Recent N* Results Using an Effective Lagrangian Model

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Review of The Model.

Progress in Spin-$\frac{5}{2}$ nucleon resonance.

Progress in $\pi N \to \pi\Delta$ channel.

Conclusions.

collaboration :

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Motivation

1. Is it worth working on spin-$\frac{5}{2}$ nucleon resonance in effective lagrangian?
YES!

1. it is lorentz covariant,
2. proper kinematics structure,
3. proper spin structure.

2. $\pi N \rightarrow \pi Delta$ channel inclusion

1. $\pi\pi N$ is prevalent decay channel of baryon resonances
2. Scalar isovector "ζ meson" not adequate to treat $\pi\pi$ in framework
3. Rigorously, one must describe $\pi\pi N$ in terms of $\rho N, \pi\Delta, \pi P_{11}(1440)$, and σN as "underlying final states"

Implementing $\pi\Delta$ channel will enable better extraction of all resonances having relatively large branching ration into $\pi\Delta$

Review of the Model

1. Originally started by Feuster and Mosel (PRC 59, 460 (1999))
2. Based on :
   - Effective Lagrangian
   - Coupled-Channels
   - K-matrix approximation

   nucl-th/0008023, 0008024
3. Channels that are included :

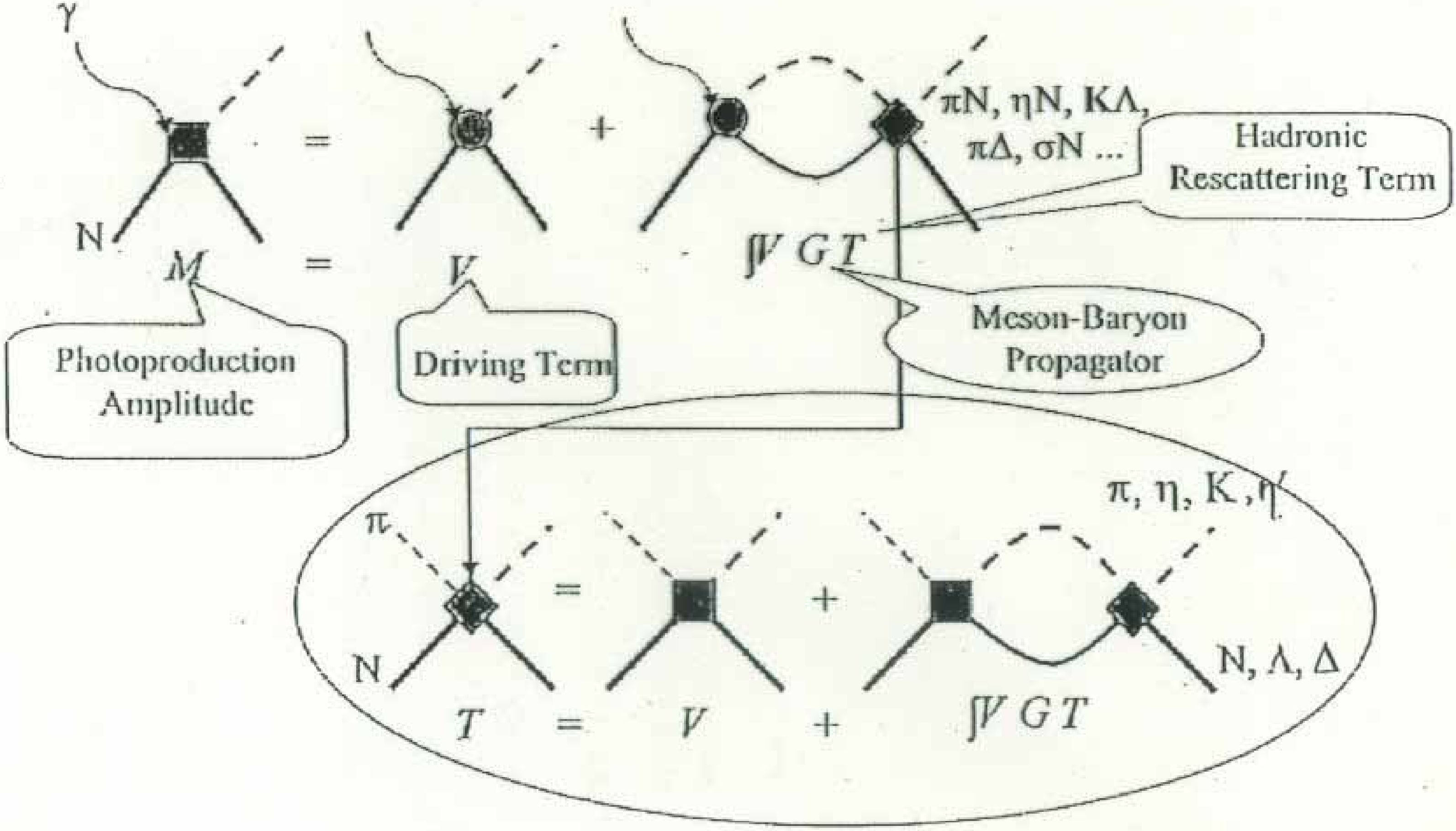
$$\pi N \to \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma, \text{ and } (\eta' N)$$
$$\text{and}$$
$$\gamma N \to \pi N, \eta N, K\Lambda, K\Sigma, \text{ and } (\eta' N).$$

