Excited Baryons from Bayesian Priors and Overlap Fermions

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- Physics Motivation
- Brief Review
- Simulation Details
- Fitting Procedure
- Results
- Conclusion

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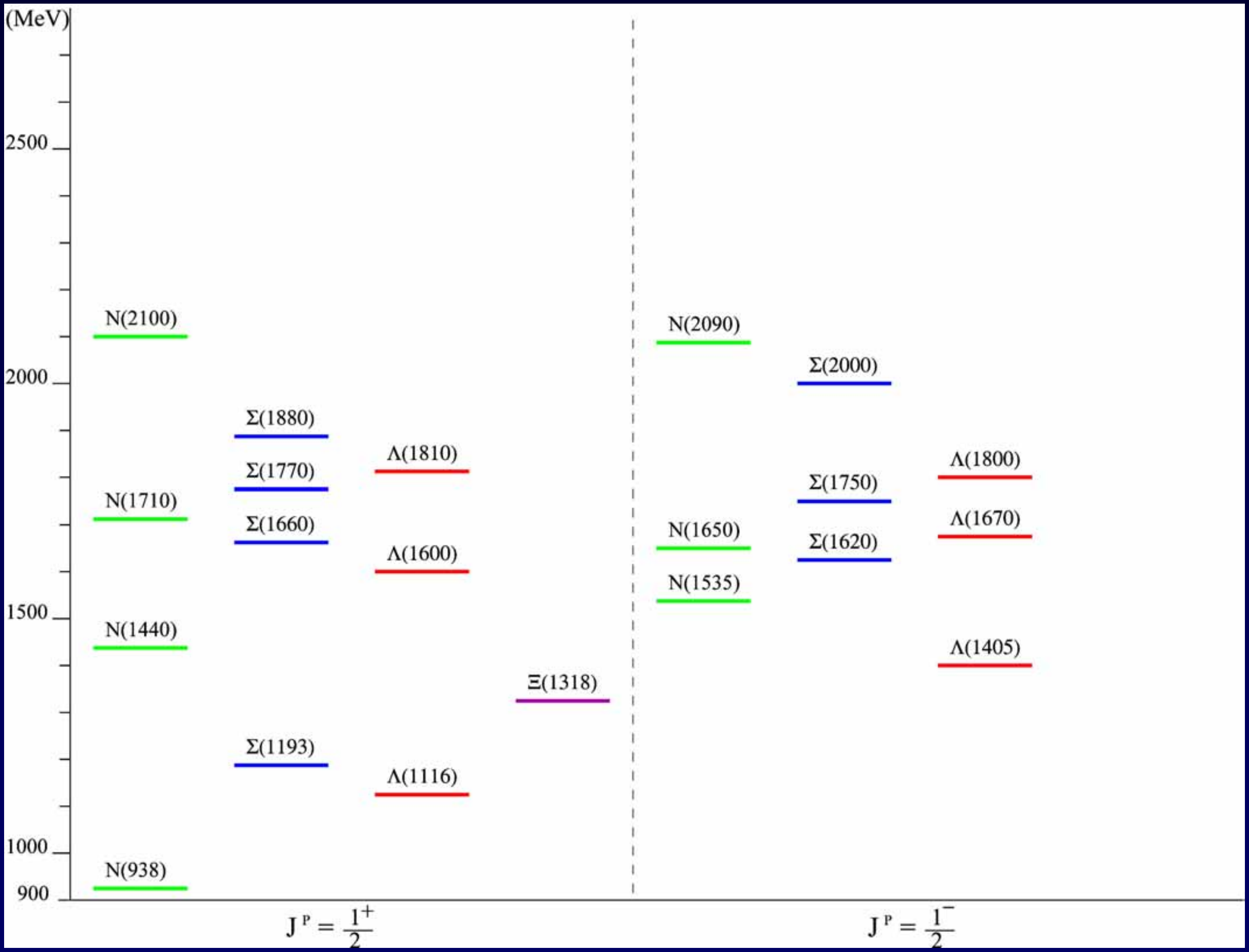
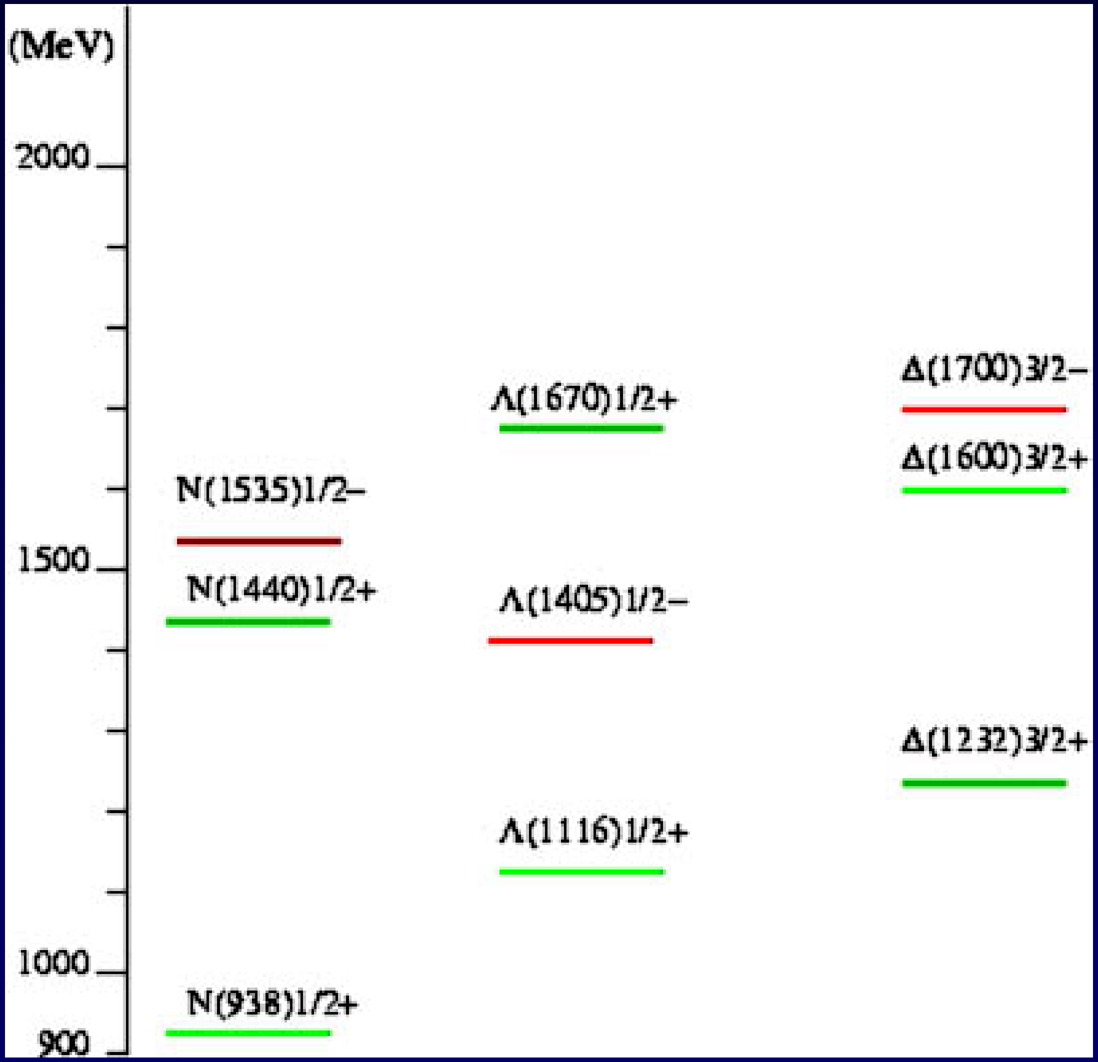
Observed baryon spectrum: spin 1/2



Chiral symmetry breaking, no parity doubling

Ordering of low-lying excited states



Hyperfine Interaction of Quarks in Baryons

versus

- Color-spin

$$\lambda_1^C \cdot \lambda_2^C \, \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

