Phys 3707, Assignment 1 – Operators and Fields

1. Prove that $\hat{n}_i$ and $\hat{n}_j$ commute for fermions.

2. Show that $\{\hat{\psi}(x), \hat{\psi}^\dagger(y)\} = \delta(x-y)$ for fermionic fields.

3. Derive a form for the number operator in terms of fields. Using the field representation, show that $[\hat{H}, \hat{N}] = 0$ and thus particle number is conserved.

4. [1.2.b F&W] Prove that a fermionic system interacting via a central spin-independent potential which is locally attractive and finite will collapse.

5. **one-body Schrödinger Equation** Try to re-establish contact with the Schrödinger equation (in momentum space) by deriving it from the second quantised momentum space Hamiltonian in the case of $N = 1$. To get this to work you will need to add a one-body term $\hat{W} = \sum_{kq} W(q) a^\dagger_{k+q} a_k$. What does $\hat{V}$ do? Write the terms you are getting diagrammatically. Can you interpret them? Why is it necessary to add $\hat{W}$? What happens when you try this with $N = 2$? (do the full calculation if you want, but it is ok to just draw the diagrams you think will contribute and discuss their meaning)

6. **coherent states**

Let

$$|c\rangle = \mathrm{e}^{-|c|^2/2} \sum_n \frac{c^n}{\sqrt{n!}} |n\rangle$$

where c is a complex number and $|n\rangle = \frac{(a^\dagger_{k\lambda})^n}{\sqrt{n!}} |0\rangle$. Prove that

(i) $|c\rangle$ is normalized

(ii) $a_{k\lambda}|c\rangle = c|c\rangle$

(iii) $n = \langle c|\hat{n}|c\rangle = |c|^2$.

[A coherent state is a minimum uncertainty wavepacket for the SHO, and thus represents the 'most classical' wavefunction which can be formed.]