Phys 3707, Assignment 7 – Magnetism

1. **Magnetism in the DEG**

Consider the degenerate electron gas (jellium model) with $N_\pm$ electrons in the $\pm$ spin state and consider the ground state energy $E(N_+, N_-)$. Show that $E(N, 0) < E(N/2, N/2)$ when $k_F a_0 < c/\pi$. Determine the constant c.

2. **Symmetries in the aHF**

(a) Show that the z-component of the total spin is a conserved quantity for the anisotropic d-dimensional Heisenberg ferromagnet.

(b) Show that the system is invariant under spatial translations.

3. **Magnon-Phonon Mixing**

In general a spin system can also support phonon collective modes. Since these modes share the quantum numbers of magnons, they may interact and mix. We consider this phenomenon with the following simple Hamiltonian:

$$H = \sum_k [\omega_k a_k^\dagger a_k + \Omega_k b_k^\dagger b_k + v_k (a_k b_k^\dagger + a_k^\dagger b_k)]$$

where v_k is a coupling coefficient and $a^\dagger$ and $b^\dagger$ are magnon and phonon creation operators.

(a) Show that a rotation to new operators

$$\begin{pmatrix} a_k \\ b_k \end{pmatrix} = R \begin{pmatrix} A_k \\ B_k \end{pmatrix}$$

preserves the canonical commutation relations.

(b) Determine the angle which diagonalizes H.

(c) Assume that the dispersions ω and Ω cross. Find ω_A and ω_B at this point.