EXTRA S_{11} AND P_{13} IN THE HYPERCENTRAL CONSTITUENT QUARK MODEL (hCQM)

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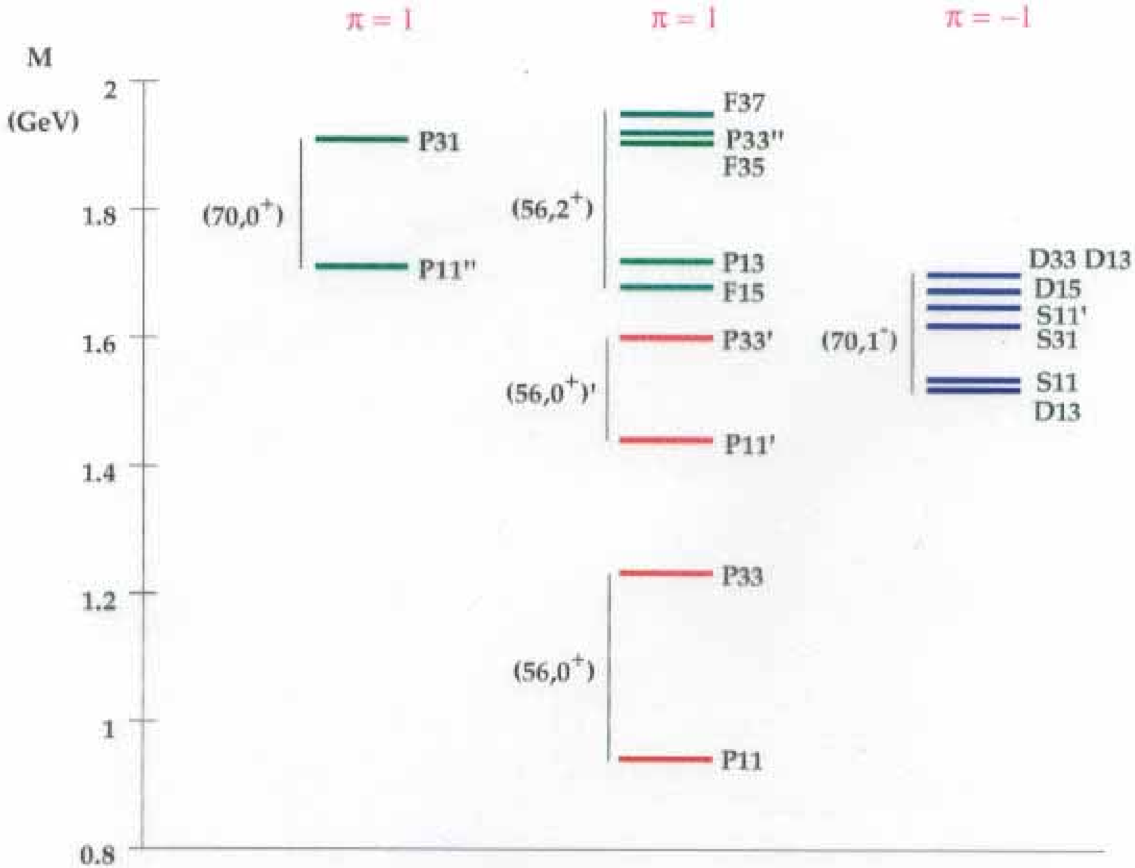
NSTAR 2002

PITTSBURGH 11 Oct. 2002

4 * PDG
3 *


π = 1
π = 1
π = −1
M (GeV)
2
1.8
1.6
1.4
1.2
1
0.8
(70,0+)
P31
P11''
(56,2+)
F37
P33''
F35
P13
F15
(56,0+)'
P33'
P11'
(56,0+)
P33
P11
(70,1-)
D33 D13
D15
S11'
S31
S11
D13

3q Wave function

$$\Psi_{3q} = \Theta_{colour} \times \chi_{spin} \times \Phi_{iso} \times \psi_{space}$$

$$SU(3)_c \quad \underbrace{SU(2) \quad SU(3)_f}_{\downarrow} \quad O(3)$$

spin-flavour independent quark interaction

$$SU(6)$$

Θ colour singlet $\rightarrow$ antisymm.

$SU(6)$
$O(3)$ wf: same symmetry

for 3 objects

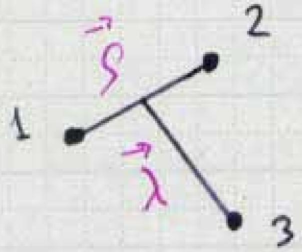
A
MA
MS
S

χ $\quad S = \frac{1}{2}, \frac{3}{2}$

Φ $\quad T = \frac{1}{2}, \frac{3}{2}$

ψ space

Jacobi coordinates



$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2)$$

$$\vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$$

- six degrees of freedom

$$\rho, \Omega_\rho, \lambda, \Omega_\lambda$$

- new coordinates $\rightarrow$ hyperspherical

$x = \sqrt{\rho^2 + \lambda^2}$ hyperradius ("size")

$t = \operatorname{arctg}(\rho/\lambda)$ hyperangle ("form")

$+ \; \Omega_\rho, \Omega_\lambda$

- $$H_{3q} = 3m - \frac{1}{2m}\underbrace{(\Delta_\rho + \Delta_\lambda)}_{\frac{\partial^2}{\partial x^2} + \frac{5}{x}\frac{\partial}{\partial x} - \frac{L^2(\Omega_\rho, \Omega_\lambda, t)}{x^2}} + V(\vec{\rho}, \vec{\lambda})$$

$\rightarrow C_2(O(6))$

- L^2 eigenfunctions $Y_{[\gamma, \ell_\rho, \ell_\lambda]}(\Omega_\rho, \Omega_\lambda, t)$

$$\propto Y_{\ell_\rho m_\rho} Y_{\ell_\lambda m_\lambda} P_{(\gamma \ell_\rho \ell_\lambda)}(t)$$

Jacobi polin.

eigenvalues $\gamma \cdot (\gamma + 4)$ $\quad \gamma = 2n + \ell_\rho + \ell_\lambda$

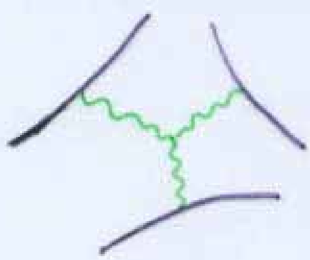
- wf basis $\psi(x) \cdot Y_{[\gamma, \ell_\rho, \ell_\lambda]}(\Omega_\rho \Omega_\lambda t)$

