Phys 3707, Assignment 2 – Green Functions

1. [F&W 3.2] Derive

$$e^{iS}\mathcal{O}e^{-iS} = \mathcal{O} + i[S,\mathcal{O}] + \frac{i^2}{2!}[S,[S,\mathcal{O}]] + \frac{i^3}{3!}[S,[S,[S,\mathcal{O}]]] + \dots$$

to the order shown. Use this result to prove that $c_I(t) = c e^{-i\omega t}$.

2. Prove that $\theta(x) = -\int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \frac{e^{-i\omega x}}{\omega + i\epsilon}$.

3. Verify formulas (7.27) and (7.26) by computing E and N for a free fermion gas.

4. [F&W 3.8] Derive the Lehmann representation for $D(k,\omega)$, which is the Fourier transform of

$$iD(x,y) = \frac{\langle\psi_0|T[\tilde{n}_H(x)\tilde{n}_H(y)]|\psi_0\rangle}{\langle\psi_0|\psi_0\rangle}$$

with the density fluctuation operator defined by

$$\tilde{n}(x) = \psi_\alpha^\dagger(x)\psi_\alpha(x) - \frac{\langle\psi_0|\psi_\alpha^\dagger(x)\psi_\alpha(x)|\psi_0\rangle}{\langle\psi_0|\psi_0\rangle}$$

Show that $D(k,\omega)$ is a meromorphic with poles in the second and fourth quadrants of the complex ω plane. Introduce retarded and advanced functions and construct a Lehmann representation for their Fourier transforms. Discuss the analytic properties and derive dispersion relations analogous to Eq. (7.70).