The reaction $\bar{p}p \to \pi^+\pi^-$ in a constituent quark model: relativistic aspects of the pion wavefunctions.

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Bruno EL-Bennich[1,2]

and

Willem Kloet[1]

1. Rutgers University, New Jersey

2. Université Pierre et Marie Curie, LPTPE-LPNHE Paris

- Motivation :

- LEAR data on $\frac{d\sigma}{d\Omega}$ and A_{ON} for 67 MeV – 875 MeV

  Hasan et al., Nucl. Phys. B 378 (1992)

  → strong left-right asymmetry observed in $\bar{p}p \to \pi^+\pi^-$, K^+K^-

- New facilities at CERN (Super LEAR ?) and projects (proposals) at Fermilab.

  → more intensive study of the $\bar{p}p$ system, protonium, antihydrogen . . . .

- Of theoretical interest : is the annihilation range as short ($1\,\mathrm{fm} \lesssim$) as baryon exchange and microscopic quark models want to make us believe ?

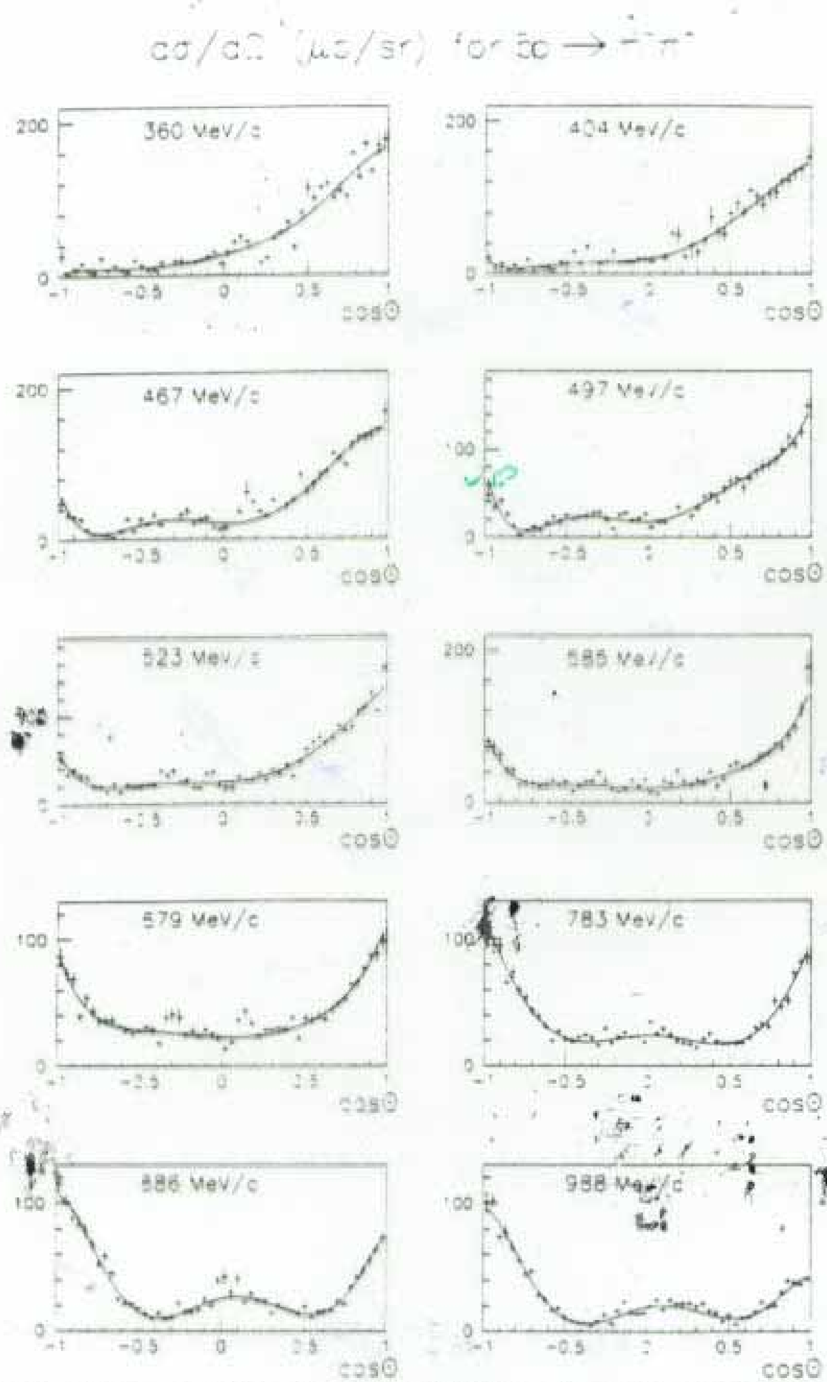


Fig. 14. $d\sigma/d\Omega(\bar{p}p \to \pi^-\pi^+)(\mu b/sr)$ from 360 to 988 MeV/c from this experiment. Smooth curves are fits with Legendre series.