4. Invariant Energy: 1.2 – 2.0 GeV.
5. All 2π final states are parametrized through the coupling to a scalar isovector ζ-meson with mass $m_\zeta = 2m_\pi$.
6. New features are:
   - spin-$\frac{5}{2}$ nucleon resonance, ←
   - $\sigma N, \pi P_{11}(1440)$, and $\pi\Delta$ channel.

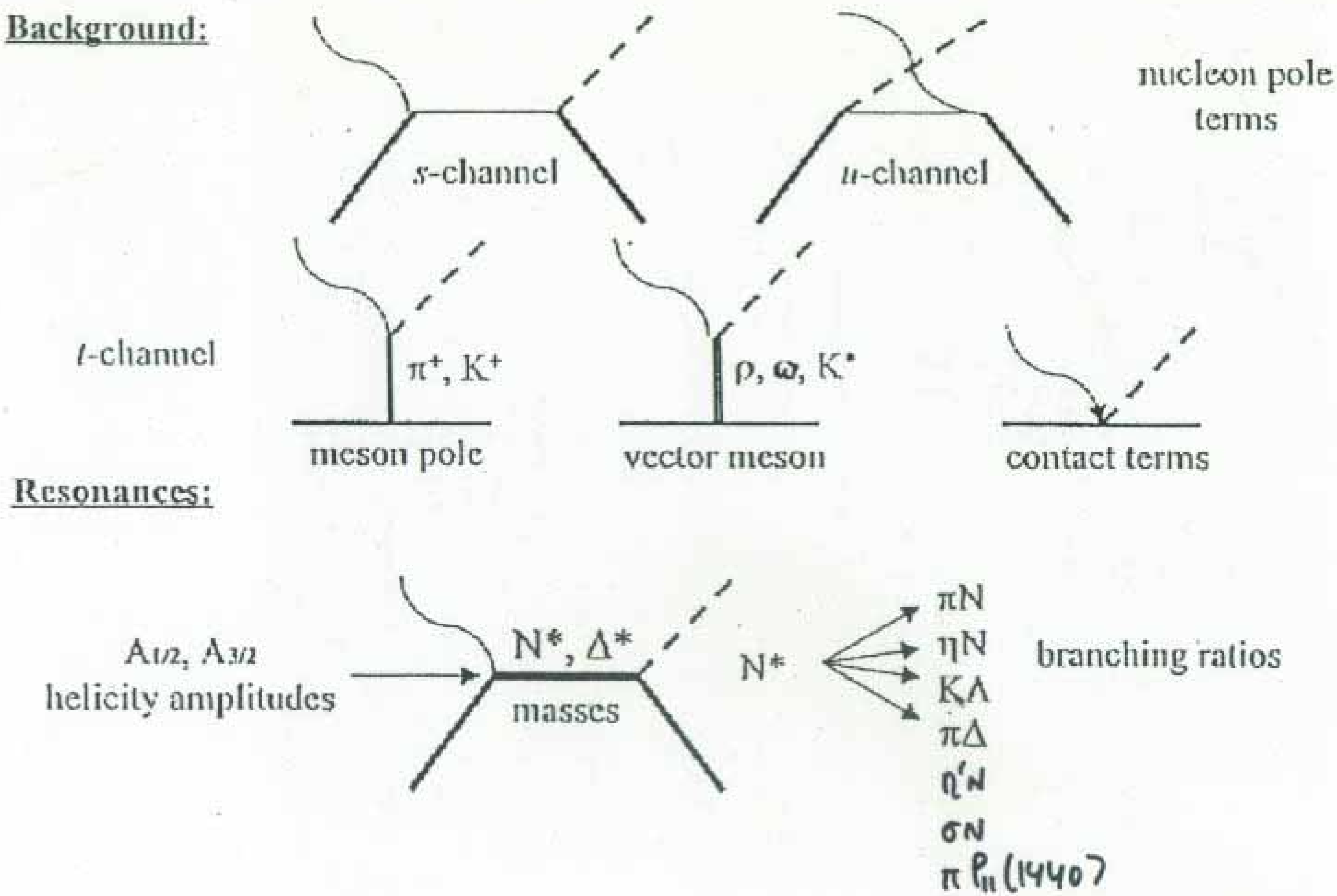
Gregor Penner → next talk in

"Vector meson production and nucleon resonance analysis in the Giessen coupled-channel model"

Scattering equation in the model



Driving terms



Papers that are important to the study of spin-$\frac{5}{2}$ nucleon resonance:

1. C. Fronsdal Nuovo Cimento **IX** 416 (1958),

2. A. Aurilia and H. Umezawa Phys. Rev. **182** 182, (1969),

3. P. Carruthers Phys. Rev. **152** 1345 (1966),

4. J. G. Rushbrooke Phys. Rev. **143** 1345 (1966),

5. S. Weinberg Phys. Rev. **133** B1318 (1964) and **134** B882 (1964),

