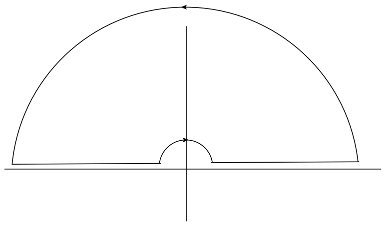
Phys 3707, Assignment 5 – Complex Analysis & Eqn 14.26

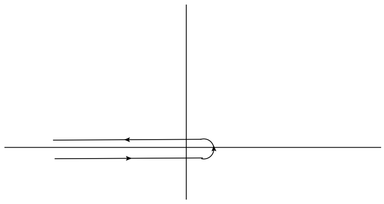
(i) Remind yourself about the properties of the logarithm of a complex number, in particular think about branch cuts. To help, show that $\log(1+i)^2 = 2\log(1+i)$ but $\log(-1+i)^2 \neq 2\log(-1+i)$. Explain why.

(ii) Establish branch cuts for $\log(z-1)$ and $\log(z^2-1)$.

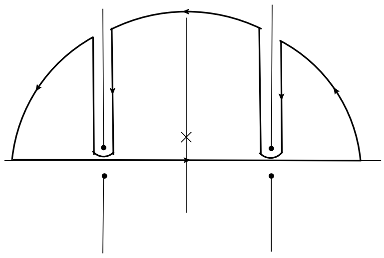
(iii) As further warm up, evaluate $\int_0^\infty dx \frac{\log(x)}{x^2+1}$ and $\int_0^\infty dx \frac{\log(x)}{x^4+1}$. Hint: you may use the definite integrals $\int_0^\infty \frac{dx}{x^2+1} = \frac{\pi}{2}$ and $\int_0^\infty \frac{dx}{x^4+1} = \frac{\pi}{2\sqrt{2}}$. Carefully define your branch cut. Consider the following contour.



(iv) Argue that an integral along a contour such as shown below is zero unless the contour encloses a branch cut.



(v) We now focus on obtaining Eq. 14.26. Argue that the following contour is a good one to use.



(vi) Verify the equation after 14.24.

(vii) Use Eq. 14.8 and the argument at the top of page 179 to obtain Eqs. 14.25 and 14.26.

(viii) Show explicitly that retaining terms of order $u/2k_F$ (or $v/2k_F$) leads to corrections (and not major modifications) of Eq. 14.26.