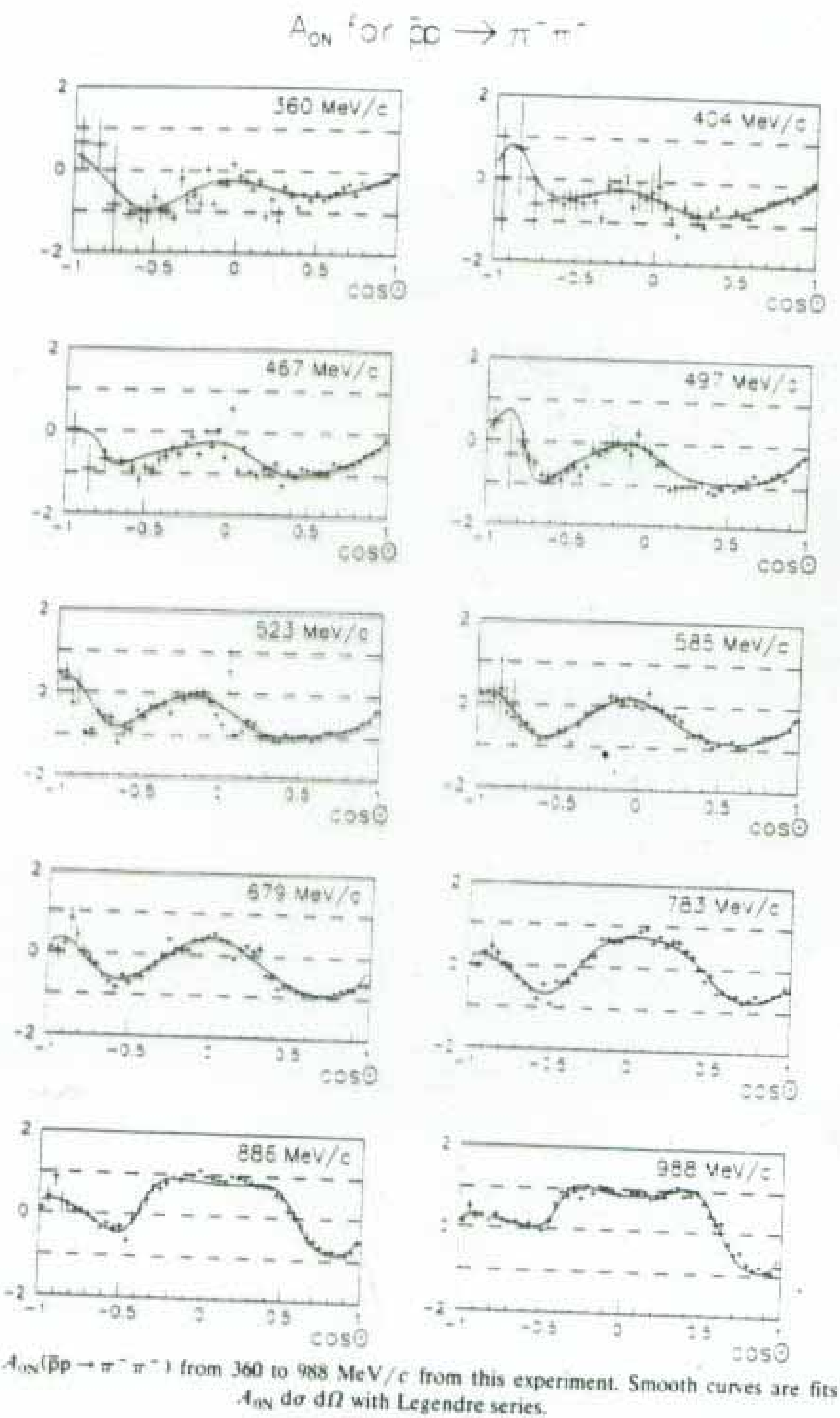


Fig. 16. $A_{ON}(\bar{p}p \to \pi^-\pi^+)$ from 360 to 988 MeV/c from this experiment. Smooth curves are fits to $A_{0N}\, d\sigma/d\Omega$ with Legendre series.

- A few generalities about the $\bar{p}p \to \pi^+\pi^-$ annihilation.

- two-meson channels are the simplest
  (but $\bar{p}p \to \pi^+\pi^-, K^+K^-$ provides only 1% of total branching ratio)

- Since the $|\pi^+\pi^-\rangle$ state is even :

  if pions are in a $J = L_{\pi\pi}$ even state
  then : $I = 0$

  if $J = L_{\pi\pi}$ odd then : $I = 1$

- Baryon number $B = 0$ in reaction, $I_3 = 0$

- The constituent quark model – Assumptions

→ Initial proton and antiproton each made of quarks and antiquarks respectively.

