Hadron-Hadron Reactions in the Quark-Interchange Model

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1. Introduction
2. Quark-interchange model of Barnes & Swanson
3. Relativistic generalisation

In this workshop, many meson-baryon reaction data are presented:

$\pi p \to \eta n$,

$K p \to \pi \Lambda$, ...

Hadron-hadron reaction theories are needed

— to understand the reaction mechanism

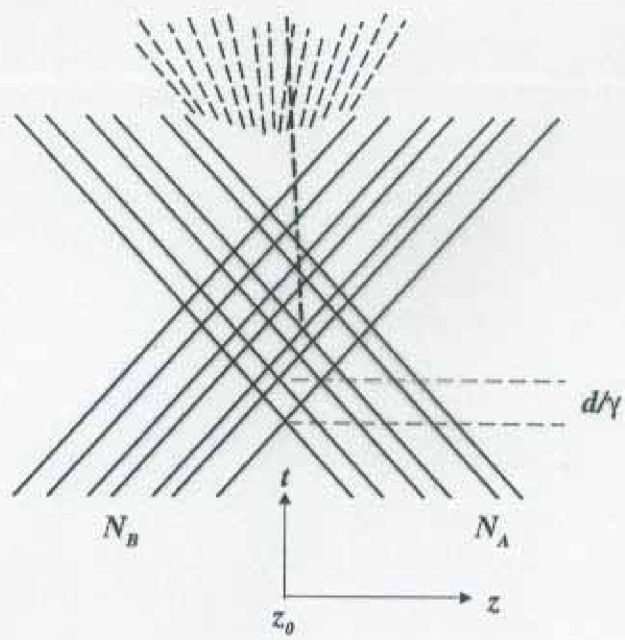
— to calculate the non-resonant part of the phase shift

— to calculate reaction cross sections for which there are no experimental data

Dissociation by collision with medium particles

$$h + J/\psi \longrightarrow (C\bar{q}) + (q\bar{C}) + X$$

J/ψ (or its precursor) is absorbed by collision with nucleons at high energies and with produced π, ρ, K, and other particles at low kinetic energies.



It is important to know the J/ψ dissociation cross section in collisions with medium particles.

Different hadron-hadron reaction theories

1. Parton model

2. Effective Lagrangian Model

3. Quark-interchange model of Barnes & Swanson

Reaction of Two Mesons

Each particle is subject to a constraint:

$$H_i = \frac{\boldsymbol{p}_i^2}{2m_i} + \frac{1}{2}\sum_{j \neq i} V_{ij}(x_{ij}), \qquad i = 1, 2, 3, 4$$

Total Hamiltonian is

$$H = \frac{\boldsymbol{p}_1^2}{2m_1} + \frac{\boldsymbol{p}_2^2}{2m_2} + \frac{\boldsymbol{p}_3^2}{2m_3} + \frac{\boldsymbol{p}_4^2}{2m_4} + V_{12} + V_{13} + V_{14} + V_{23} + V_{24} + V_{34}$$

There are two ways to split this Hamiltonian

$$H = H_0 + V_{\text{int}}$$

$$H_0(\text{prior}) = \frac{\boldsymbol{p}_1^2}{2m_1} + \frac{\boldsymbol{p}_2^2}{2m_2} + V_{12} + \frac{\boldsymbol{p}_3^2}{2m_3} + \frac{\boldsymbol{p}_4^2}{2m_4} + V_{34}$$

$$V_{\text{int}}(\text{prior}) = V_{13} + V_{14} + V_{23} + V_{24}$$

$$H_0(\text{post}) = \frac{\boldsymbol{p}_1^2}{2m_1} + \frac{\boldsymbol{p}_4^2}{2m_2} + V_{14} + \frac{\boldsymbol{p}_3^2}{2m_3} + \frac{\boldsymbol{p}_2^2}{2m_4} + V_{23}$$