6. D. M. Brudnoy Phys. Rev. Lett. **14** 273 (1965), Phys. Rev. **145** 1229 (1966), and Phys. Rev. **179** 1388 (1969),

7. P. R. Auvil and J. J. Brehm Phys. Rev. **145** 1243 (1966),

8. Y. Renard Nucl. Phys. **B40** 499 (1971).

SPIN-$\frac{5}{2}$ Nucleon Resonance.

In Rarita-Schwinger formalism a spin-$\frac{5}{2}$ function is a second-rank symmetric tensor-spinor, $u_{\alpha\beta}^{(m)}(p)$. The positive-energy projection operator is defined by

$$X_{\alpha\beta,\rho\sigma} \equiv \sum_{m=-5/2}^{5/2} u_{\alpha\beta}^{(m)}(p)\bar{u}_{\rho\sigma}^{(m)}(p). \tag{1}$$

The projection operator is uniquely determined by the four conditions

$$u_{\alpha\beta}^{(m)}(p) = u_{\beta\alpha}^{(m)}(p) \tag{2}$$

$$(\not{p} + m)X_{\alpha\beta,\rho\sigma}(p) = 0 \tag{3}$$

$$\gamma_\alpha X_{\alpha\beta,\rho\sigma}(p) = 0 \tag{4}$$

$$X_{\alpha\beta,\mu\nu}(p)X_{\mu\nu,\rho\sigma}(p) = X_{\alpha\beta,\rho\sigma}(p) \tag{5}$$