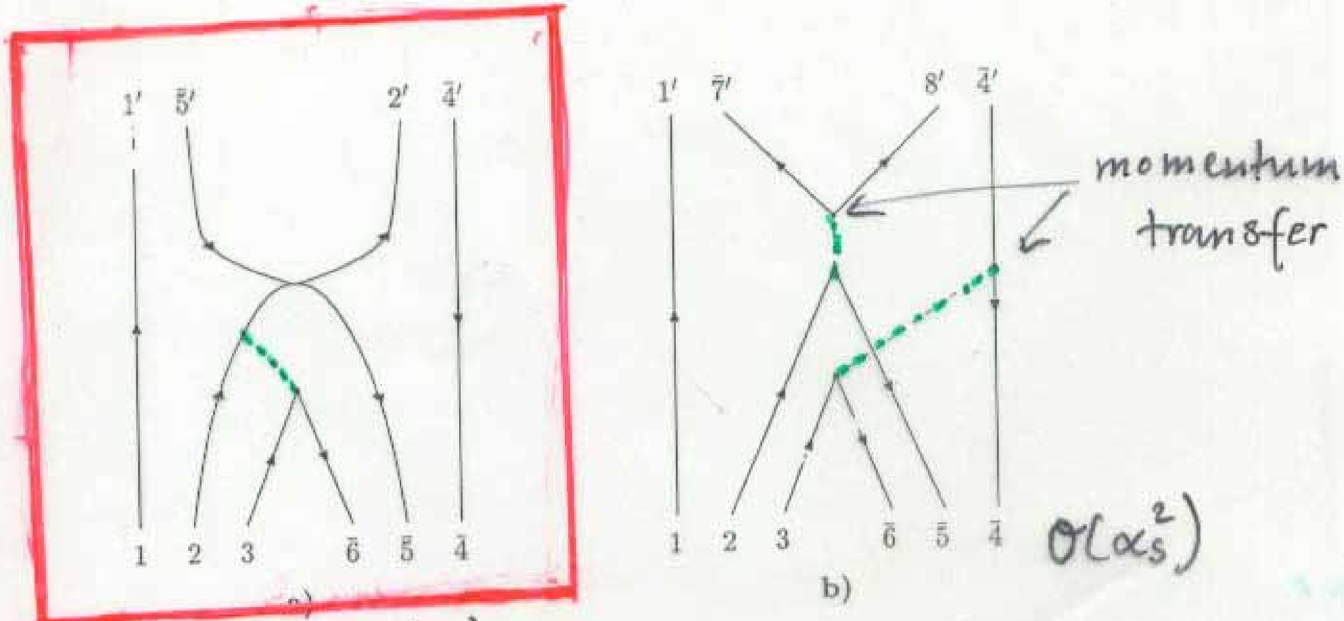
→ Quark interaction connects initial $\bar{p}p$ pair to final $\pi^+\pi^-$ state in various ways.

→ In order to achieve this, one uses Feynman diagrams of QCD with exchanges of "effective objects". However, this does <u>not</u> represent the first orders of perturbation theory in QCD!

[We rather speak of an expansion of the $\bar{q}q$ annihilation into terms of increasing J^{π}.]

References: Le Yaouanc et al. (1973) (early work)
Green, Niskanen (1984, 1986), Faessler, Furui,

— QUARK MODEL —



$O(\alpha_s)$

Fig. 1a: *Rearrangement diagram* **Fig. 1b**: *Annihilation diagram*
The numbers without bars stand for the quarks, those with bars for the antiquarks, the dashed line represents either the exchange of the effective "vacuum" $^3P_0$ or "gluon" $^3S_1$ state.

$\Rightarrow$ - - - - - : $J^\pi = 0^+, 1^-, \cdots$

The transition operators are obtained from the Feynman diagrams. For the Dirac spinors we use a semi-relativistic reduction with $\vec{p}^2 \ll m^2$:

$$\boxed{\begin{aligned} u(p) &= \sqrt{\frac{E+m}{2m}} \begin{pmatrix} 1 \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \end{pmatrix} \chi \simeq \begin{pmatrix} 1 \\ \frac{\vec{\sigma}\cdot\vec{p}}{2m} \end{pmatrix} \chi \\ v(p) &= \sqrt{\frac{E+m}{2m}} \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \\ 1 \end{pmatrix} \chi \simeq \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{2m} \\ 1 \end{pmatrix} \chi \end{aligned}}$$

At the vertices momentum conservation is imposed. In particular, in the $^3P_0$ mechanism we allow momentum transfer from the annihilated $\bar{q}q$ state to any of the other (anti)quarks on equal footing with the $^3S_1$ mechanism. In the $^3S_1$ case each vertex provides a factor $\boxed{\frac{1}{2}\lambda^i\gamma^\mu}$ where the λ^i are the Gell-Mann color matrices. Furthermore, one uses the approximation that the effective mass M of the exchanged object is much larger than the four momentum q:

$$\boxed{D_F = -\frac{g^{\mu\nu}}{q^2 - M^2} \simeq \frac{g^{\mu\nu}}{M^2}}$$

This produces a short-ranged interaction, as the propagator reduces to a point-like operator.

The transition amplitudes

Following the Feynman rules the following operators are obtained for the $^3S_1$ and $^3P_0$ case

$$\begin{aligned}\hat{V}(^3P_0) &= \bar{v}(p_{\bar{6}})u(p_3) = \frac{1}{2m}\chi_{\bar{6}}^{\dagger}\,\vec{\sigma}\cdot(\vec{p}_{\bar{6}}-\vec{p}_3)\,\chi_3 \\ &\quad \times\delta(\vec{p}_{\bar{6}}+\vec{p}_3+\vec{p}_1-\vec{p}_{1'})\,\delta(\vec{p}_{2'}-\vec{p}_2) \\ &\quad \times\delta(\vec{p}_{\bar{4}'}-\vec{p}_{\bar{4}})\,\delta(\vec{p}_{\bar{5}'}-\vec{p}_{\bar{5}})\end{aligned}$$

