Higher spin N^* as relativistic fields

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Outline

- Motivation for a field theory of N^* baryons

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- The consistency condition:
  local (gauge) invariances of higher-spin fields
- Consistent πNN* and γNN* couplings
- Applications to Compton scattering

1. Motivation

- QCD is the fundamental theory of the strong interaction.
- At (relatively) low energies one can describe the strong-interaction physics in terms of hadronic DOF. => Look for an effective theory which matches low-energy QCD.
- Since QCD is a relativistic field theory its effective theory can be found in the same framework.
- Rel. potential models and the chiral EFT are examples. An effective Lagrangian as a starting point.

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Lagrangian of low-energy strong interaction, hadronic DoF

$$\mathcal{L}_{eff} = \partial_\mu \pi^* \partial^\mu \pi - m_\pi^2 \pi^* \pi + \overline{N}(i\partial\!\!\!/ - m_N)N + (g_A/f_\pi)\overline{N}\gamma_\mu\gamma_5 N \partial^\mu \pi + \cdots$$

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&\quad + \mathcal{L}_\rho + \mathcal{L}_{\pi\pi\rho} + \mathcal{L}_{\rho NN} + \cdots
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& + \mathcal{L}_\rho + \mathcal{L}_{\pi\pi\rho} + \mathcal{L}_{\rho NN} + \cdots \\
& + \mathcal{L}_\Delta + \mathcal{L}_{\pi N\Delta} + \mathcal{L}_{\pi\Delta\Delta} + \cdots
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1. Motivation

Hadron	Field	Lorentz property
Pion:	$\pi(x)$	pseudo-scalar
Nucleon:	$N^{(\alpha)}(x)$	spinor
Rho-meson:	$\rho^{\mu}(x)$	vector
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In general,

Spin-s meson: $h^{\mu_1 \cdots \mu_s}(x)$

Spin-s baryon: $\psi^{\mu_1 \cdots \mu_{s-1/2}}(x)$

2. The problem of higher spin fields

$h^{\mu_1 \cdots \mu_s}(x)$, $\psi^{\mu_1 \cdots \mu_{s-1/2}}(x)$ have many more components than is needed to describe particles with

$$\#\ \text{Spin DOF} = \begin{cases} 2s+1\,, & m \neq 0 \\ 2\,, & m = 0 \end{cases}$$

So constraints (symmetries) must be imposed.

2. The problem of higher spin fields

Local (gauge) symmetries reduce the # DOF. With

$$\delta\mathcal{L} = 0, \quad \text{under} \quad \delta h^{\mu_1\cdots\mu_s}(x) = \partial^{\{\mu_1}\phi^{\mu_2\cdots\mu_s\}}$$

it is generally possible to reduce to only 2 DOF.

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E.g., EM field, A_μ: 4 components, but $\mathcal{L} = -(1/4)F^2$ is invariant under $\delta A_\mu = \partial_\mu\phi$, so that # DOF =2. Mass term, m^2A^2, provides # DOF =3.

2. The problem of higher spin fields

Free spin-3/2 field

Spin-3/2 particle is described by a 16-component ψ_μ. The Lagrangian,

$$\mathcal{L} = \bar{\psi}_\mu \gamma^{\mu\nu\alpha} \partial_\alpha \psi_\nu,$$

with $\gamma^{\mu\nu\alpha} = (\gamma^\mu \gamma^\nu \gamma^\alpha - \gamma^\alpha \gamma^\nu \gamma^\mu)/2$, describes free massless spin-3/2 particle., since due to the gauge symmetry:

$$\psi_\mu \to \psi_\mu + \delta\psi_\mu, \;\; \delta\psi_\mu = \partial_\mu \epsilon,$$

$\epsilon(x)$ is a spinor field, there are only 2 DOF left.

2. The problem of higher spin fields

Free spin-3/2 field

The mass term is introduced such that # DoF = 2s+1=4.

One finds

$$\mathcal{L} = \bar{\psi}_\mu \gamma^{\mu\nu\alpha} \partial_\alpha \psi_\nu - m \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu$$

with $\gamma^{\mu\nu} = [\gamma^\mu, \gamma^\nu]/2.$

And so on for higher s. Free formulation was done in 70's.

Interactions are problematic. (Johnson-Sudarshan, Velo-Zwanziger problems)

3. Consistency condition

Interactions must be consistent with the DOF counting.

Warranted if interactions are gauge-symmetric (both for $m = 0$ and not), $\delta \mathcal{L}_I = 0$.

Proof:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_m + \mathcal{L}_I$$

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$\delta\mathcal{L}_I = 0$ is a sufficient condition. Becomes necessary if to require smooth $m \to 0$ limit.

Ref: V. P., Phys Rev D 58 (1998) 096002.