Hypercentral hypothesis

$V(x)$

3-body potential

- motivations:
  - QCD fundamental mechanisms



  - flux-tube model

  - $\sum_{i<j} V(\vec{r}_{ij}) \cong V_0(x) + \dots$

hypercentral approximation

$O(6)$-invariance

" hypercentral potentials "

$$V = V(x)$$

$\longrightarrow$ factorization

$$\Psi(x, t, \Omega_\rho, \Omega_\lambda) = \Psi(x) \quad Y_{[\gamma, l_\rho, l_\lambda]}$$

("dynamics") ("geometry")

"hypercentral equation"

$$-\frac{1}{2m}\left[\frac{d^2\psi}{dx^2} + \frac{5}{x}\frac{d\psi}{dx} - \frac{\gamma(\gamma+4)}{x^2}\psi\right] + V(x)\psi = E\psi$$

$$\psi = \psi_{\nu\gamma}(x)$$

γ $\longrightarrow$ grand angular quantum number

ν $\downarrow$ hyperradial quantum number (hyperradial excitation)

$V(x)$: three-body force

two-body potential in hypercentral approximation

$$\left\langle \sum_{i<j} (r_{ij})^n \right\rangle_{av.} \propto x^n$$

P_{11} P_{11} P_{11}

$$\begin{pmatrix} S_{11} & D_{13} & \\ S_{11} & D_{13} & D_{15} \end{pmatrix} + \Delta$$

(N, Δ)

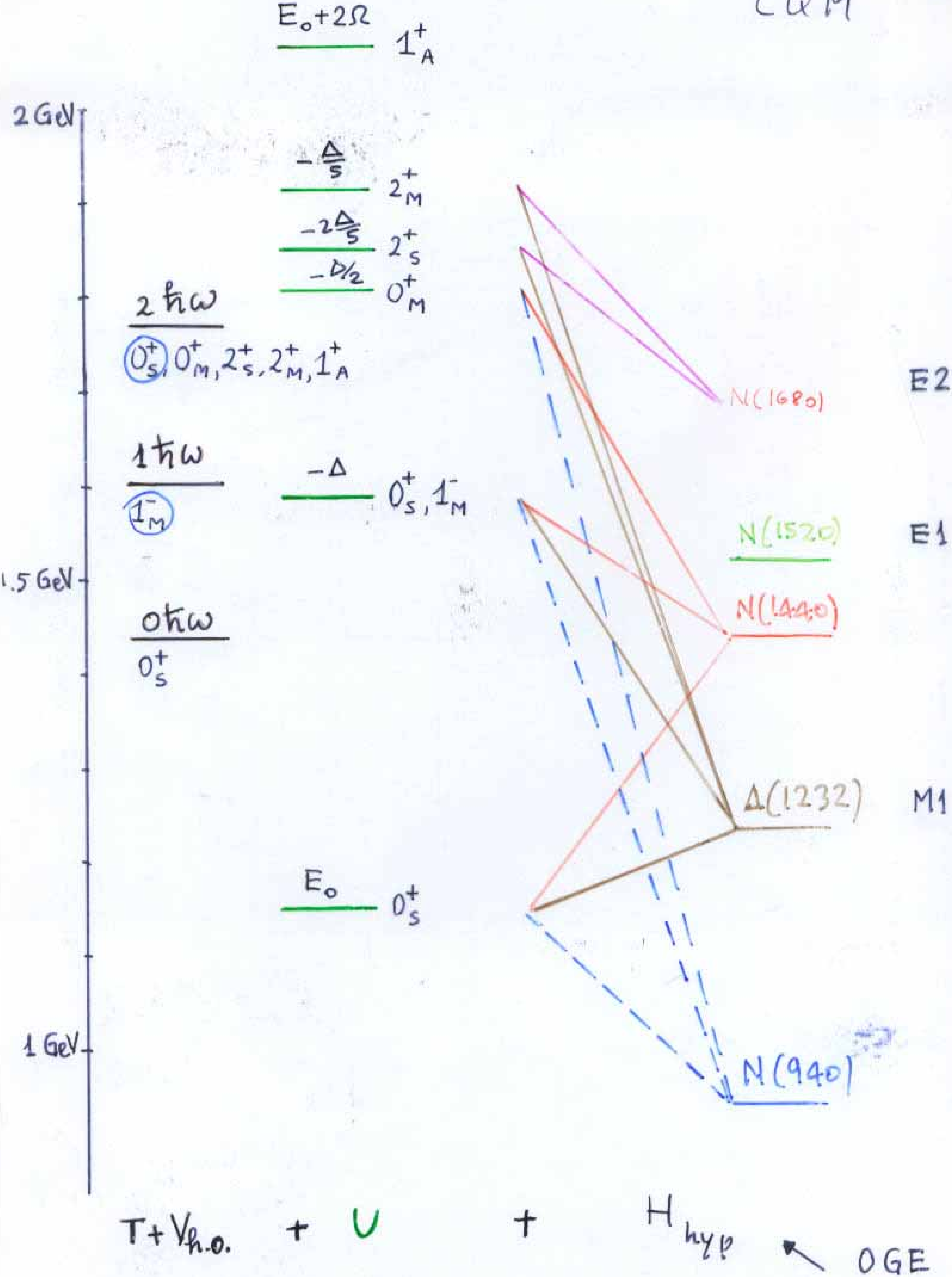
H. O.

$\omega=2$ — 2^+_S — 2^+_M — 1^+_A — 0^+_M — 0^+_S

$\omega=1$ — 1^-_M

$\omega=0$ — 0^+_S

CQM


$E_0+2\Omega$ 1^+_A
2 GeV
$-\frac{\Delta}{5}$ 2^+_M
$-2\frac{\Delta}{5}$ 2^+_S
$-\Delta/2$ 0^+_M
$2\hbar\omega$
$0^+_S, 0^+_M, 2^+_S, 2^+_M, 1^+_A$
N(1680)
E2
$1\hbar\omega$
1^-_M
$-\Delta$ $0^+_S, 1^-_M$
N(1520)
E1
1.5 GeV
N(1440)
$0\hbar\omega$
0^+_S
$\Delta(1232)$
M1
E_0 0^+_S
1 GeV
N(940)
$T+V_{h.o.}$ + U + H_{hyp}
OGE

two analytical solutions

A) harmonic oscillator

$$\sum_{i<j} \frac{1}{2} k (\vec{r}_i - \vec{r}_j)^2 = \frac{3}{2} k x^2$$

$$E = (3 + \underbrace{N}_{2\nu+\gamma}) \hbar\omega$$

B) hypercoulomb

$$V_{hyc}(x) = -\frac{\tau}{x}$$

$$E = -\frac{m\tau^2}{2\underbrace{N}_{\frac{5}{2}+\gamma+\nu}{}^2}$$

$-\frac{z}{x}$ "hypercoulomb"