$$V_{\text{int}}(\text{post}) = V_{12} + V_{13} + V_{24} + V_{34}$$

It is easy to prove that

$$\langle C(14)D(32)|V_{\text{int}}(\text{prior})|A(12)B(34)\rangle$$

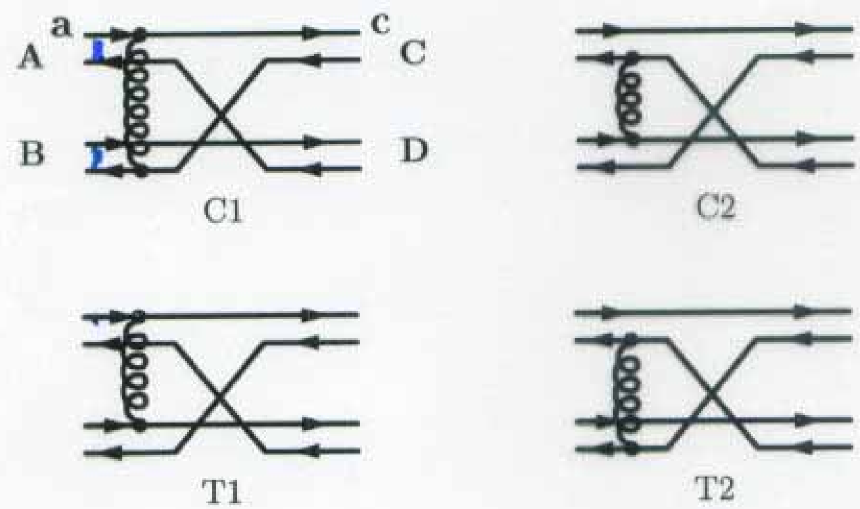
$$= \langle C(14)D(32)|V_{\text{int}}(\text{post})|A(12)B(34)\rangle$$

Barnes and Swanson Model

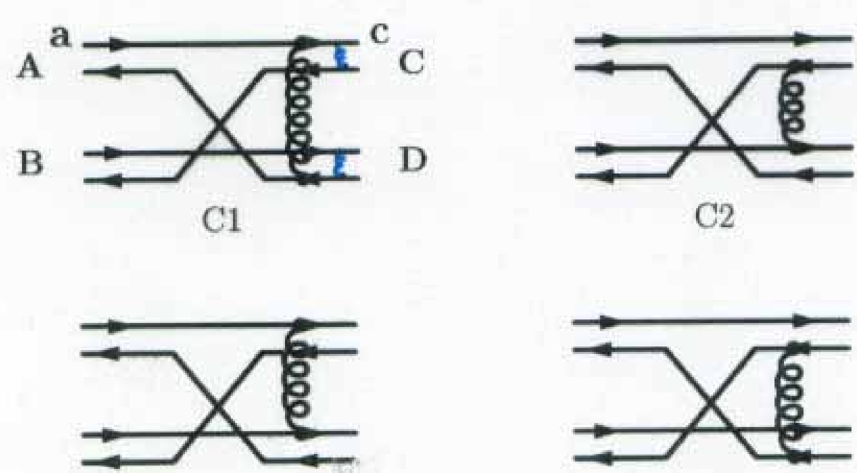
(T. Barnes and E. S. Swanson, Phys. Rev. D46, 131 (1992))

Reaction is described as a quark interchanged process after a constituent-constituent interaction.

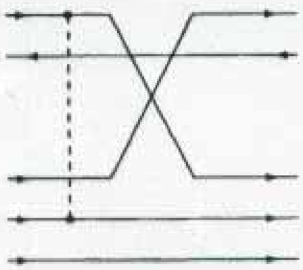
“Prior”' Diagrams included in the calculations (interaction takes place before the interchange of quarks):



There are also “‘Post” diagrams (interaction takes place after the interchange of quarks):

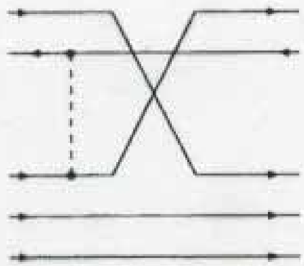


$D_2 =$



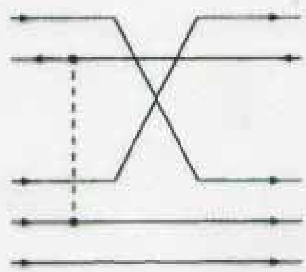
, (21)

$D_3 =$



, (22)

$D_4 =$



. (23)

The spin factor is simply the matrix element of $\vec{S}_i \cdot \vec{S}_j$ for scattering constituents i and j, evaluated between the initial and final $(q\bar{q})(qqq)$ spin states. This is 1/2 if both spins i and j are antialigned and both flip, +1/4 if the spins are aligned and neither flips, and −1/4 if they are antialigned and neither flips. All other cases give zero. All spectator spins must not flip or the overall spin factor is trivially zero.