4. Consistent πNN* and γNN*

Couple to explicitly invariant quantities, e.g., in s=3/2 case

$$G_{\mu\nu} = \partial_\mu \psi_\nu - \partial_\nu \psi_\mu, \quad \tilde{G}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta} G_{\mu\nu}$$

$\pi N \Delta$ and $\gamma N \Delta$ couplings:

$$\mathcal{L}_{\pi N \Delta} = \frac{f_{\pi N \Delta}}{m_\pi m_\Delta} \bar{\Psi}\, \gamma_5\, \gamma_\mu\, \tilde{G}^{\mu\nu}\, \partial_\nu\, \phi + \text{H.c.}$$

$$\mathcal{L}_{\gamma N \Delta} = \frac{3e}{2m_N(m_N + m_\Delta)} \left[g_M\, \bar{\Psi} T_3 G_{\mu\nu} \tilde{F}^{\mu\nu} + g_E\, \bar{\Psi} T_3 \gamma_5 G_{\mu\nu} F^{\mu\nu} \right]$$

4. Consistent couplings

The spin-3/2 (Rarita-Schwinger) propagator

$$\begin{aligned} S_{\mu\nu}(p) &= \frac{1}{\not{p} - m} P^{(3/2)}_{\mu\nu} - \frac{2}{3m^2}(\not{p} + m) P^{(1/2)}_{22,\mu\nu} \\ &+ \frac{1}{\sqrt{3}m}\left(P^{(1/2)}_{12,\mu\nu} + P^{(1/2)}_{21,\mu\nu}\right) \end{aligned} \qquad (1)$$

where $P^{(J)}_{mn,\mu\nu}$ are the spin-projection operators.

$$P^{(3/2)}_{\mu\nu}(p) = g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3p^2}(\not{p}\gamma_\mu p_\nu + p_\mu\gamma_\nu\not{p})$$

projects onto the pure spin-3/2 states

4. Consistent couplings

$$
\begin{aligned}
P^{(1/2)}_{22,\mu\nu} &= p_\mu p_\nu / p^2 \ , \\
P^{(1/2)}_{12,\mu\nu} &= p^\varrho p_\nu \sigma_{\mu\varrho} / (\sqrt{3}\, p^2) \ , \\
P^{(1/2)}_{21,\mu\nu} &= p_\mu p^\varrho \sigma_{\varrho\nu} / (\sqrt{3}\, p^2) \ ,
\end{aligned}
$$

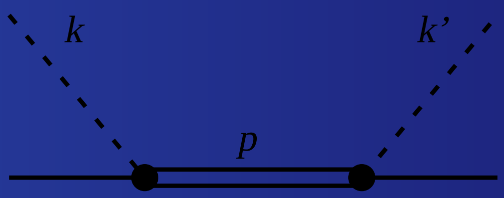
project onto the spin-1/2 sector, which however decouples **if the coupling is HS gauge-invariant,** since the vertex satisfies the transversality condition:

$$p_\mu \Gamma^\mu(p,k) = 0, \quad => \Gamma^\mu \, P^{(1/2)}_{\mu\nu} \Gamma^\nu = 0.$$

4. Consistent πNN* and γNN*

A typical amplitude:

$$\Gamma^{\mu}(p,k')\,S_{\mu\nu}(p)\,\Gamma^{\nu}(p,k) =$$



$$= \frac{(f_{\pi N\Delta}/m_{\pi})^2}{\not p - m}\,\frac{p^2}{m^2}\,P^{(3/2)}_{\mu\nu}(p)\,k'^{\mu}k^{\nu}$$

$P^{(3/2)}(p)$ the spin-3/2 projection operator
— nonlocal, but $p^2\,P^{(3/2)}$ is local!

4. Consistent πNN* and γNN*

Easily generalized to other spins. Transversality:

$$p_{\{\mu_1} \Gamma^{\mu_1 \cdots \mu_{s-1/2}\}} = 0.$$

Full propagator can then be replaced by the highest spin term (drop the lower-spin sector).

Ref: V. P. and J. A. Tjon, PRC (2001); V. P. and O. Scholten, (in prep).

4. Equivalence theorem

“Conventional” vs “HS gauge-invariant couplings”.