$$\begin{aligned}\hat{V}(^3S_1) &= \bar{v}(p_{\bar{6}})\gamma_\mu u(p_3)\,\frac{g^{\mu\nu}}{M_g^2}\,\bar{u}(p_{1'})\gamma_\nu u(p_1) \simeq \\ &\simeq \frac{-1}{2mM_g^2}\chi_{\bar{6}}^{\dagger}\,\chi_{1'}^{\dagger}[2\,\vec{p}_1\cdot\vec{\sigma}_{\bar{6}3}+ \\ &\quad +i(\vec{p}_1-\vec{p}_{1'})\cdot(\vec{\sigma}_{1'1}\times\vec{\sigma}_{\bar{6}3})]\,\chi_1\,\chi_3 \\ &\quad \times\delta(\vec{p}_{\bar{6}}+\vec{p}_3+\vec{p}_1-\vec{p}_{1'})\,\delta(\vec{p}_{2'}-\vec{p}_2) \\ &\quad \times\delta(\vec{p}_{\bar{4}'}-\vec{p}_{\bar{4}})\,\delta(\vec{p}_{\bar{5}'}-\vec{p}_{\bar{5}})\end{aligned}$$

where second and higher order terms in $\vec{p}/m$ are ignored and m is the (anti)quark mass. The operators are used to connect the incoming $\bar{p}p$ state with the final pion states. In order to do this, the wave functions of the quarks are needed.

• Transition Amplitudes

The Feynman diagrams yield effective operators $\hat{V}(^3P_0)$ and $\hat{V}(^3S_1)$. The annihilation potentials are then obtained from the T-matrix elements:

$$T^{SP} = \langle \phi_\pi^a \, \phi_\pi^b \, ; jm \,|\, \hat{V}(^3P_0/^3S_1) \,|\, \psi_{\bar{p}} \, \psi_p \, , lsjm \rangle$$

The total amplitude T depends on quark momenta (→ non-local), spin, isospin as well as color.

$$\Rightarrow \quad T = \sum_i^4 T_{SP}^i \times T_{SI}^i \times T_C^i$$

Including both annihilation mechanisms:

$$T_{tot.} = T(^3P_0) + \lambda\, T(^3S_1)$$

λ is relative strength → <u>only</u> parameter!

- Pion $\bar{q}q$ wave functions.

How to realize phenomenologically the confinement of (anti)quarks ?

→ Describe pions, proton and antiproton by Gaussian wave functions

→ amounts to solving Dirac equation with scalar/vector oscillator type potential.

(can be compared with MIT bag model)

→ Maruyama, Gutsche, Faessler '91)

Therefore one has for instance for the pion :

$$\phi_\pi = N_\pi \exp\left\{-\frac{\beta}{2}\sum_i(\vec{r}_i - \vec{r}_\pi)^2\right\} X_\pi(\text{spin, isospin, color})$$

$\vec{r}_i$: quark coordinates $\qquad$ $\vec{r}_\pi$: pion coordinate

$$\beta = 3.23\,\text{fm}^{-2} \Rightarrow \langle r_\pi \rangle = 0.48\,\text{fm}$$

- Relativistic Effects

In $\bar{p}p \to \pi^+\pi^-$ the kin. energy of the pions $\underline{E_{kin.} \gg m_\pi c^2}$!

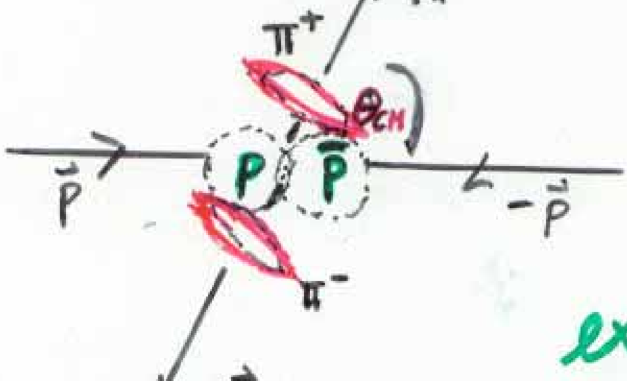
For example: $T_{lab} = 300\,\mathrm{MeV} \Rightarrow E_{CM} = \sqrt{s} \simeq 1900\,\mathrm{MeV}$

$$\Rightarrow \gamma = \frac{E_{CM}}{2m_\pi c^2} \simeq 6.9 \;!$$

$$\left(\frac{v}{c} \simeq 0.98\right)$$

Hence, the pions seen in the center-of-mass frame are far from being symmetrical spheres!

→ Distorted to Pancakes (Crêpes, Dosas ....)



⇒ Gaussians deformed.

$\exp\{-\sum_i \frac{\beta}{2}\vec{p}_i^{\,2}\} \xrightarrow{L}$

- Lorentz boosts of the pion spinors / wavefunction

Generally: helicity states transform as

$$|\vec{p},\lambda\rangle \xrightarrow{\ell} |\vec{p}\,';\lambda'\rangle = \mathcal{D}^{s}_{\lambda'\lambda}\left[r(\ell,\vec{p})\right] |\ell^{-1}\vec{p};\lambda'\rangle$$

where $\mathcal{D}_{\lambda'\lambda}$ is a $2s+1$ dim. representation of the Wick ~~helicity~~ rotations; ℓ a Lorentz transformation.