		P_{11}		P_{11}		P_{11}
$\omega = 2$	2^+_S	2^+_M	1^+_A	0^+_M	1^-_M	0^+_S
$\omega = 1$	0^+_S	1^-_M	$\sim \begin{pmatrix} S_{11} & D_{13} & D_{15} + \Delta \\ S_{11} & D_{13} & \end{pmatrix}$			
	P_{11}					
$\omega = 0$	0^+_S	(N, Δ)				

(M.M. Giannini, F. Iachello, E. Santopinto)

- O(7) symmetry

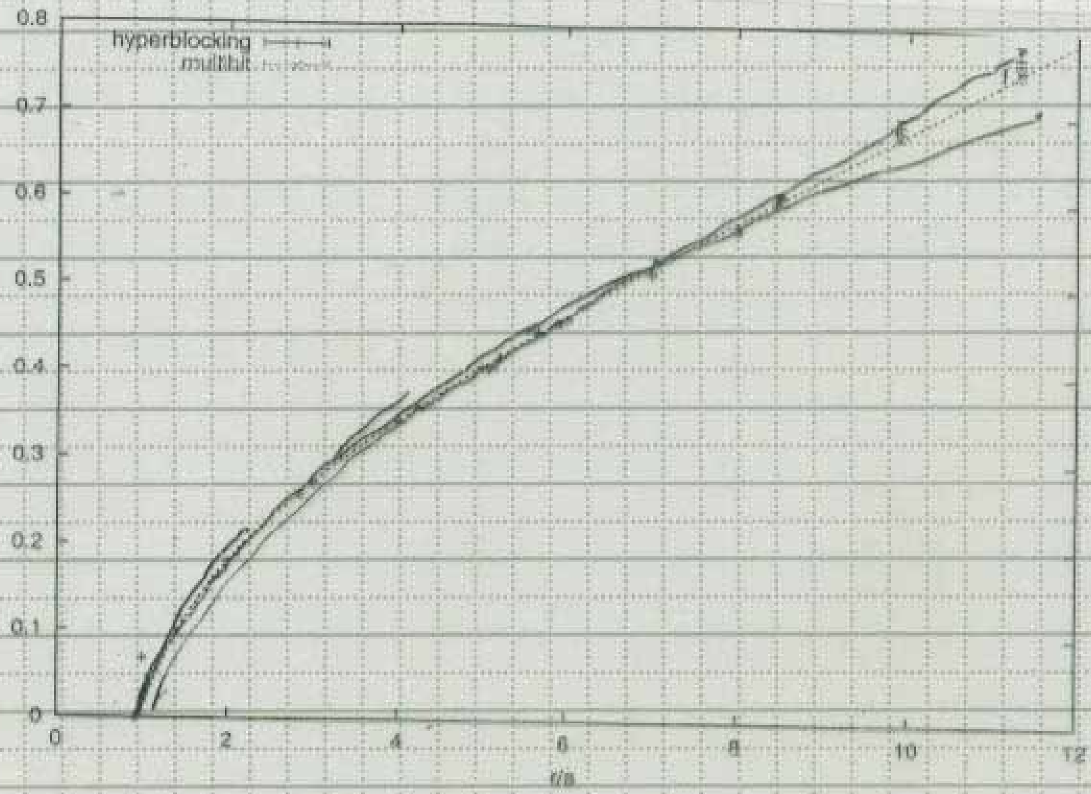
S_3 :

S (symmetric)

M (mixed symmetric)

A (antisymmetric)

LATTICE QCD $q\bar{q}$ potential



$$V_{q\bar{q}}(r) = -\frac{A_{q\bar{q}}}{r} + \sigma_{q\bar{q}}\, r + C_{q\bar{q}}$$

(Cornell pot.)

G. Bali et al.
Phys. Rev. D51, 5165 (1995)

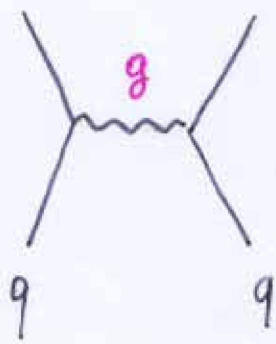
HYPERCENTRAL MODEL

$$V_{3q} = V(x) + H_{hyp}$$

- $V(x) = -\frac{\tau}{x} + \alpha x$ $\qquad x = \sqrt{\rho^2 + \lambda^2}$

- $H_{hyp} = \sum_{i<j} V^s(\vec{r}_i, \vec{r}_j)\ \vec{\sigma}_i \cdot \vec{\sigma}_j + \text{tensor}$