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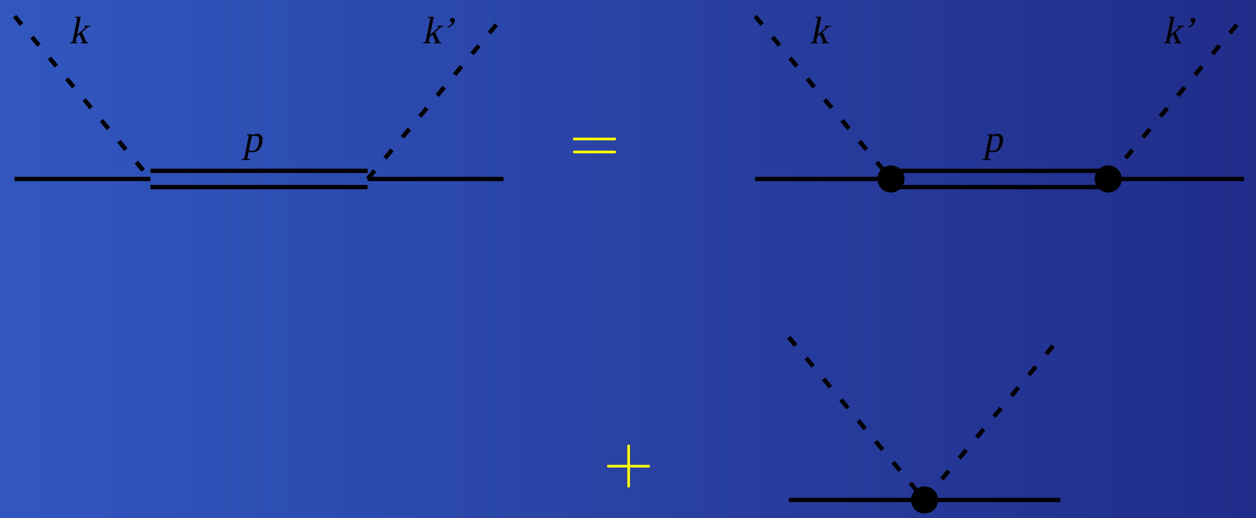
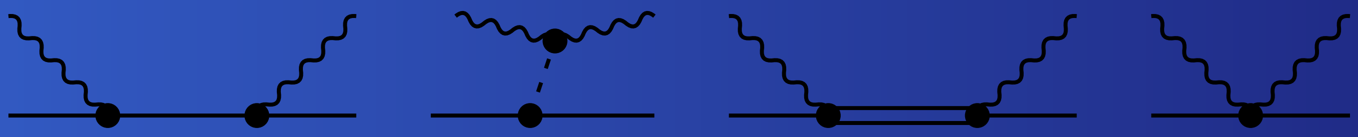
In general are not equal. But (for $m \neq 0$) can be related by field redefinitions, so made equivalent up to contact terms.



Ref: V. P., PLB 503 (2001).

5. Compton scattering on the nucleon

V. P. and O. Scholten, NPA (1995). ELA: tree Born and Delta graphs, using conventional $\gamma N \Delta$ $(G_1,\, z_1,\, G_2,\, z_2)$.



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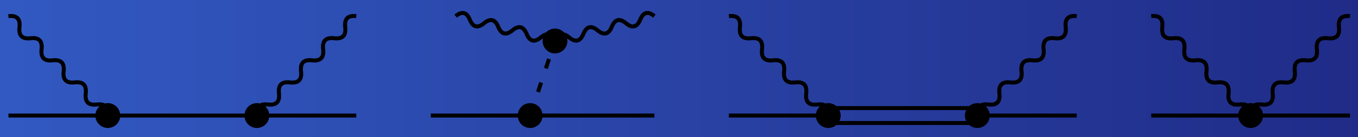
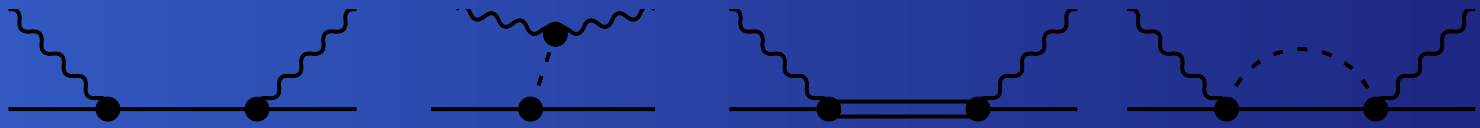


V. P. and D. Phillips, (in prep). ChPT in "δ-expansion" :

Born $+$ one-πN loops $+\Delta$ with gauge-invariant (g_M, g_E) .



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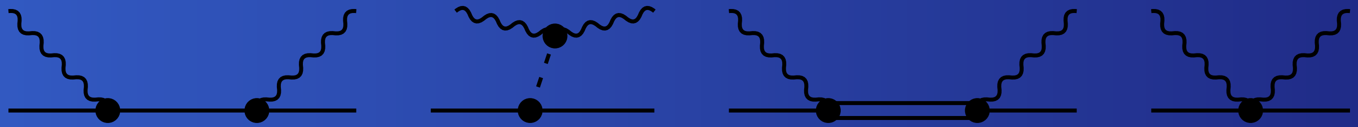