The color factor can be evaluated using the states (9), (10) and standard trace techniques, as in (51) of Ref. [16]. The result for each diagram is

$$I_{\text{color}}(D_1) = +4/9 \ , \tag{24}$$

$$I_{\text{color}}(D_2) = -2/9 \ , \tag{25}$$

$$I_{\text{color}}(D_3) = -4/9 \ , \tag{26}$$

$$I_{\text{color}}(D_4) = +2/9 \ . \tag{27}$$

D. "Diagram weights" for KN scattering

We conventionally write the meson-baryon h_{fi} matrix elements as row vectors which display the numerical coefficient of each diagram's spatial overlap integral. Thus,

$$h_{fi} = \left[\ w_1, \ w_2, \ w_3, \ w_4 \ \right] \tag{28}$$

represents

$$h_{fi} = w_1 \, I_{\text{space}}(D_1) \ + w_2 \, I_{\text{space}}(D_2) \ + w_3 \, I_{\text{space}}(D_3) + w_4 \, I_{\text{space}}(D_4) \ . \tag{29}$$

This notation is useful because the diagram weights $\{w_i\}$ are group theoretic numbers that obey certain symmetries, whereas the spatial overlap integrals are complicated functions that depend on the specific spatial wave functions rather than the symmetries of the problem.

As an illustration, our practice subamplitude h^{eg}_{fi} is

$$h^{eg}_{fi} = \sqrt{2} \cdot \left(\frac{1}{2}\right) \cdot \left(-\frac{2}{9}\right) \cdot I_{\text{space}}(D_2) \ , \tag{30}$$

(using the spin and color matrix elements given above), which we abbreviate as

$$h^{eg}_{fi} = \left[\ 0, \ -\frac{\sqrt{2}}{9}, \ 0, \ 0 \ \right] . \tag{31}$$

This completes our detailed derivation of h^{eg}_{fi} for the sub-

The interaction $V_{ij}(r_{ij})$ for for q-$\bar{q}$ is

$$V_{ij}(r) = -\frac{\boldsymbol{\lambda}(i)}{2}\cdot\frac{\boldsymbol{\lambda}^\dagger(j)}{2}\{V_{\text{color-Coulomb}}(r) + V_{\text{linear}}(r) + V_{\text{spin-spin}}(r) + V_{\text{con}}\}$$
$$= -\frac{\boldsymbol{\lambda}(i)}{2}\cdot\frac{\boldsymbol{\lambda}^\dagger(j)}{2}\left\{\frac{\alpha_s}{r} - \frac{3b}{4}r - \frac{8\pi\alpha_s}{3m_i m_j}\boldsymbol{S}_i\cdot\boldsymbol{S}_j\left(\frac{\sigma^3}{\pi^{3/2}}\right)e^{-\sigma^2 r^2} + V_{con}\right\}.$$