→ Translating this into the language of field operators; one finds for spinors:

$$u(\vec{p};\lambda) \xrightarrow{\ell} \mathcal{D}^{s}_{\lambda\lambda'}\, u(\ell^{-1}\vec{p};\lambda')$$

And for a transformation from the π-rest frame to the C.M. frame:

$$u(\vec{p};\lambda) \xrightarrow{\ell^{-1}} \mathcal{D}^{\frac{1}{2}}_{\frac{1}{2}\frac{1}{2}}(\theta_{\text{wick}})\, u(\ell^{-1}p;\lambda') =$$

$$= \sqrt{\frac{E+m}{4m}\left\{1+\frac{\gamma}{(\ell^{-1}p)}\left(p-\beta E\cos\delta\right)\right\}} \begin{pmatrix} \chi \\ \frac{\vec{\sigma}\cdot(\ell^{-1}\vec{p})}{E+m}\chi \end{pmatrix}$$

Partial Wave Analysis, Part II

Spin-Momentum Operators and Spherical Integrations

As for the exponential part of the new transition amplitudes, one has additional spin operators linear in the relative pion and proton-antiproton coordinates. In both the ${}^3S_1$ and ${}^3P_0$ mechanisms an additional term of the form

$$(\vec{\sigma}\cdot\hat{R}')(\vec{R}\cdot\hat{R}') = (\vec{\sigma}\cdot\hat{R}')R\cos\theta \tag{1}$$

appears times a factor that depends on the parameters α, β and γ. $\hat{R}'$ is a unit vecor along the relative pion coordinates and θ the angle between this vector and the relative proton-antiproton coordinates. Using the spherical harmonic addition theorem and $P_1(\theta) = \cos\theta$ we can rewrite this more generally.

$$(\vec{\sigma}\cdot\hat{R}')R\cos\theta = R\left(\frac{4\pi}{3}\right)^{\frac{3}{2}}\left(\sum_m \sigma_m Y_{1m}(\hat{R}')\right)\left(\sum_n Y^*_{1n}(\hat{R})Y_{1n}(\hat{R}')\right) \tag{2}$$

The sums m and m' rund from -1 to $+1$ and σ^m denotes the three components:

$$\sigma_0 = \sigma_z \qquad \sigma_1 = -\frac{\sigma_x + i\sigma_y}{\sqrt{2}} \qquad \sigma_{-1} = \frac{\sigma_x - i\sigma_y}{\sqrt{2}} \tag{3}$$

These operators will be sandwiched between the pion and the Paris $\bar{p}p$ wave functions. The proton and antiproton spin wave functions act on these spin operators and we know how to compute the matrix elements of the σ^m

$$\langle \chi^\dagger_{-m_{\bar{p}}}|\sigma_m|\chi_{m_p}\rangle \propto \left(\begin{array}{cc|c} \frac{1}{2} & 1 & \frac{1}{2} \\ -m_{\bar{p}} & m & m_p \end{array}\right) \tag{4}$$

where the right-hand side is a Clebsch-Gordan coeffcient. The S matrix elements involve integrals of the form (the subscript V refers to the vacuum ${}^3P_0$ case)

$$\begin{aligned}
&\int R^2 R'^2 \mathrm{d}R\,\mathrm{d}R'\,\mathrm{d}\Omega\,\mathrm{d}\Omega'\,\exp(AR^2+BR'^2)\,\varphi_j^{\pi\pi}(R')Y^*_{jm}(\Omega')\times \\
&\times 4\pi i\left[A_V\sqrt{\frac{4\pi}{3}}R'\sum_{m'}Y_{1m'}(\Omega')\,\sigma_{m'}\sum_{\text{odd }\lambda,\mu}Y^*_{\lambda\mu}(\Omega')Y_{\lambda\mu}(\Omega)f_\lambda(R,R')+\right. \\
&\quad + B_V\sqrt{\frac{4\pi}{3}}R\sum_{m'}Y_{1m'}(\Omega')\,\sigma_{m'}\sum_{\text{even }\lambda,\mu}Y^*_{\lambda\mu}(\Omega')Y_{\lambda\mu}(\Omega)f_\lambda(R,R')+ \\
&\quad \left. + C_V\left(\frac{4\pi}{3}\right)^{\frac{3}{2}}R\sum_{m'}Y_{1m'}(\Omega')\,\sigma_{m'}\sum_n Y^*_{1n}(\Omega)Y_{1n}(\Omega')\sum_{\text{even }\lambda,\mu}Y^*_{\lambda\mu}(\Omega')Y_{\lambda\mu}(\Omega)f_\lambda(R,R')\right] \\
&\times \sum_{lm_lm_s}\Psi_l^{\bar{p}p}(R)\,Y_{lm_l}(\Omega)\left(\begin{array}{cc|c} l & s & j \\ m_l & m_s & m \end{array}\right)\chi^\dagger_{-m_{\bar{p}}}(-)^{m_{\bar{p}}+\frac{1}{2}}\chi_{m_p}\left(\begin{array}{cc|c} \frac{1}{2} & \frac{1}{2} & s \\ m_p & m_{\bar{p}} & m_s \end{array}\right)
\end{aligned} \tag{5}$$

Results

Comparing theoretical predictions of $d\sigma/d\Omega$ and A_N with experiment — does relativity help us better understand the annihilation process ?