- One-gluon exchange
- Isgur and Karl, PRD18, 4187 (1978)
- Capstick and Isgur, PRD34, 2809 (1986)

- Flavor-spin

$$\lambda_1^F \cdot \lambda_2^F \, \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

- Goldstone boson exchange
- Glozman and Riska, Phys. Rep. 268, 263 (1996)
- K.F. Liu et al, PRD59, 112001 (1999)

Lattice studies of N* spectrum

- Wilson-OPE
  - Leinweber, PRD51, 6383 (1995)
- Tadpole-improved, anisotropic actions (Dχ34,D234)
  - Lee, Leinweber, heplat/9809095, heplat/0011060, heplat/0110164
- Doman-wall fermion
  - Sasaki, Blum, Ohta, heplat/0004252, heplat/0011010, heplat/0102010
- NP-improved clover
  - Richards, heplat/0011025, Gockeler et al, heplat/0106022
- Fat-link clover
  - Adelaide group, heplat/0202022
- Overlap fermions
  - F.X. Lee for the Kentucky collaboration, heplat/0208070

Baryon Interpolating Fields

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+):$

$$\chi_1 = \varepsilon_{abc}(u^{aT} C\gamma_5 d^b)u^c$$

$$\chi_2 = \varepsilon_{abc}(u^{aT} C d^b)\gamma_5 u^c$$

Negative parity (multiply by γ_5): $\chi_1^- = \gamma_5\chi_1 , \quad \chi_2^- = \gamma_5\chi_2$

Non-relativistic limit:

$\chi_1 \rightarrow (\text{big - big - big}) \rightarrow O(1)$ (couples to nucleon)

$\chi_2 \rightarrow (\text{big - small - small}) \rightarrow O(p^2 / E^2)$ (couples to ?)

$\chi_1^- \rightarrow (\text{big - big - small}) \rightarrow O(p / E)$ (couples to $\frac{1}{2}^-$ state)

$\chi_2^- \rightarrow (\text{big - small - big}) \rightarrow O(p / E)$ (couples to $\frac{1}{2}^-$ state)

In the spectrum : $\mathrm{N}^*(1535)\,\frac{1}{2}^-$ and $\mathrm{N}^*(1620)\,\frac{1}{2}^-$.

Quark mass coverage

No.	$m_0 a$	$m_\pi a$	$m_\rho a$	m_π / m_ρ	$m_\pi L$
1	1.20000	1.5766	1.9524	0.808	25.23
2	1.00000	1.4679	1.7807	0.824	23.49
3	0.80000	1.3047	1.5676	0.832	20.88
4	0.60000	1.1039	1.3511	0.817	17.66
5	0.40000	0.8731	1.1361	0.769	13.97
6	0.32200	0.7739	1.0517	0.736	12.38
7	0.26833	0.7013	0.9938	0.706	11.22
8	0.22633	0.6410	0.9501	0.675	10.26
9	0.18783	0.5820	0.9142	0.637	9.31
10	0.15633	0.5300	0.8843	0.599	8.48
11	0.12950	0.4823	0.8501	0.567	7.72
12	0.10850	0.4423	0.8407	0.526	7.08
13	0.08983	0.4040	0.8194	0.493	6.46
14	0.07583	0.3730	0.8070	0.462	5.97
15	0.06417	0.3460	0.7966	0.434	5.54
16	0.05367	0.3190	0.7945	0.402	5.10
17	0.04433	0.2940	0.7818	0.376	4.70
18	0.03617	0.2700	0.7749	0.348	4.32
19	0.03033	0.2520	0.7697	0.327	4.03
20	0.02567	0.2350	0.7637	0.308	3.76
21	0.02333	0.2270	0.7613	0.298	3.63
22	0.02100	0.2170	0.7591	0.286	3.47
23	0.01867	0.2080	0.7567	0.275	3.33
24	0.01750	0.2030	0.7561	0.268	3.25
25	0.01633	0.1980	0.7553	0.262	3.17
26	0.01400	0.1870	0.7534	0.248	2.99

- 16^3x28 lattice with 1/a=0.978 GeV or a=0.20 fm from f_π
- Strange quark mass set at No. 6 from ϕ(1020) input.
- Smallest pion mass is about 180 MeV.
- Physical m_π / m_ρ =0.18
- Box size is 3 times the smallest pion Compton wavelength

Conventional Curve Fitting

$$\text{Data}: \quad \{G_k(t_i)\}, \text{configuration } k = 1, N, \text{ time slice } i = 1, N_t$$

$$\text{average } \overline{G}(t_i) = \tfrac{1}{N} \sum_{k=1}^{N} G_k(t_i)$$

$$\text{Theory}: \quad G_{\text{th}}(t) = \sum_{n=1}^{\infty} A_n e^{-E_n t}$$

$$\text{minimize } \chi^2(A_n, E_n) = \sum_{i,j=1}^{N_t} \left[\overline{G}(t_i) - G_{th}(t_i)\right] C_{ij}^{-1} \left[\overline{G}(t_j) - G_{th}(t_j)\right]$$

$$\text{covariance matrix}: C_{ij} = \frac{1}{N(N-1)} \sum_{k=1}^{N} \left[G_k(t_i) - \overline{G}(t_i)\right]\left[G_k(t_j) - \overline{G}(t_j)\right]$$

Problem: the procedure is intrinsically singular so one is forced to fit at large times where the ground state dominates.

Constrained Curve Fitting: theory

(see Lepage, heplat/0110175 and Morningstar, heplat/0112023)

Bayes's probability theorem: $P(A \mid B) = \dfrac{P(B \mid A)\, P(B)}{P(A)}$

Translation to our problem: $P\left(\rho \mid \overline{G}\right) = \dfrac{P\left(\overline{G} \mid \rho\right) P(\rho)}{P\left(\overline{G}\right)} \propto P\left(\overline{G} \mid \rho\right) P(\rho)$

The best solution is from maximization of this probability: $\dfrac{\partial}{\partial \rho} P\left(\rho \mid \overline{G}\right) = 0$

Assume:

χ^2 probability: $P\left(\overline{G} \mid \rho\right) = e^{-\chi^2/2}$

prior probability: $P(\rho) = e^{-\chi^2_{prior}/2}$

(maximum entropy method: $P(\rho) = e^{\alpha S}$)

Then the final probaility is $P\left(\rho \mid \overline{G}\right) = e^{-\chi^2_{aug}/2}$ where $\chi^2_{aug} = \chi^2 + \chi^2_{prior}$

Note: if no prior, this amounts to a simple χ^2 minimization, which does not work.

Constrained Curve Fitting: practice

$$\text{minimize } \chi^2_{aug} = \chi^2 + \chi^2_{prior}$$

$$\chi^2_{prior} = \sum_n \frac{(A_n - \tilde{A}_n)^2}{\tilde{\sigma}^2_{A_n}} + \sum_n \frac{(E_n - \tilde{E}_n)^2}{\tilde{\sigma}^2_{E_n}}$$

$\{A_n, E_n\}$ are fit parameters as in $G_{\text{th}}(t_i) = \sum_n A_n e^{-E_n t_i}$

$\{\tilde{A}_n \pm \tilde{\sigma}_{A_n}, \tilde{E}_n \pm \tilde{\sigma}_{E_n}\}$ are input parameters (Bayesian priors)

1) Goal: fit as many data points in $\overline{G}(t_i)$ and as many terms in G_{th}.
2) Use prior knowledge, like $\tilde{A}_n > 0,\ \tilde{E}_n - \tilde{E}_{n-1} > 0$
3) Seek guidance for priors from a subset of data (empirical Bayes method).
4) Un-constrain the term of interest to have conservative error bars.