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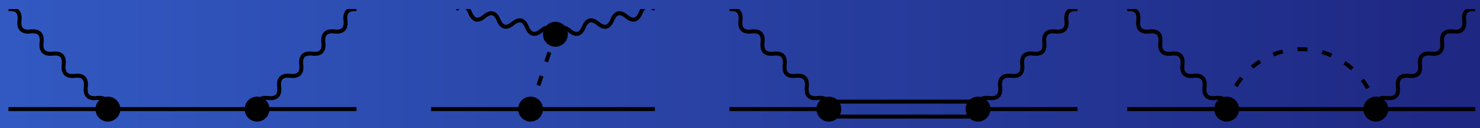
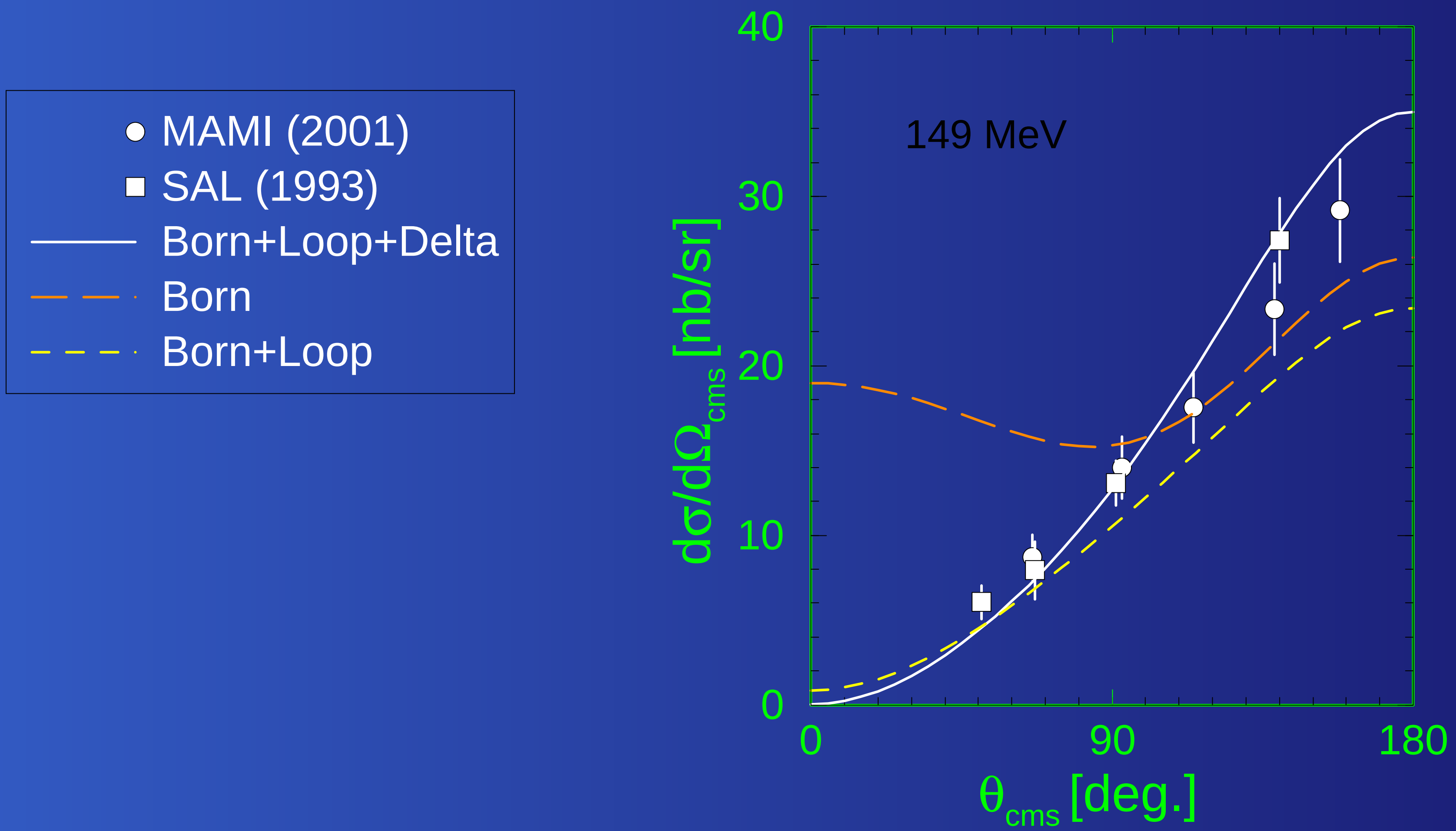


Difference: spin-1/2 bkgd contributions ($\gamma\gamma NN$ contact term) are replaced by the πN loops. 2 less parameters.

5. Compton diff. cross-section



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- Work in progress: D. Phillips – Compton scattering on p and D, J .A. Tjon and G. Caia, L. Wright – pion photo- and electro-production on N.