The observables measured by Hasan *et al.* in this reaction are differential cross sections and analyzing powers. Both observables are expressed in the two helicity amplitudes $F_{++}(\theta)$ and $F_{+-}(\theta)$, which fully describe the reaction $\bar{p}p \to \pi^+\pi^-$.

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}(|F_{++}|^2 + |F_{+-}|^2)$$
$$A_N\frac{d\sigma}{d\Omega} = \mathrm{Im}(F_{++}F^*_{+-})$$

quark model

The amplitudes depend on transition integrals

$$\Rightarrow \quad I_{Jl} = \int\int R'^2 dR' R^2 dR \boxed{\Phi_J^{\pi\pi}(R')} \boxed{T_{Jl}} \boxed{\Psi_{Jl}^{\bar{p}p}(R)}$$

where the T_{Jl} originate from the angular momentum decomposition of the annihilation potentials. As initial state $\Psi_{Jl}^{\bar{p}p}$, we choose the $\bar{p}p$ given by the 1998 Paris model.

↳ B. EL-Bennich, Loiseau, Lacombe, Vinh Mau

• Conclusion :

→ The annihilation range of the $\bar{p}p$ system is still an object of hot debate!

→ Geometric alterations in the interaction due to relativity **increases** this range by distorting pion wave functions.

→ One obtains a richer angular dependence of the annihilation potentials and therefore higher partial wave contributions

→ First fits look encouraging - implement full **relativistic change.**

W. Kloet and G. Bathas, Phys. Rev. C 47 (1993)

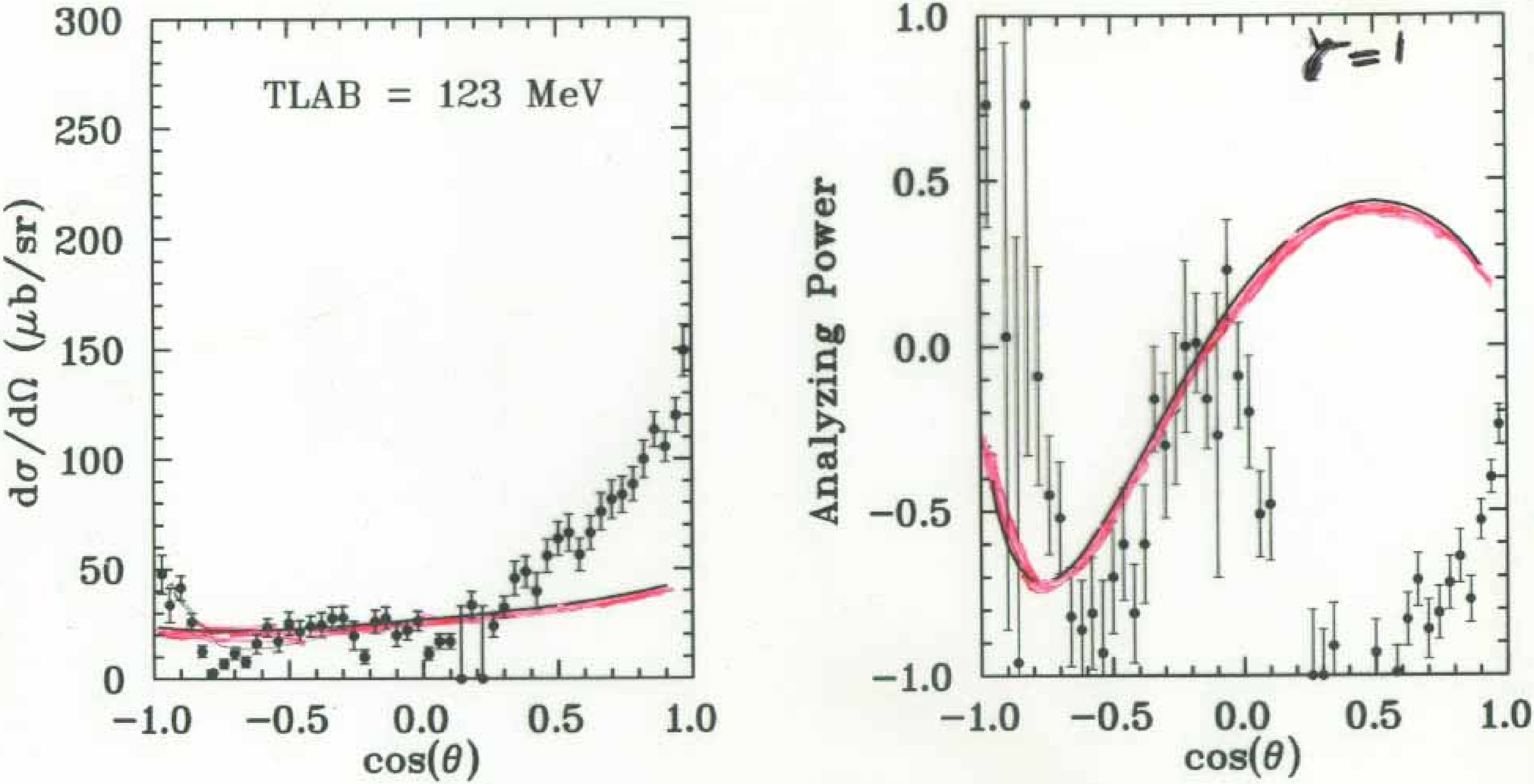


FIG. 3. P(BAR)P − P(−)P(+)

PION plane wave versus J−PION range 0 3

Result of chi−square minimization for minasymp.f

SIGNORM= 0.325E+05
LAMBDA= −0.907 PHASE= 0.