(See heplat/0208055 by S.J. Dong et al)

Baryon Two-point Function

$$G(t)=\sum_{\vec{x}}\langle vac\,|\,T\left[\chi_1(x)\overline{\chi_1}(0)\right]|\,vac\rangle$$

$$=(1+\gamma_4)[A_+e^{-m_+(t-t_0)}+bA_-e^{-m_-(N_t+t_0-t)}]+(1-\gamma_4)[bA_+e^{-m_+(N_t+t_0-t)}+A_-e^{-m_-(t-t_0)}]$$

Fixed boundary condition (b=0)

$$G(t)=(1+\gamma_4)A_+e^{-m_+(t-t_0)}+(1-\gamma_4)A_-e^{-m_-(t-t_0)}$$

upper components: $G_U(t)=2A_+e^{-m_+(t-t_0)}$

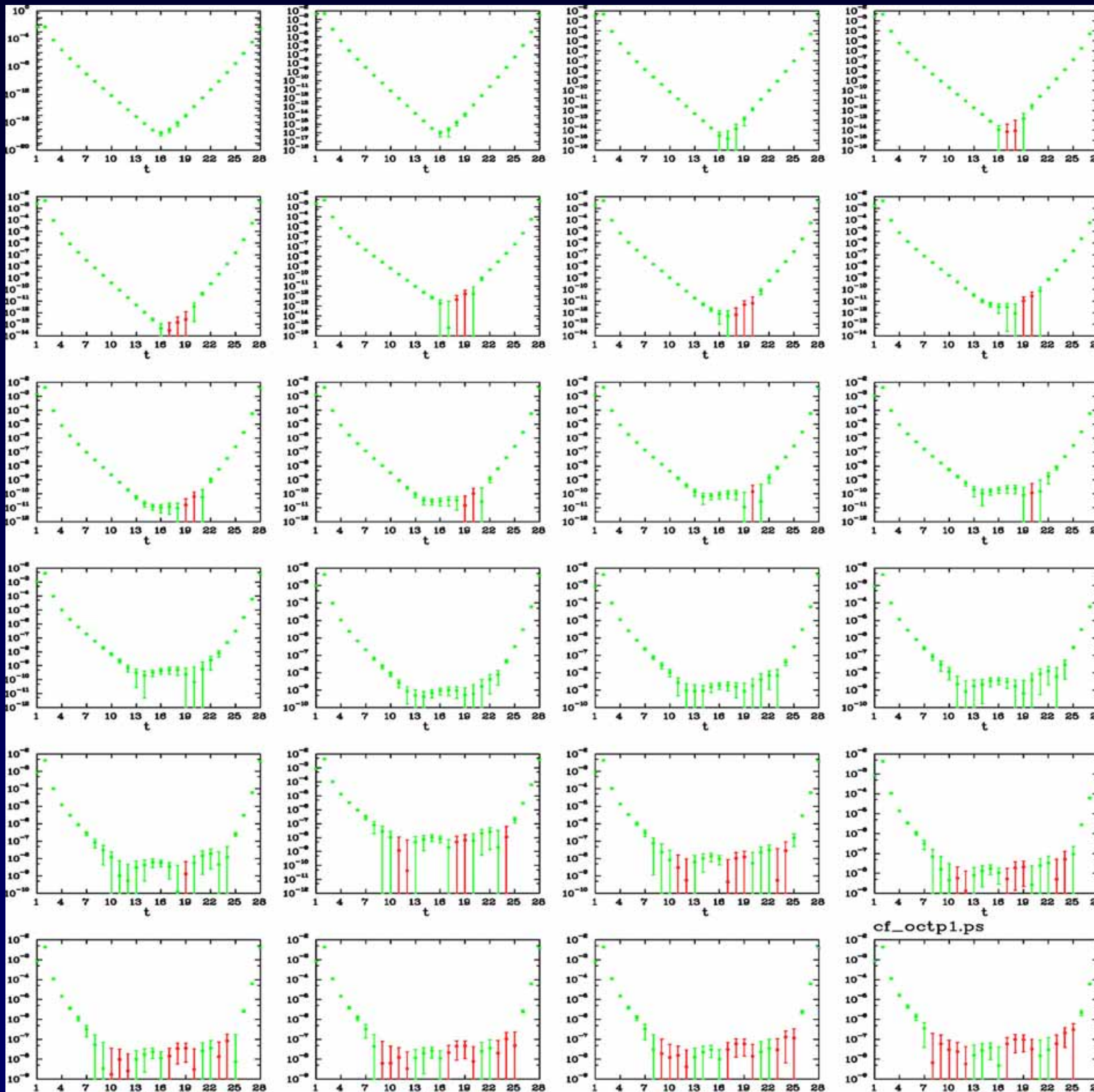
lower components: $G_L(t)=2A_-e^{-m_-(t-t_0)}$

Anti-periodic boundary condition (b= -1)

$$G_U(t)=2A_+e^{-m_+(t-t_0)}-2A_-e^{-m_-(N_t+t_0-t)}$$

$$G_L(t)=-2A_+e^{-m_+(N_t+t_0-t)}+2A_-e^{-m_-(t-t_0)}$$

Nucleon from χ_1 operator: $G_U(t) = 2A_+ e^{-m_+(t-t_0)} - 2A_- e^{-m_-(N_t+t_0-t)} + \cdots$



Upper components:

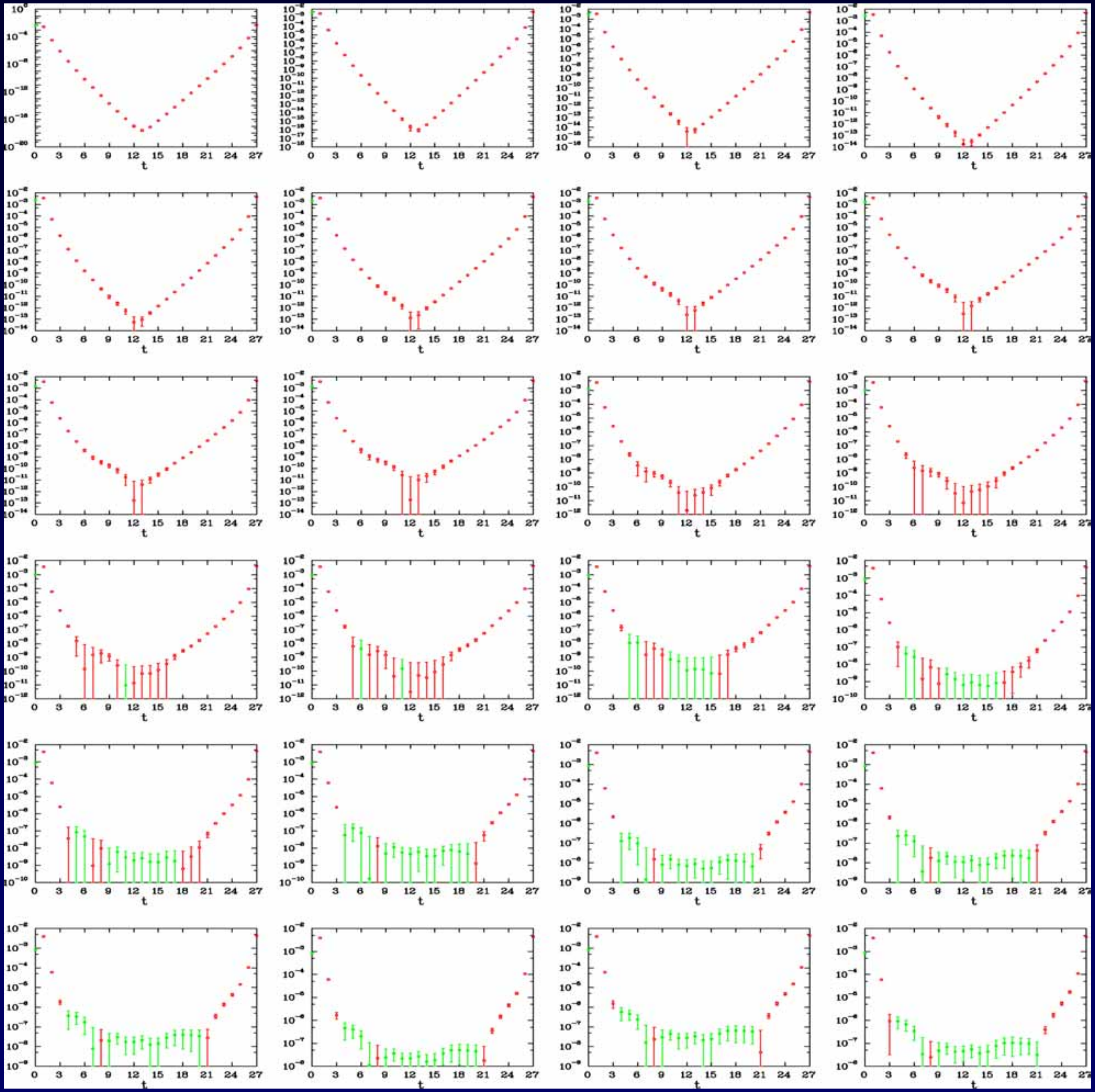
Left half is dominated by m_+

Right half is dominated by m_-

A_+ is positive.