$$\delta(\vec{r}_i - \vec{r}_j)$$



$\rightarrow \langle r_p^2 \rangle^{1/2} \sim 0.48\ \text{fm}$

$\underline{3}$ parameters

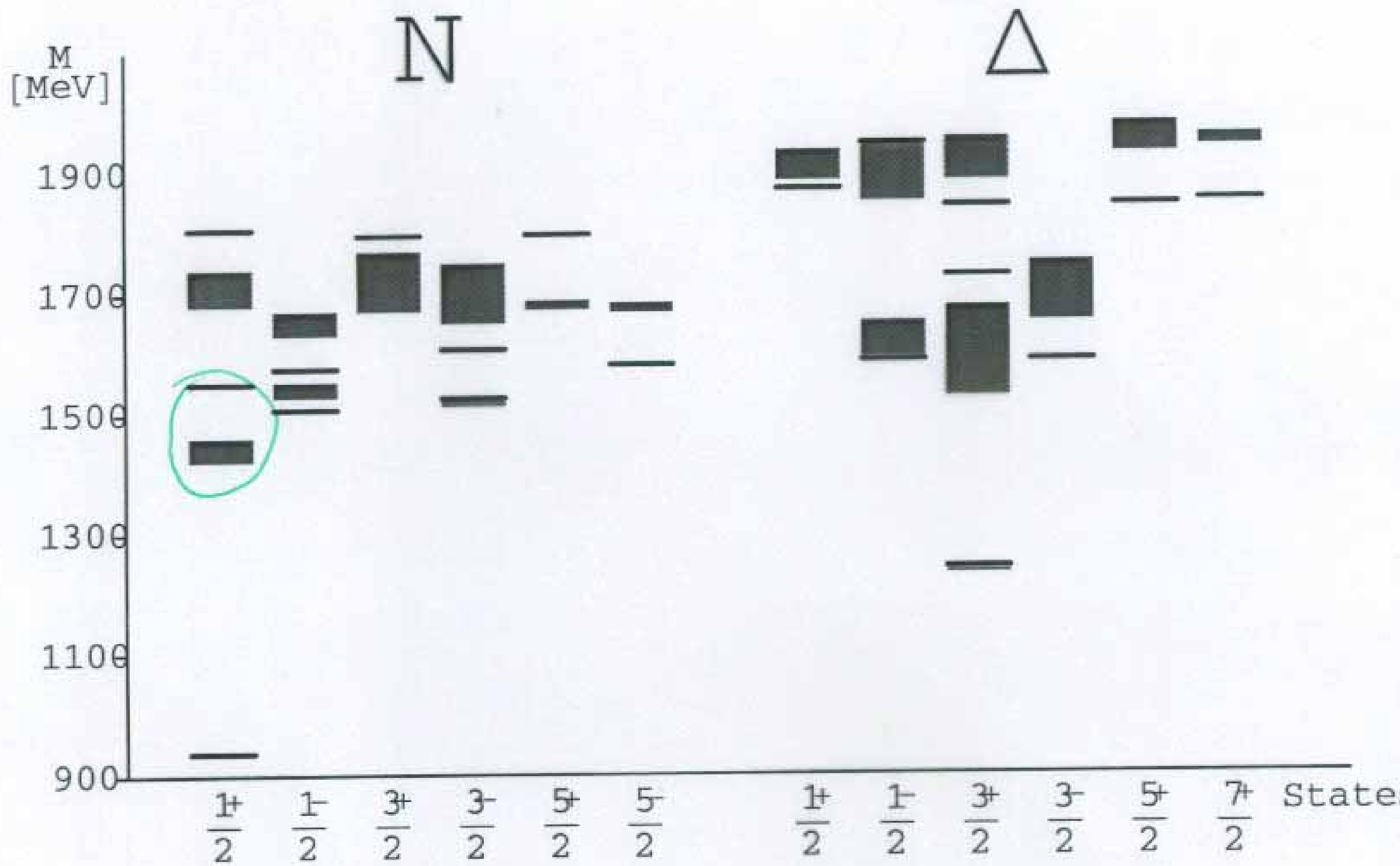
τ

α $\qquad \leftarrow$ fixed to the spectrum

strength of H_{hyp}

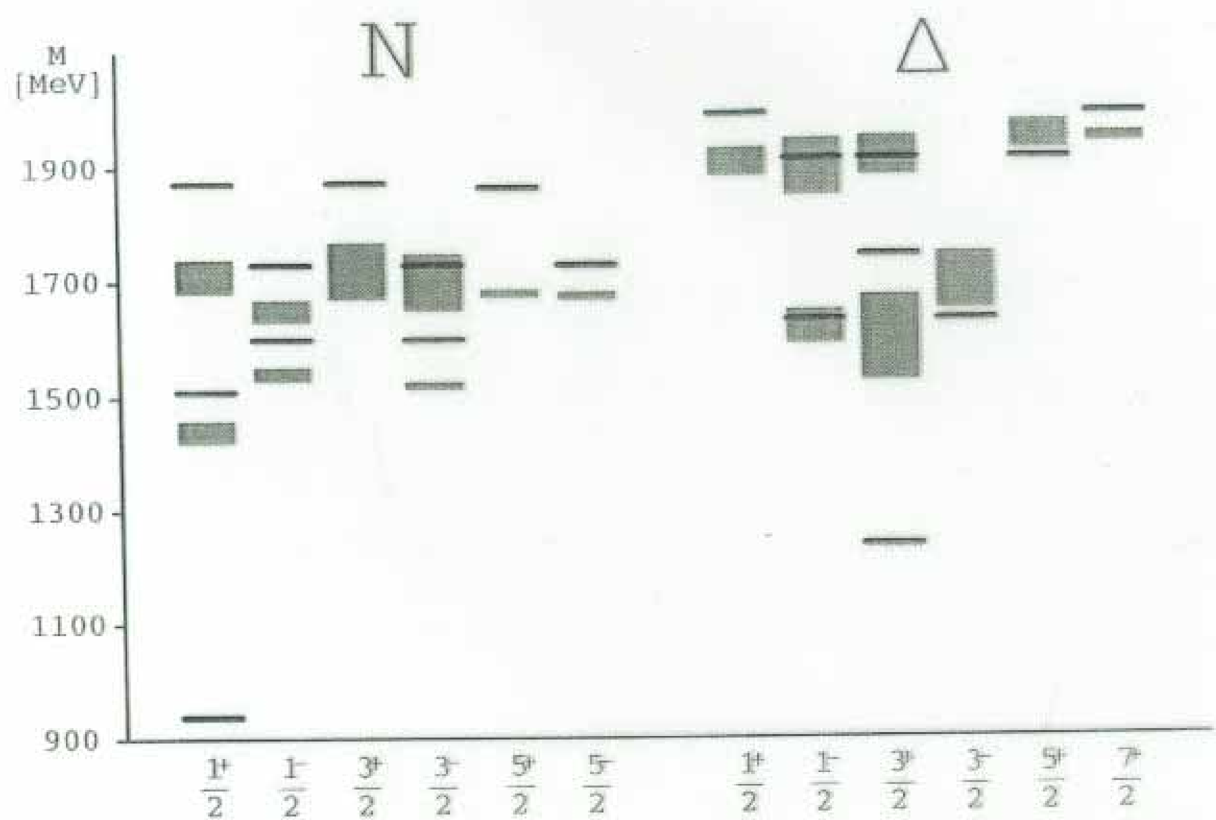
$\left[m = \frac{M}{3} \right]$

Genoa Group



M. Ferraris et al., Phys. Lett. B 364 (1995) 231 hCQM with $T = \sum_{i=1}^{3} \sqrt{m^2 + p_i^2}$

(H_{hyp})