J	Eta	Delta
0	0.6638	363.0
1	1.000	184.1
2	0.8375	162.2
3	0.4919	−19.99

ALPHA = 2.800 BETA = 3.230

		χ^2	χ^2/N
Dsigma	50	905.	18.1
Analys	50	0.387E+04	77.3

b) Comme a) mais tenant compte des effets relativiste dans les fonctions d'onde (partie spatiale)

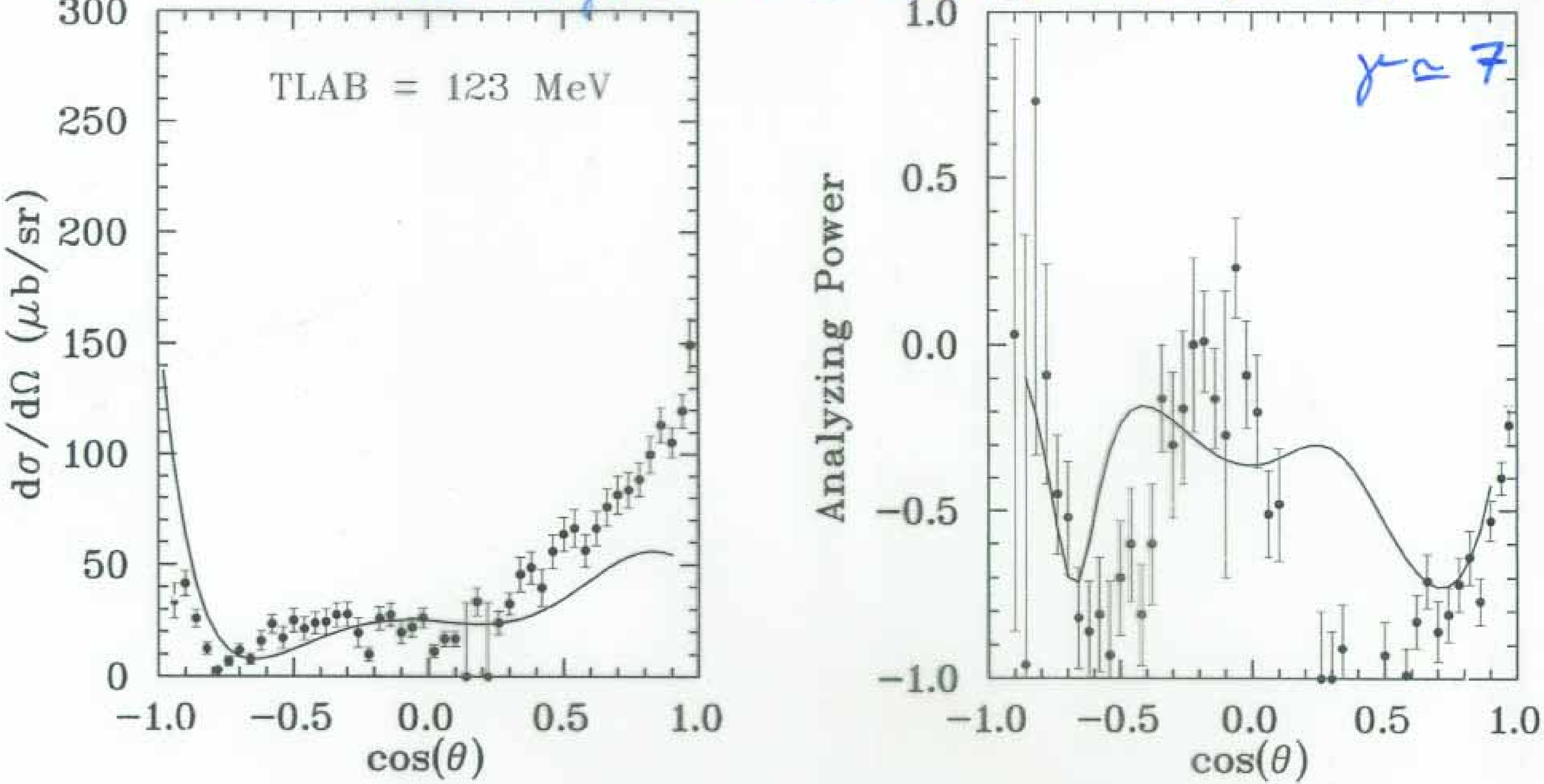


FIG. 1. P(BAR)P - P(-)P(+)