A_- is negative.

Nucleon from χ_1 operator: $G_L(t) = -2A_+ e^{-m_+(N_t+t_0-t)} + 2A_- e^{-m_-(t-t_0)} + \cdots$



Lower components:

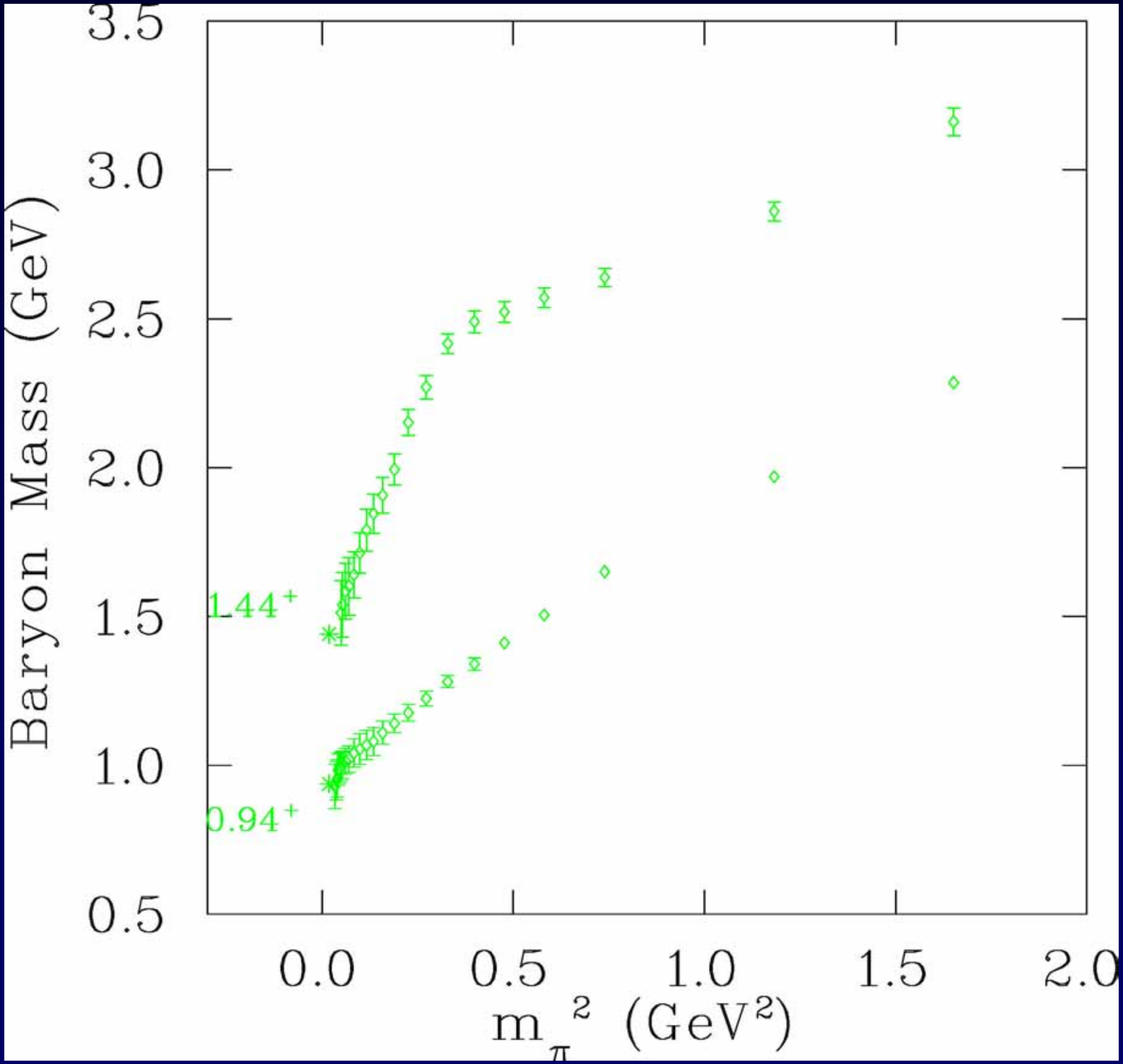
left half m_-

right half m_+

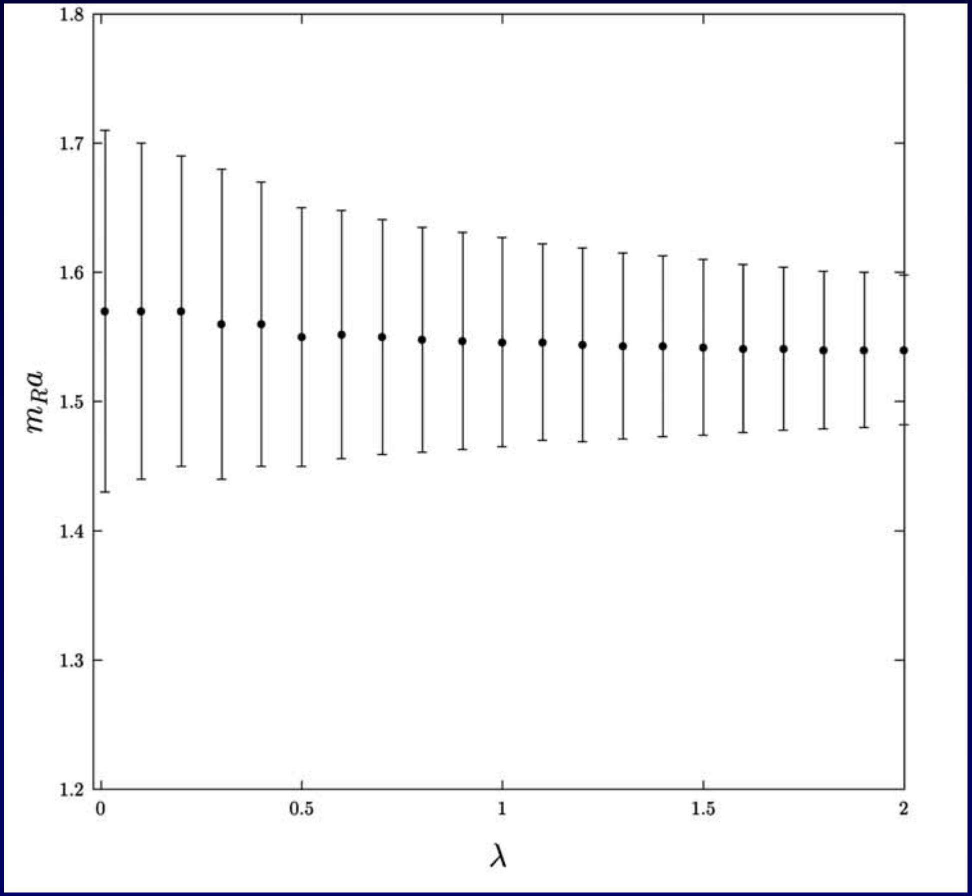
A_- is negative.