$$X_{\alpha\beta,\rho\sigma} \equiv$$
$$\sum_{\substack{\rho\leftrightarrow\sigma\\ \alpha\leftrightarrow\beta}} \Big[\frac{1}{10m^4} p_\alpha p_\beta p_\rho p_\sigma - \frac{1}{10m}(p_\alpha p_\beta p_\sigma \gamma_\rho - p_\rho p_\sigma p_\alpha \gamma_\beta) + \frac{1}{10m^2} p_\alpha \gamma_\beta p_\sigma \gamma_\rho$$
$$+\frac{1}{20m^2}(p_\alpha p_\beta g_{\sigma\rho} + p_\sigma p_\rho g_{\alpha\beta}) - \frac{2}{5m^2} p_\alpha p_\sigma g_{\beta\rho} - \frac{1}{10}\gamma_\alpha \gamma_\sigma g_{\beta\rho}$$
$$-\frac{1}{10m}(\gamma_\sigma p_\beta g_{\alpha\rho} - \gamma_\alpha p_\sigma g_{\beta\rho}) - \frac{1}{20} g_{\alpha\beta} g_{\sigma\rho} + \frac{1}{4} g_{\alpha\rho} g_{\beta\sigma}\Big]\Big(\frac{m - \not p}{2m}\Big)$$

where

$$\sum_{\substack{\rho\leftrightarrow\sigma\\ \alpha\leftrightarrow\beta}} T_{\alpha\beta\rho\sigma} \equiv T_{\alpha\beta\rho\sigma} + T_{\beta\alpha\rho\sigma} + T_{\alpha\beta\sigma\rho} + T_{\beta\alpha\sigma\rho} \tag{6}$$

The interaction lagrangian of $\frac{5}{2}^+ \to \frac{1}{2}^+ + 0^-$ is

$$L = \frac{g_{5/2-1/2-0}}{m_\pi^2} \bar{\psi}(x)\gamma_5 \psi_{\alpha\beta}(x)\partial_\beta \partial_\alpha \varphi(x), \tag{7}$$

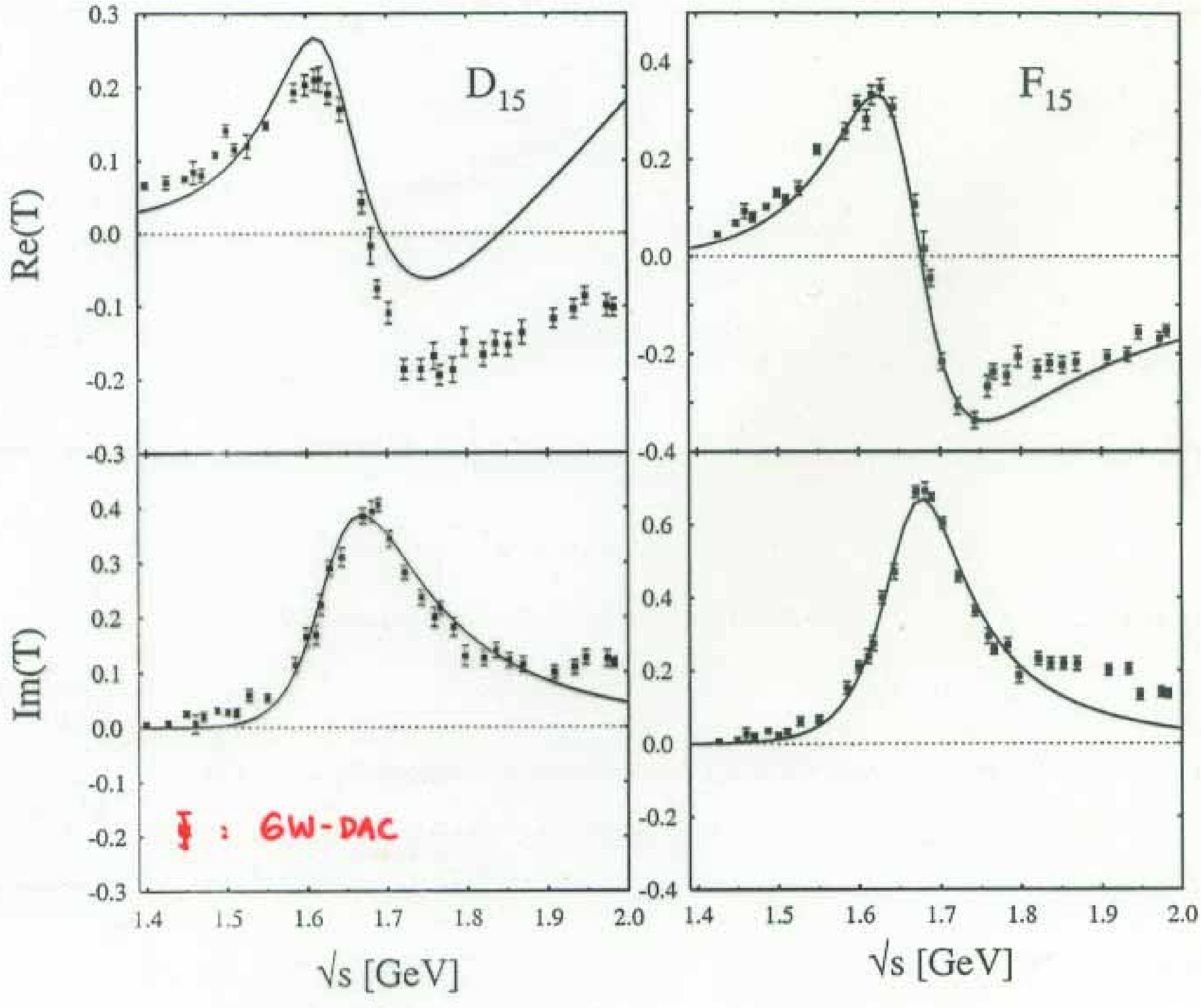
m_π the mass of the pion, $g_{5/2-1/2-0}$ is the coupling constant.