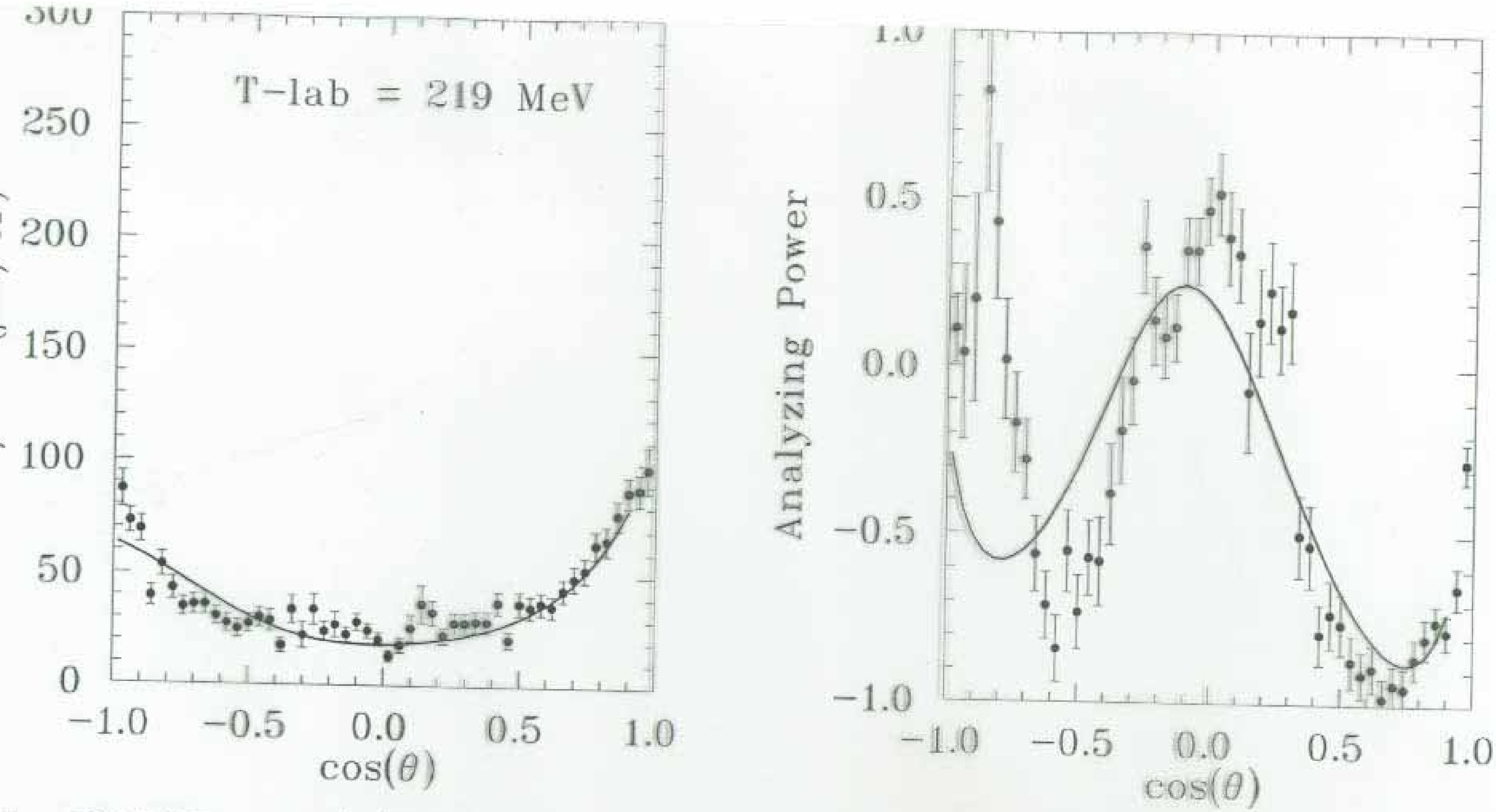
PION plane wave versus J-PION range 0 4

Result of chi-square minimization for minasymp.f

ALPHA = 2.7815 BETA = 3.9930
SIGNORM= 268585.5
LAMBDA =-0.4407 PHASE = 1.603073

J	Eta	Delta
0	0.29441	39.093
1	0.99836	115.39
2	0.95040	-14.893
3	0.77319	-0.69346
4	1.0000	0.

	N-data	Chi2	Chi2/data
Dsigma	50	792.	15.8
Analyzi	50	462.	9.25



G. P(BAR)P − PI(−)PI(+)' plane wave PI−PI versus FSI for J−range 0 4
Chi−square minimization (minasymp.f)

b = 220. Plab = 679. Wtot = 1983.
PHA = 2.9643 BETA = 3.2433
NORM= 487659.0
MBDA = −0.0738 PHASE = 5.262

	N data	Chi2	Chi2/data
gma	50	122.	2.44
alyzi	50	246.	4.91

J	Eta	Delta
0	0.8931	22.555
1	0.7446	−0.922
2	0.5740	260.460
3	0.9997	−19.964
4	0.9997	6.542

fjplpl		fjplmi	
3.342	3.345	0.000	0.000
2.174	−0.556	1.929	1.257
0.151	0.856	0.271	−1.843
0.107	−0.063	0.135	−0.054
[illegible]		[illegible]	−0.152