G(t) changes sign at small quark masses: quenched artifacts

Positive-parity N(1/2+) from χ_1



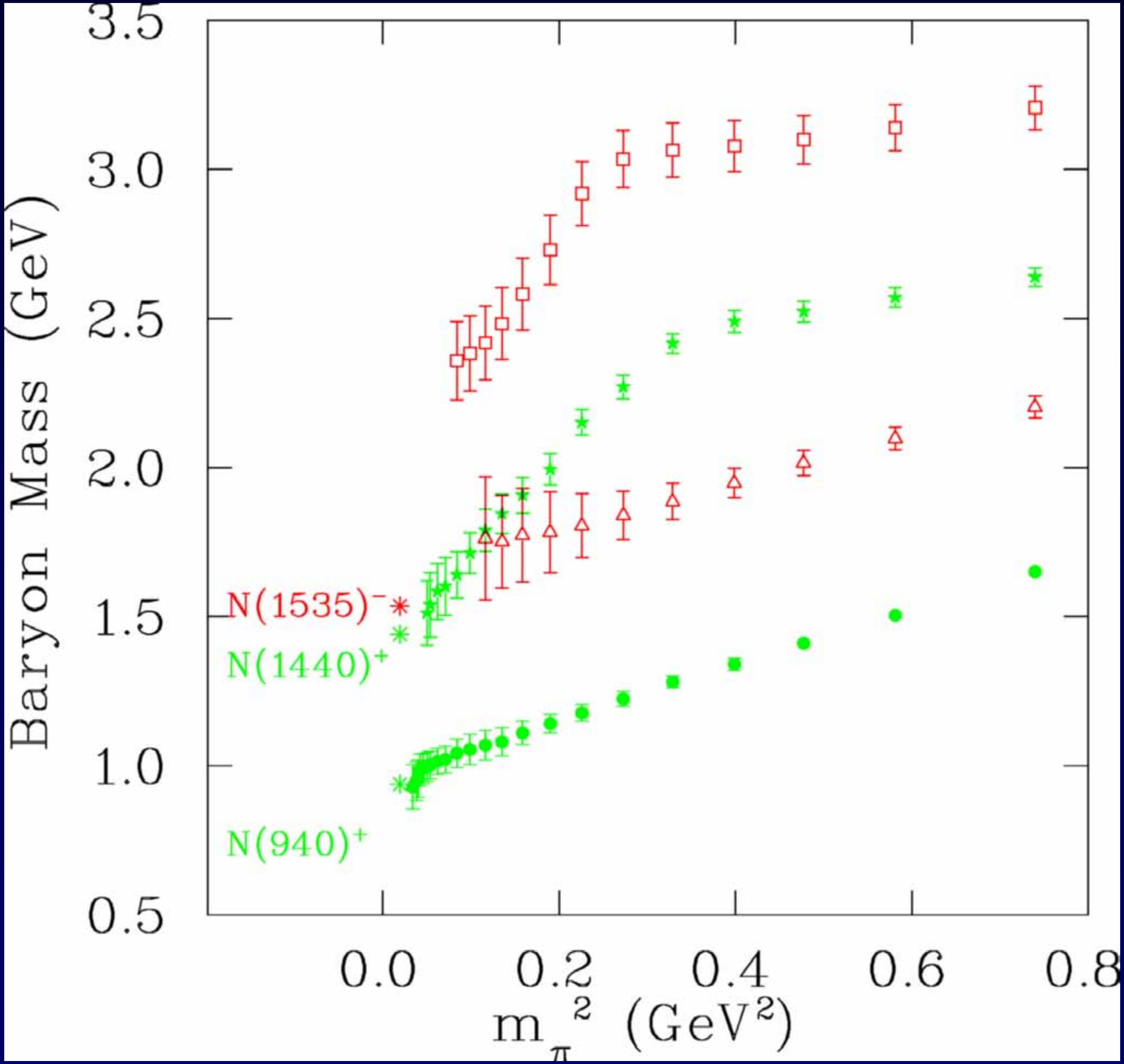
Sensitivity to priors



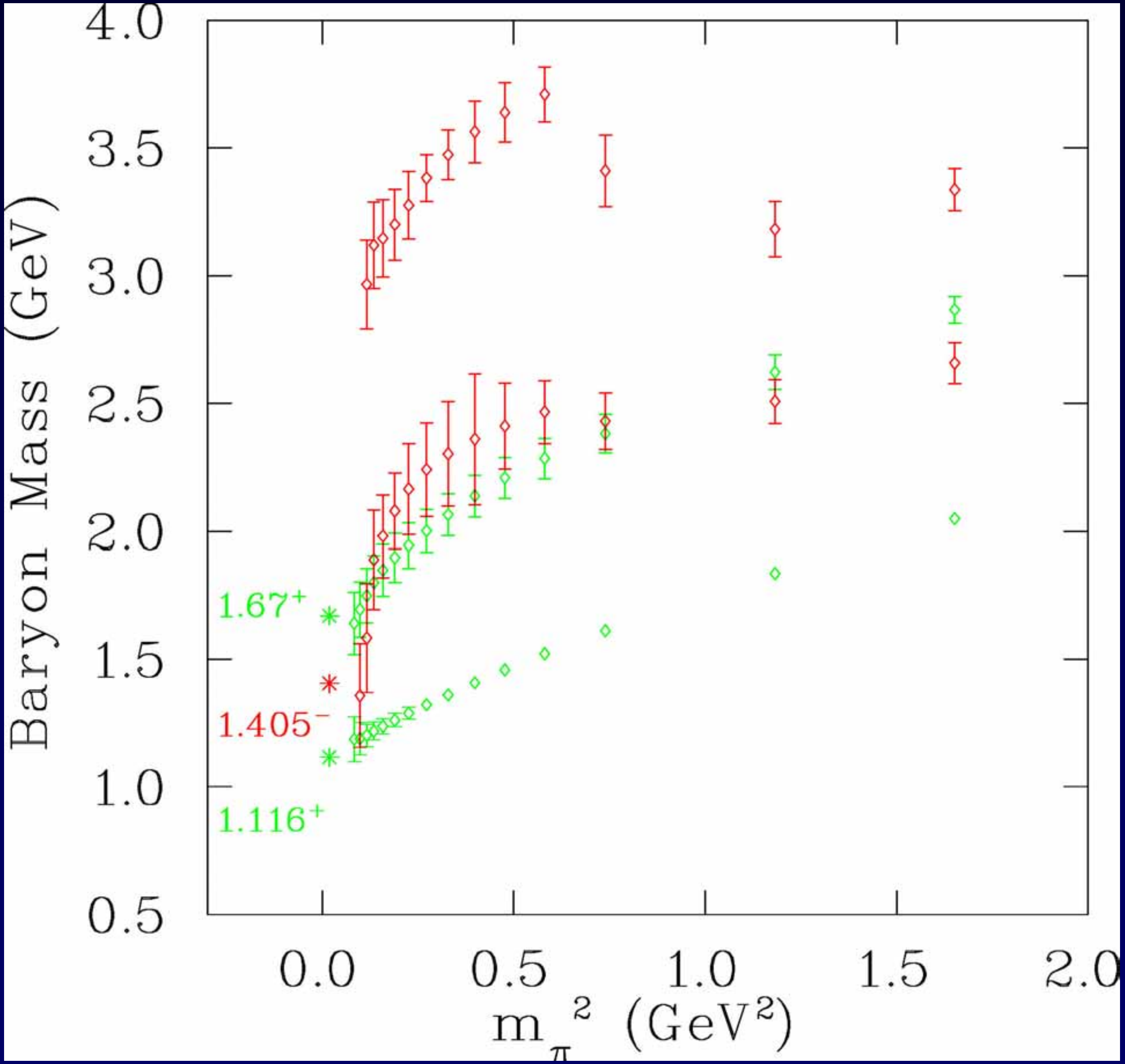
- Examle:
- N'(1/2+) at $m_\pi / m_\rho = 0.30$
- Vary the Roper prior

$$\chi^2_{prior} = \lambda \left[\frac{(A_2 - \tilde{A}_2)^2}{\tilde{\sigma}^2_{A_2}} + \frac{(E_2 - \tilde{E}_2)^2}{\tilde{\sigma}^2_{E_2}} \right] + \cdots$$

