchange
Show that parallel photons do not interact in the limit where the exchange momentum goes to zero.

5. Scalar-scalar Gravitational Interactions
a. Derive the general amplitude for scalars (of different mass) interacting via one-graviton exchange.
b. Obtain the form of the gravitational interaction up to third order in the nonrelativistic expansion.