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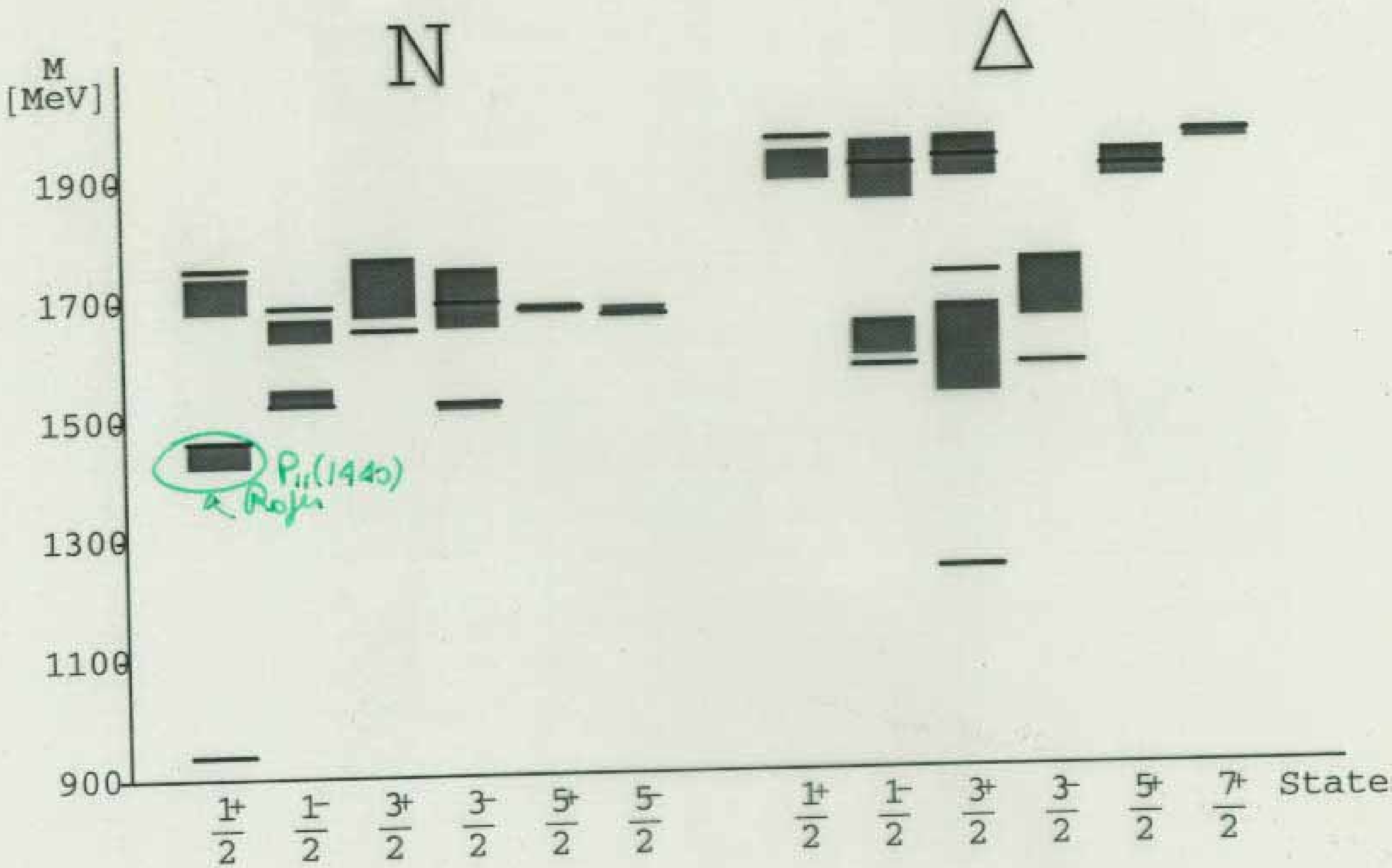
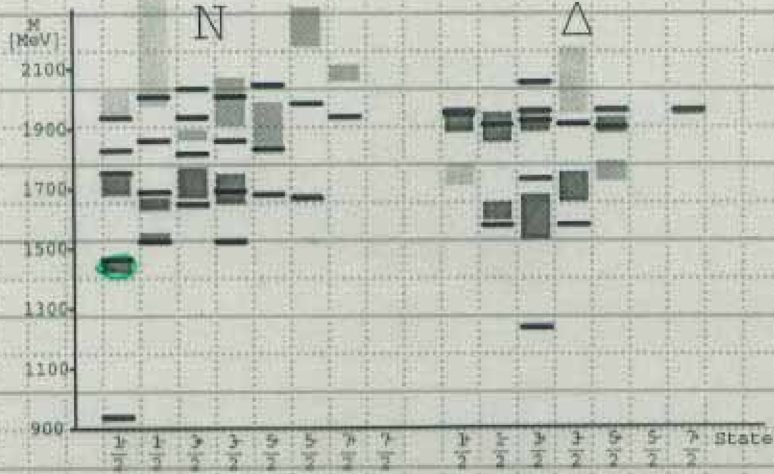
Isospin dependence

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_S + \mathcal{H}_I + \mathcal{H}_{SI} + V_T$$

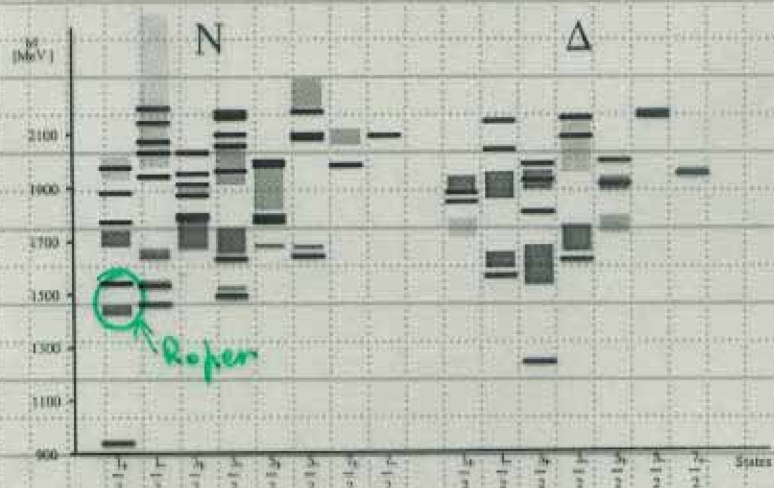
- $\mathcal{H}_0 = T - \frac{\tau}{x} + \alpha x$
- $\mathcal{H}_S = A_S \ \sum_{i<j} f_S(\mathbf{r}_{ij})(\mathbf{S}_i \cdot \mathbf{S}_j)$
- $\mathcal{H}_I = A_I \ \sum_{i<j} f_I(\mathbf{r}_{ij})(\mathbf{I}_i \cdot \mathbf{I}_j)$
- $\mathcal{H}_{SI} = A_{SI} \ \sum_{i<j} f_{SI}(\mathbf{r}_{ij})(\mathbf{S}_i \cdot \mathbf{S}_j)(\mathbf{I}_i \cdot \mathbf{I}_j)$
  - $f_k(\mathbf{r}_{ij}) = \frac{1}{(\sqrt{\pi}\sigma_k)^3} e^{-\frac{r_{ij}^2}{\sigma_k^2}} \quad k = S, I, SI$
- $V_T = \sum_{i<j} \frac{2\alpha_s}{3m^2} \left\{ \frac{1}{r_{ij}^3} \left[3\frac{(\mathbf{S}_i \cdot \mathbf{r}_{ij})(\mathbf{S}_j \cdot \mathbf{r}_{ij})}{r_{ij}^2} - \mathbf{S}_i \cdot \mathbf{S}_j \right] \right\}$



