Phys 3707, Assignment 3 – Wick's Theorem and Greens Functions

1. [Relativistic Wicks theorem]
Consider a field theory with interaction $V = \lambda \int d^4x \phi(x)^4$. Evaluate the propagator,

$$\Delta_F(x-y) = \langle \psi | T\phi_H(x)\phi_H(y)|\psi\rangle$$

to order λ^2 using the Gell-Mann–Low formula and Wick's theorem. Verify that only connected diagrams contribute. You may assume that the contraction of two fields is given by $\langle \phi_I(x)\phi_I(y)\rangle = \Delta(x-y)$. Fourier transform your result and express your answer in terms of $\Delta(k)$. Finally, express your results in terms of diagrams.

2. Show that $\epsilon(k) \approx \frac{k^2}{2m^*}$ near the band minimum for a degenerate electron gas. Determine the constant A in the expression $m^* = \frac{m}{1+Ar}$ where $r = r_0/a_0$, a_0 is the Bohr radius, and r_0 is defined as the average interparticle radius: $\Omega = 4/3\pi r_0^3 N$.

3. Compute the effective electron mass at the Fermi surface for a screened Coulomb potential, $V(r) = e^2 exp(-\mu r)/r$. Obtain the limiting form of m^* valid for large μ/k_F. Take $V_D = 0$ (the jellium model).

4. [F&W 3.14]
Use

$$G_{\alpha\beta}(k,\omega) = \frac{1}{\omega - \epsilon_k^0 - \Sigma^\star(k,\omega)}\delta_{\alpha\beta}$$

to show that the energy ϵ_k and lifetime γ_k of long-lived single-particle excitations are given by

$$\epsilon_k = \epsilon_k^0 + \Re\Sigma^\star(k,\epsilon_k)$$

and

$$\gamma_k = \left(1 - \frac{\partial \Re\Sigma^\star(k,\omega)}{\partial\omega}|_{\omega=\epsilon_k}\right)^{-1} \Im\Sigma^\star(k,\epsilon_k)$$

5. [F&W 3.11]

Consider a system of noninteracting spin-1/2 fermions in an external static potential with a Hamiltonian $H_{ex} = \int d^3x \psi^\dagger_\alpha(x) V_{\alpha\beta}(x) \psi_\beta(x)$.

(a) Use Wick's theorem to find the Feynman rules for the single-particle Green's function in the presence of the external potential.

(b) Show that Dyson's equation becomes

$$G^{ex}_{\alpha\beta}(x, y) = G^0_{\alpha\beta}(x - y) + \int d^3z G^0_{\alpha\lambda}(x - z) V_{\lambda\rho}(z) G^{ex}_{\rho\beta}(z, y).$$

(c) Express the ground state energy in a form analogous to Eqn (7.23).