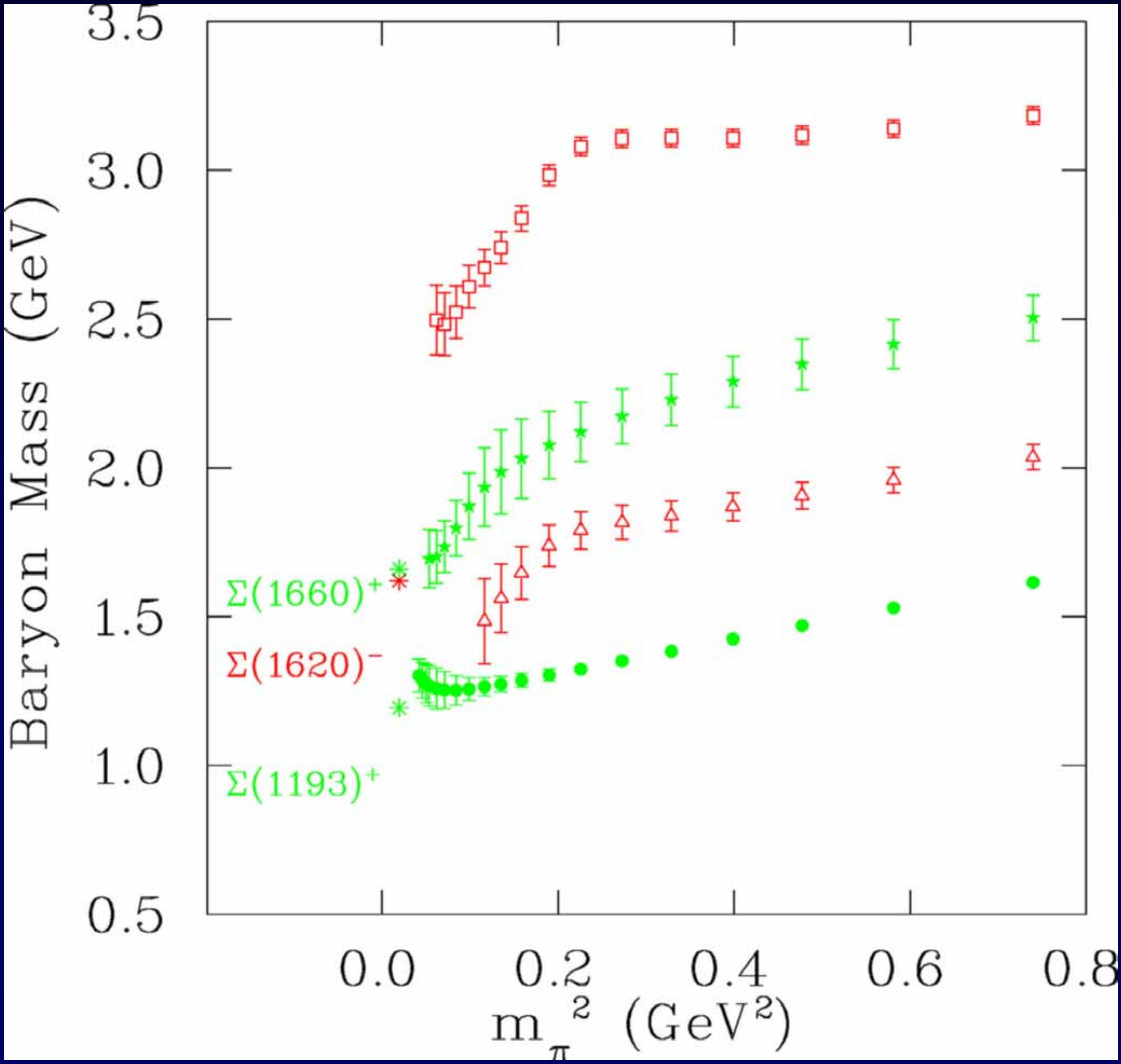
Level ordering in the nucleon channel



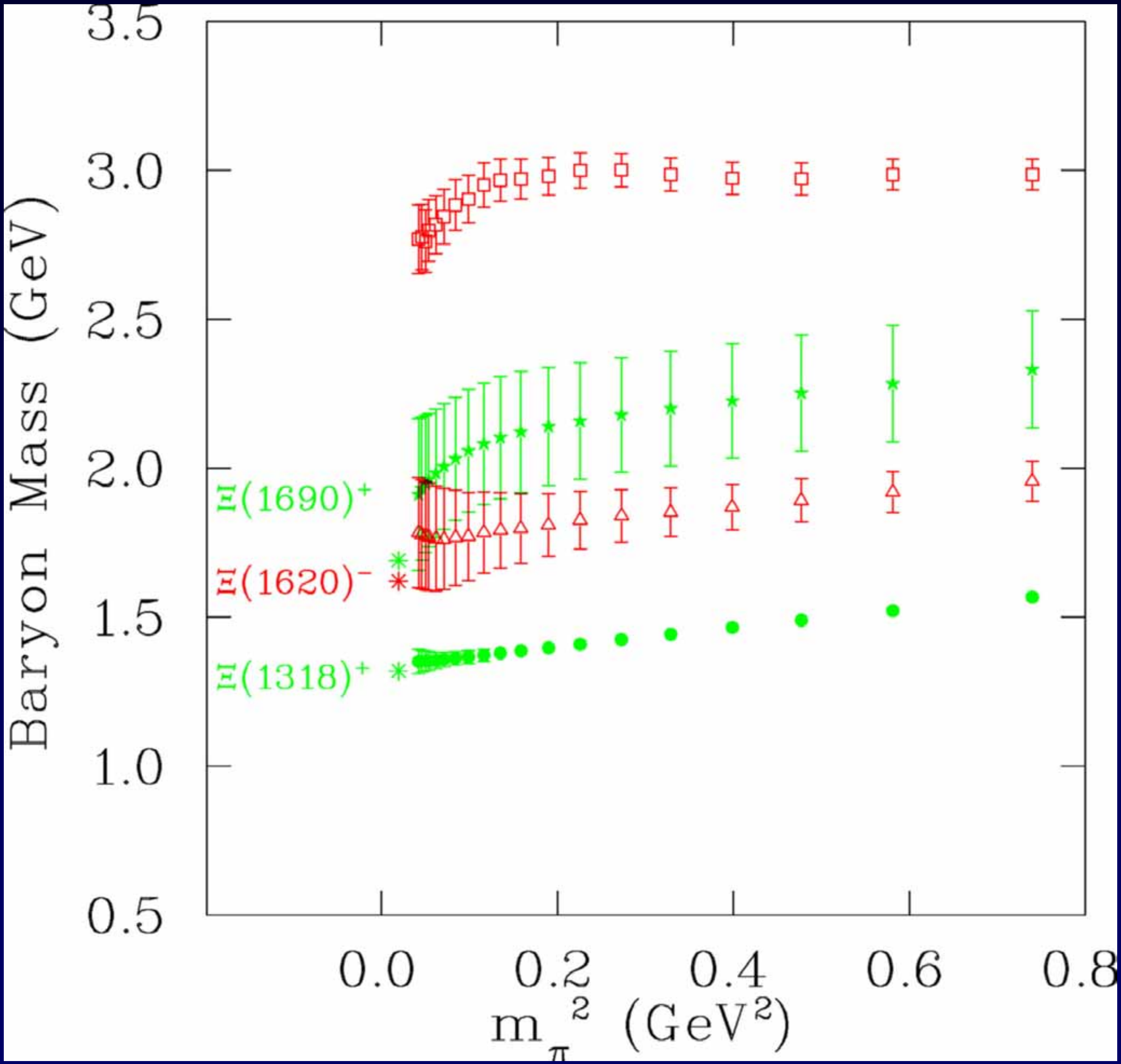
Level ordering in the Λ (1/2) channel



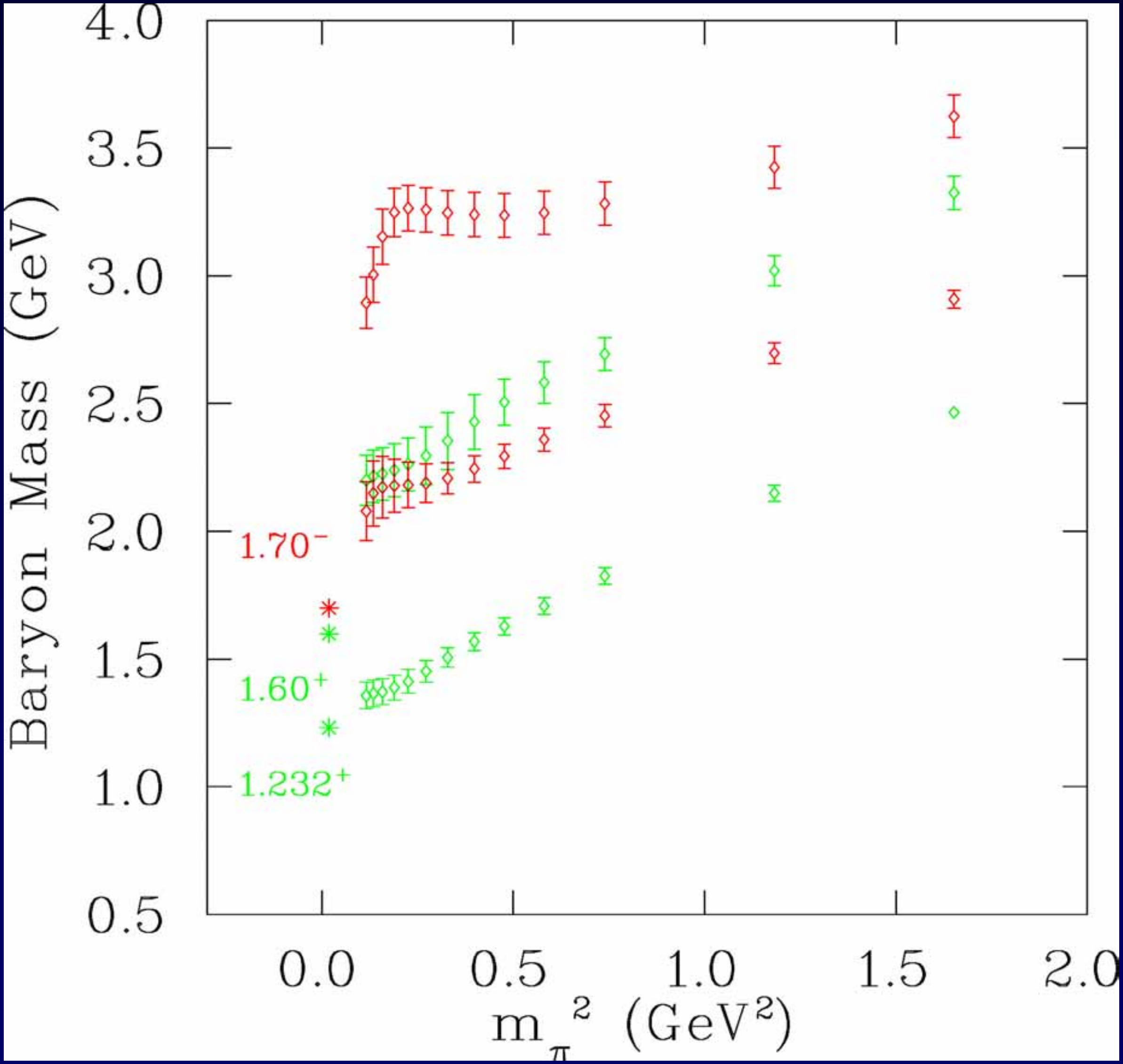
Level ordering in the Σ(1/2) channel



Level ordering in the Ξ(1/2) channel



Level ordering in the Δ channel



Conclusion

- Constrained curve-fitting offers an exciting new tool in the study of baryon spectroscopy on the lattice.
  - It allows a systematic and more reliable determination of the 1^{st} excited state.

- Coupled with the overlap fermion action which allows access to quark masses close to the physical limit with exact chiral symmetry, we found that
  - The Roper state is observed as the 1^{st} excited state of the nucleon from the standard nucleon interpolating field (χ_1).
  - The level orderings in the low-lying spin-1/2 sector are largely consistent with experiment, including the $\Lambda(1405)$.
  - Dramatic cross-overs take place in the small quark mass region of $m_\pi \sim 300$ to 400 MeV.

Conclusion continued

- Physics implications:
  - The Roper is most likely a simple 3-quark state, rather than a 5-quark state or some other combination.
  - The results favor the flavor-spin dominant picture of hyperfine splittings in baryons, over color-spin.

- Future work
  - To further study the constrained fitting method on baryon spectroscopy
    - Scrutinize the results and sensitivities to the priors
    - Automate the fitting procedure.
    - What about quenched artifacts ($N\eta'$ hair-pins) ?
  - How reliable is the 2^{nd} excited state?

Reserve Slides

Is it a true state?

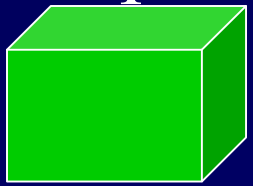
- A two-particle state would have the energy

$$E = \sqrt{p^2 + M_N^2} + \sqrt{p^2 + M_\pi^2}, \text{ where } \mathrm{p} = \frac{2\pi}{\mathrm{La}}$$

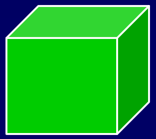
which is sensitive to the box size L.

- A true state has zero momentum which is not sensitive to L.