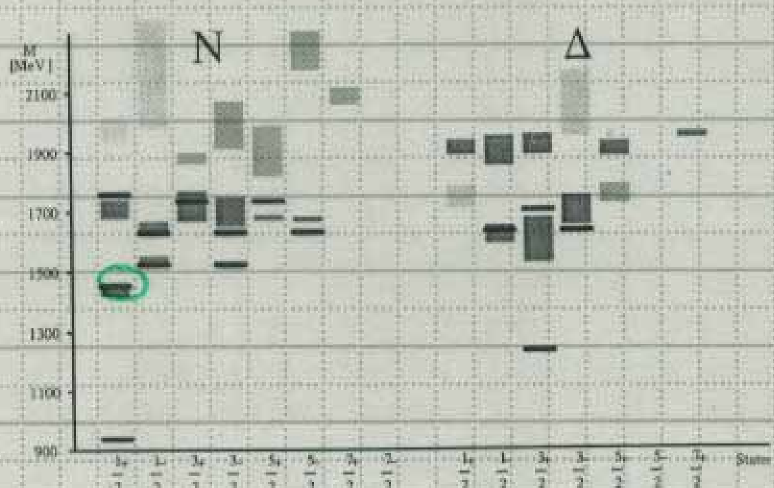
M.G., E. Santopinto, A. Vassallo, EPJ <u>A12</u>, 447 (2001)



HCQM



Capstick-Isgur



Plessas et al.

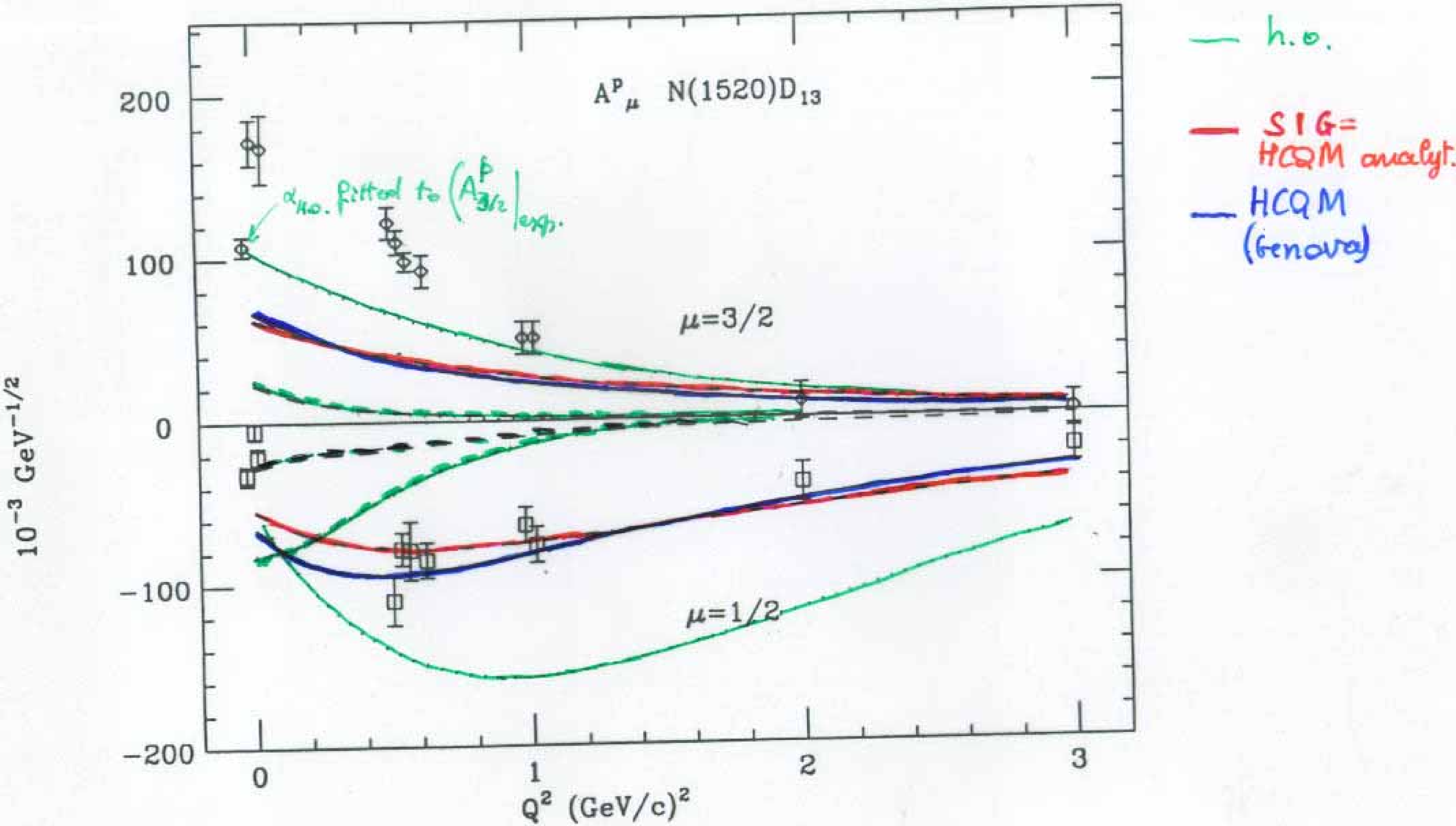
$M(\Delta) - M(N)$:

$H_{hyp} \longrightarrow 35\%$

$H_{I} \longrightarrow 15\%$

$H_{SI} \longrightarrow 50\%$

	PDG	hCQM	hCQM + Iso
S_{11}	1535	1507	1524
	1650	1574	1688
		1887	1861
	(2090)	1937	2008
D_{13}	1520	1526	1524
	1700	1606	1692
		1899	1860
	(2080)	1969	2008
P_{13}	1720	1792	1648
	1900	1835	1816
		1853	1894
		1867	1939
			2034
P_{33}	1232	1240	1232
	1600	1727	1727
		1843	1921
	1920	1856	1955
		2104	2049



Aiello et al. J. Phys. G, 24, 753 (1998)