Using the spin-5/2 propagator, this coupling gives for the decay width

$$\Gamma_{\frac{5}{2}^+ \to \frac{1}{2}^+ + 0^-} = 2\frac{E_{1/2} - m_{1/2}}{15 m_{5/2}}\Big(\frac{|g_{5/2\ \ 1/2\ \ 0}|^2}{4\pi}\Big)\Big(\frac{p}{m_\pi}\Big)^5 m_\pi. \tag{8}$$

$E_{1/2}$ is the energy, $m_{1/2}$ the mass of $\frac{1}{2}^+$, $m_{5/2}$ the mass of $\frac{5}{2}^+$, and p the three-momentum in the CM system.

T-matrix result : $\pi N \longrightarrow \pi N$



D_{15} (1675)

Mass : 1.657 GeV

Γ : 134 MeV

PDG

1.675 GeV (1.670 - 1.685 GeV)

150 MeV (140 - 180 MeV)

F_{15} (1680)

Mass : 1.676 GeV

Γ : 130 MeV

1.680 GeV (1.675 - 1.690 GeV)

130 MeV (120 - 140 MeV)

Resut in

$\pi N \longrightarrow \pi \Delta$ channel.

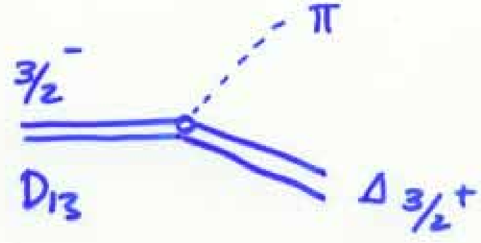
together with :

S. Karppi & K. Foe.

Resonances in $\pi N \longrightarrow \pi\Delta$

for example :

$D_{13} \rightarrow \pi\Delta$



2 possible transitions

$\ell = 0, 2$

- DS 13
- DD 13

Relativistic Vertex

$$\frac{g_{N^*\Delta\pi}}{m_\pi}\,\bar{\Psi}_{N^*\nu}\,\vec{T}\,\gamma^\mu\Delta^\nu\,\partial_\mu\phi + \text{H.C.}$$

(Krehl. et. al. Phys. Rev C62 025207 (2000)) non. rel $\Downarrow$ reduction

$$-\frac{g_{N^*\Delta\pi}}{m_\pi}\,\chi^\dagger_{3/2,\Delta}\left(1+\frac{(\bar{S}^\dagger\cdot\bar{p})(\bar{S}\cdot p)}{m_{N^*}(E_{N^*}+m_{N^*})}\right)\chi_{3/2,N^*}$$

→ relativistic vertex leads to both $\ell=0$, $\ell=2$ transitions with one coupling constant!