- A simple test: check L dependence.
  - 16^3x28 at a=0.2 fm



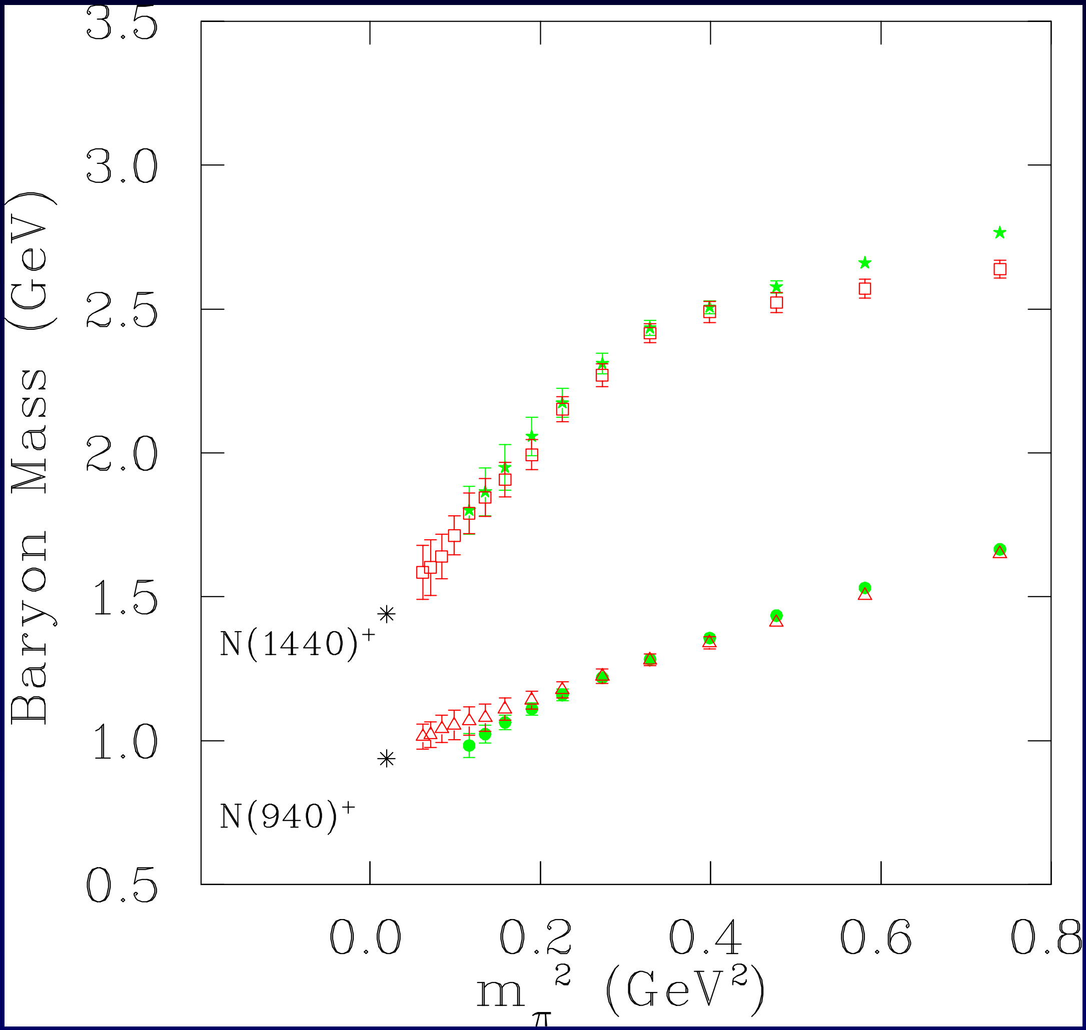
  - 12^3x28 at a=0.2 fm



Box size dependence

16^3x28

12^3x28

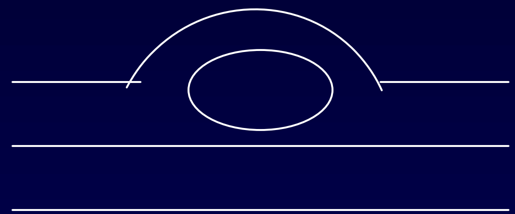


- The Roper state survives the L test.

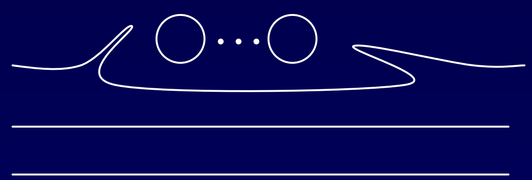
What about quenched effects?

$N \to N\eta'$

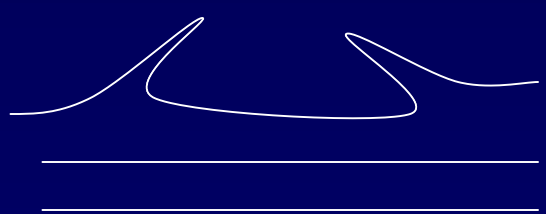
(a)



(b)



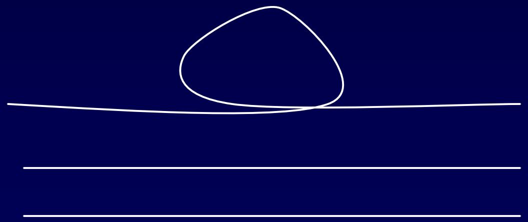
(c)



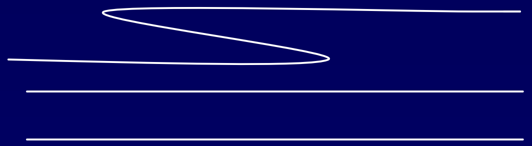
Hairpin signatures:
negative contribution to G(t).

Meson cloud
(chiral dynamics)

(c)

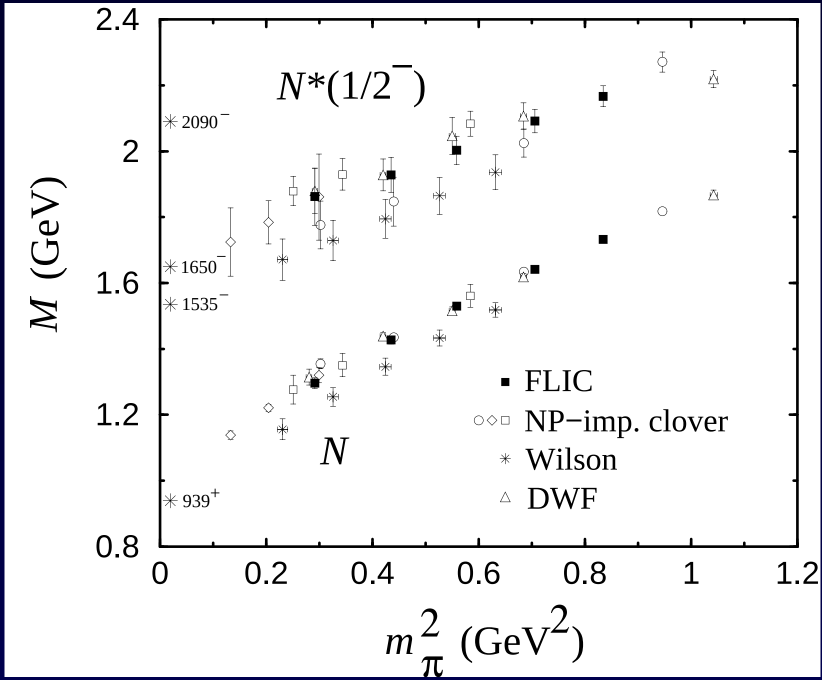


(d)

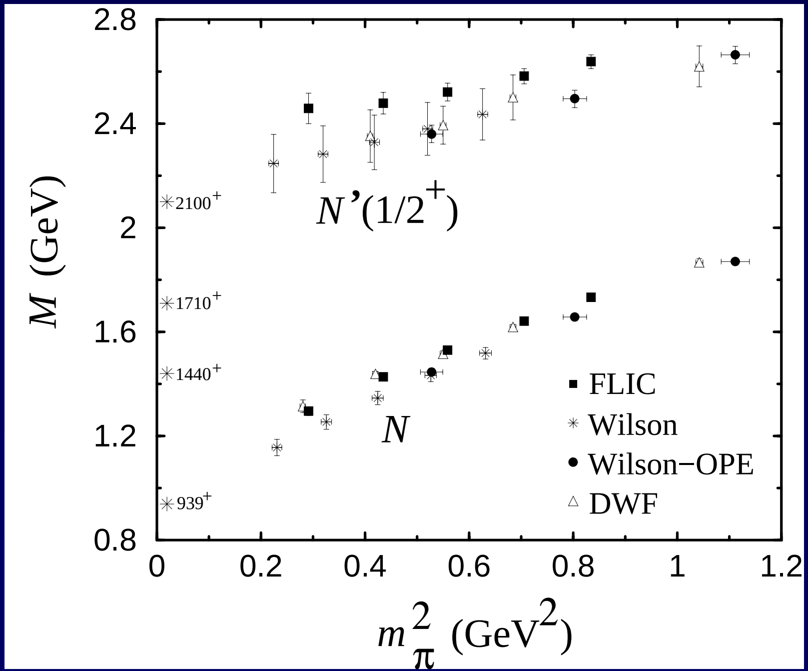


The story so far ...

- Negative-parity splitting is consistent with experiment.



- Little evidence of the Roper state N*(1440)1/2+ from χ_2.



- Question: what is the 1st excited state of χ_1 ?

Simulation Details

- Iwasaki gauge action

$$S_G = \beta(c_0\, \text{plaq} + c_1\, \text{rect}) \text{ with } \beta = 6/\text{g}^2, c_0 = 3.648, c_1 = -0.331$$

- 16^3x28 lattice at β=2.264, 80 configurations.
- a=0.175 fm or 1/a=1.126 GeV from string tension
- a=0.20 fm or 1/a=0.978 GeV from f_π

- Overlap fermions (Neu98, Lus98)

$$D(m_0) = \left(\rho + \frac{m_0 a}{2}\right) + \left(\rho - \frac{m_0 a}{2}\right)\gamma_5 \varepsilon(H)$$

$$\text{where } \varepsilon(H) = H\sqrt{H^2} \text{ is matrix sign function}$$

$$\text{and } H = \gamma_5 D_w \text{ and we use } \rho = 1.368$$

- Exact chiral symmetry: no O(a) error, even O(a^2) error mild, no exceptional configurations, critical-slowing down mild. See Dong et al, PRL85, 5051 (2001) Multi-mass algorithm: we did 26 quark masses.
  - total cost is cost of smallest mass plus 10% overhead.