Helicity Amplitudes
(Photocouplings)

$$\gamma + N \longrightarrow N^*,\ \Delta$$

$$A_{\frac{1}{2}} = \langle B, J', J'_z = \tfrac{1}{2} |\, H^t_{em} \,| N, J = \tfrac{1}{2}, J_z = -\tfrac{1}{2} \rangle$$

$$A_{\frac{3}{2}} = \langle B, J', J'_z = \tfrac{3}{2} |\, H^t_{em} \,| N, J = \tfrac{1}{2}, J_z = +\tfrac{1}{2} \rangle$$

$$H^t_{em} = -\sum_{i=1}^{3} \left[\frac{e_i}{2m_i} \left(\vec{p}_i \cdot \vec{A}_i + \vec{A}_i \cdot \vec{p}_i \right) + 2\mu_i \vec{s}_i \cdot \left(\vec{\nabla} \wedge \vec{A}_i \right) \right]$$

e_i quark charge

m_i " mass

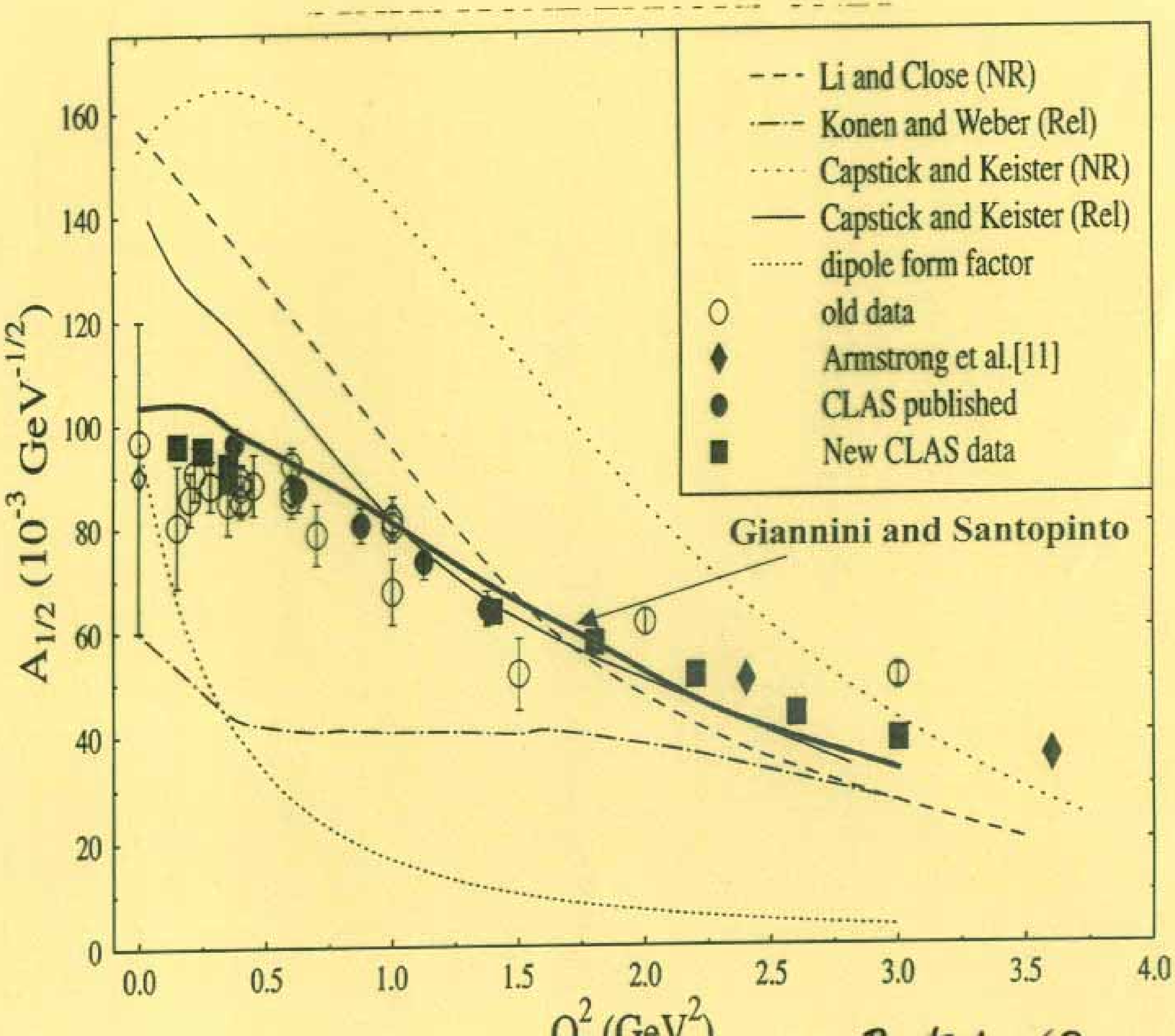
μ_i " magnetic moment

s_i " spin

p_i " momentum

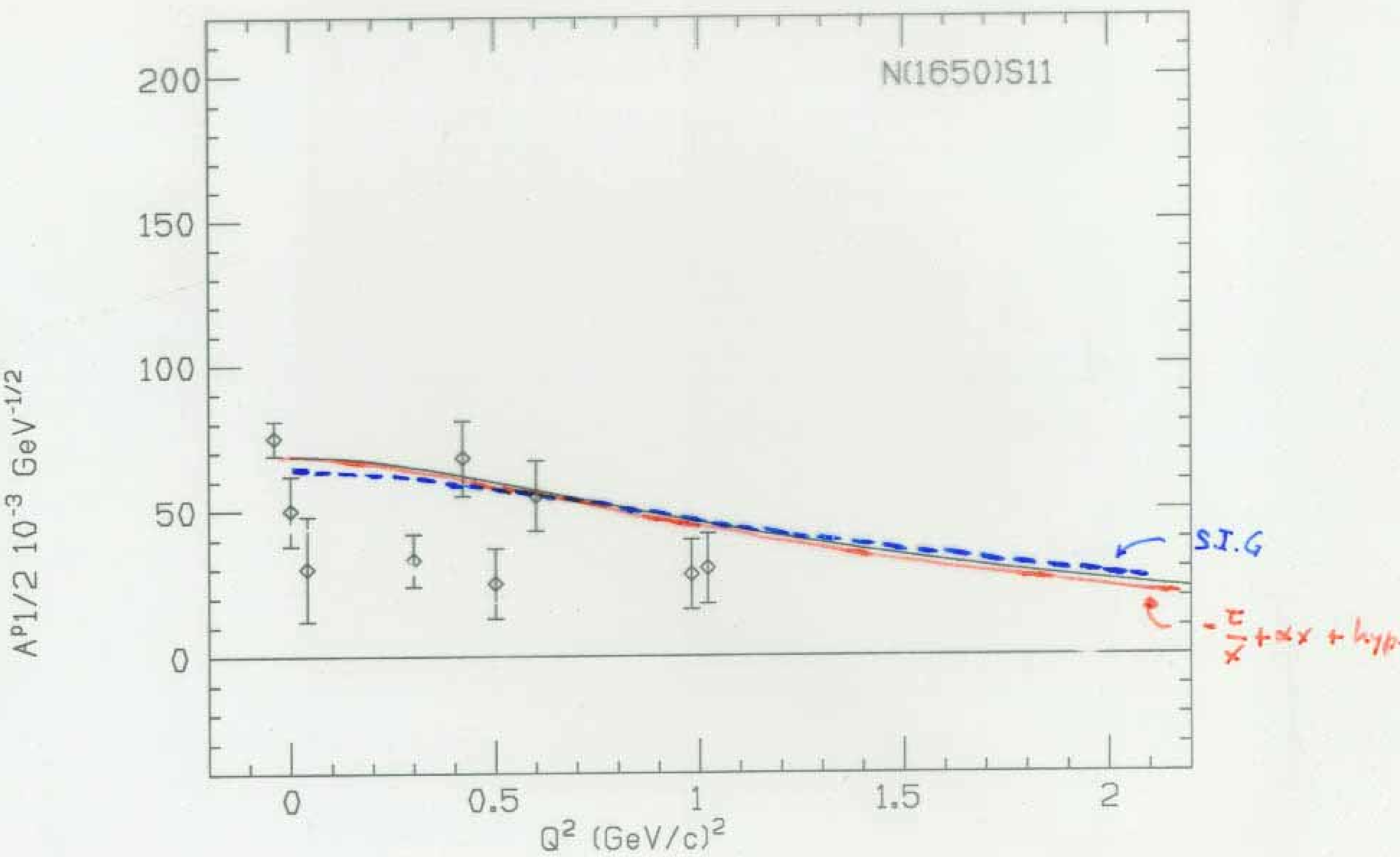
$\vec{A}_i = \vec{A}(\vec{r}_i)$ photon field

Photocoupling amplitude $A_{1/2}$



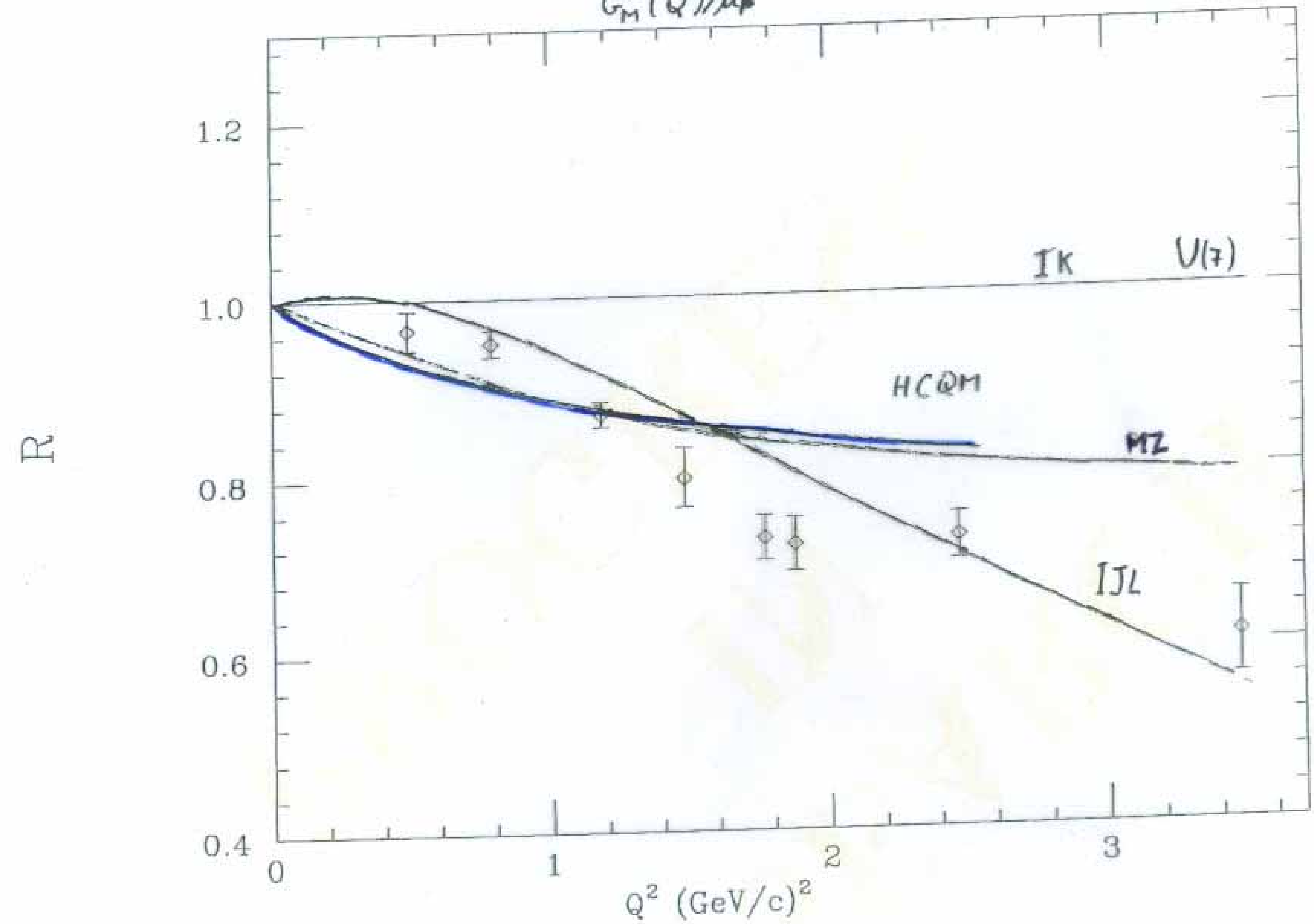
J. Phys. G: 24, 753 (1998)

Burkert (Baryons'02)


N(1650)S11
$A^p 1/2\ 10^{-3}\ GeV^{-1/2}$
200
150
100
50
0
0
0.5
1
1.5
2
$Q^2\ (GeV/c)^2$
S.I.G
$-\frac{\tau}{x} + \alpha x$ + hyp.

$$R = \frac{G_E^p(Q^2)}{G_M^p(Q^2)/\mu_p}$$

PRC 62, 025208 (2000)



T_{NR} , boosts, current expansion

Genova, Sept. 2002