In Contrast :

$$-\left(\tilde{f}_{N^*\Delta\pi}+\frac{\tilde{g}_{N^*\Delta\pi}}{\mu^2}(\bar{S}^\dagger\cdot\bar{p})(\bar{S}\cdot\bar{p})\right)$$

(Oset et.al. N.P.A 1996)

μ = mass of pion

$\bar{p}$ = pion momentum in the N^* rest frame

$\bar{S}$: the $\frac{1}{2}$ to $\frac{3}{2}$ spin transition operators.

→ needs 2 independent coupling constants!

Form of explicit T-matrix, from basic principles
("Dirac-space parameterization")

- Helps develop explicit T-matrix, is easier to work with
- Developed as follows:
  - Begin as general parameterization of 5 bilinear covariants (scalar, vector, tensor, axial vector, & pseudoscalar)
  - Use Dirac eqn & conservation of four-momentum to eliminate redundant & non-physical terms

$\pi N \rightarrow \pi N: \quad T = A + B(\not{q} + \not{q}')$

$\pi N \rightarrow \pi \Delta: \quad T_\mu = A_1 q_\mu + A_2 q_\mu' + (B_1 q_\mu + B_2 q_\mu')(\not{q} + \not{q}')$

T_μ must be Lorentz four-vector because the matrix element, $w^\mu(p') \, T_\mu \, u(p)$, must be a Lorentz scalar, where $w^\mu(p')$ is a Rarita-Schwinger spinor.

$$\pi N \rightarrow \pi N$$
$$q, p \rightarrow q', p'$$

Proposed formalism for partial-wave amplitudes

- Helicity T-matrix elements, as both:
  - Sandwiching of explicit T-matrix
  - Sum of partial-wave amplitudes of definite parity
- Developed in 3 steps:
  - (1) Sandwich explicit T-matrix: $<\hat{p}',\lambda'|\, T\, |\hat{p},\lambda>$
  - (2) Expand $|\hat{p},\lambda>$ as $|j,m_j,\lambda>$ and use conservation of j:

$$<\hat{p}',\lambda'|\, T\, |\hat{p},\lambda> = \sum_j <j, m_j, \lambda'|\, T\, |j,m_j,\lambda>\, d^{\,j}_{\lambda,\lambda'}(\theta)$$

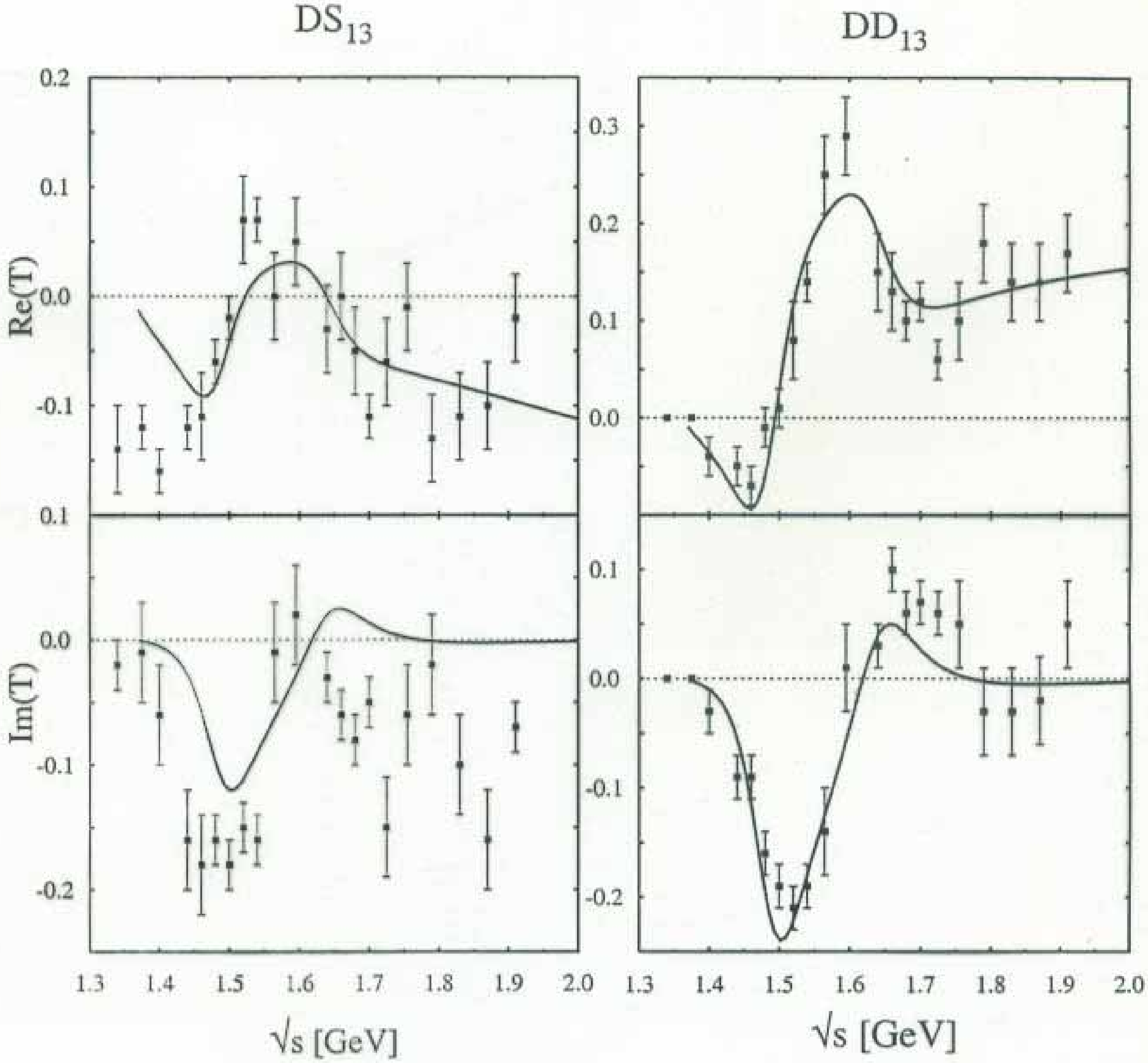
  - (3) Convert right-hand side to LS basis (definite parity). For $\pi N \rightarrow \pi N$:

$$<\hat{p}',\pm 1/2|\, T\, |\hat{p},1/2> = \sum_j \left\{ <j\text{-}1/2|T^j|j\text{-}1/2> \pm <j+1/2|T^j|j+1/2> \right\} d^{\,j}_{1/2,\pm 1/2}(\theta)$$

Partial wave, $L_i = L_f = j-1/2$

Partial wave, $L_i = L_f = j+1/2$

T-matrix result in $\pi N \to \pi \Delta$



D_{13} (1520)

Mass : 1.509 GeV

$\Gamma_{\pi\Delta}$: 25 MeV

PDG :

1.515 – 1.530 GeV

15 – 16 MeV

D_{13} (1700)

Mass : 1.660 GeV

1.650 – 1.750 GeV

Conclusions

- Spin-$\frac{5}{2}$ N^* difficult in effective Lagrangian framework!
  - all partial waves must be fitted simultaneusly.

- $\pi\Delta$ is important part of $\pi\pi N$ channel!
  - relativistic $D_{13} \rightarrow \pi\Delta$ vertex gives both $l = 0, 2$ couplings.
  - needed: $\gamma N \rightarrow \pi\Delta$ partial wave analysis.