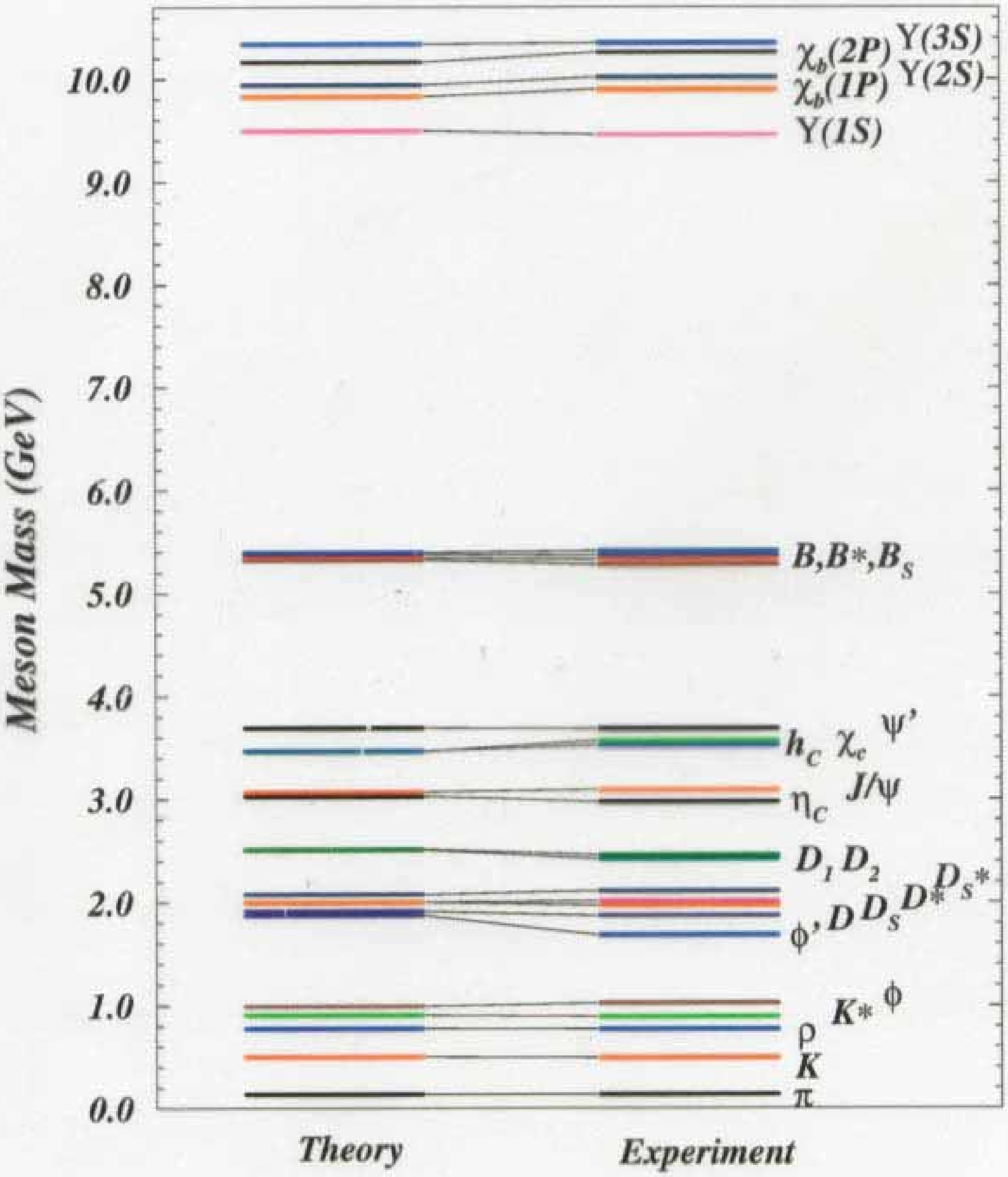
We found the following set of parameters

$$\alpha_s = \frac{12\pi}{(33-2n_f)\ln(A+Q^2/B^2)}, \quad A = 10, \quad B = 0.31 \text{ GeV},$$
$$b = 0.18 \text{ GeV}^2, \quad \sigma = 0.897 \text{ GeV}, \quad m_u = m_d = 0.334 \text{ GeV},$$

$m_s = 0.575$ GeV, $m_c = 1.776$ GeV, $m_b = 5.102$ GeV,

$V_{con} = 0.620$ GeV

and $Q^2 = s_{ij} =$ (bound state mass)^2.



Meson Mass (GeV)
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
χb(2P) Υ(3S)
χb(1P) Υ(2S)
Υ(1S)
B,B*,Bs
hc χc ψ'
ηc J/ψ
D1 D2
φ', D Ds D* Ds*
K* φ
ρ
K
π
Theory
Experiment

The differential cross section for $A(12) + B(34) \rightarrow C(14) + D(32)$ is given by

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s} \frac{\hbar^2}{|\mathbf{p}_A^*|^2} |\mathcal{M}_{fi}|^2$$

where and the matrix element $\mathcal{M}_{fi}$ is related to T_{ij} by

$$\mathcal{M}_{fi} = (2\pi)^3 \sqrt{2E_A^* 2E_B^* 2E_C^* 2E_D^*}\, T_{fi}$$

$$T_{ij} = \langle \text{final state} | V_{\text{int}} | \text{initial state} \rangle$$

$$= \langle C(14)\ D(32) | V_{\text{int}} | A(12)\ B(34) \rangle$$

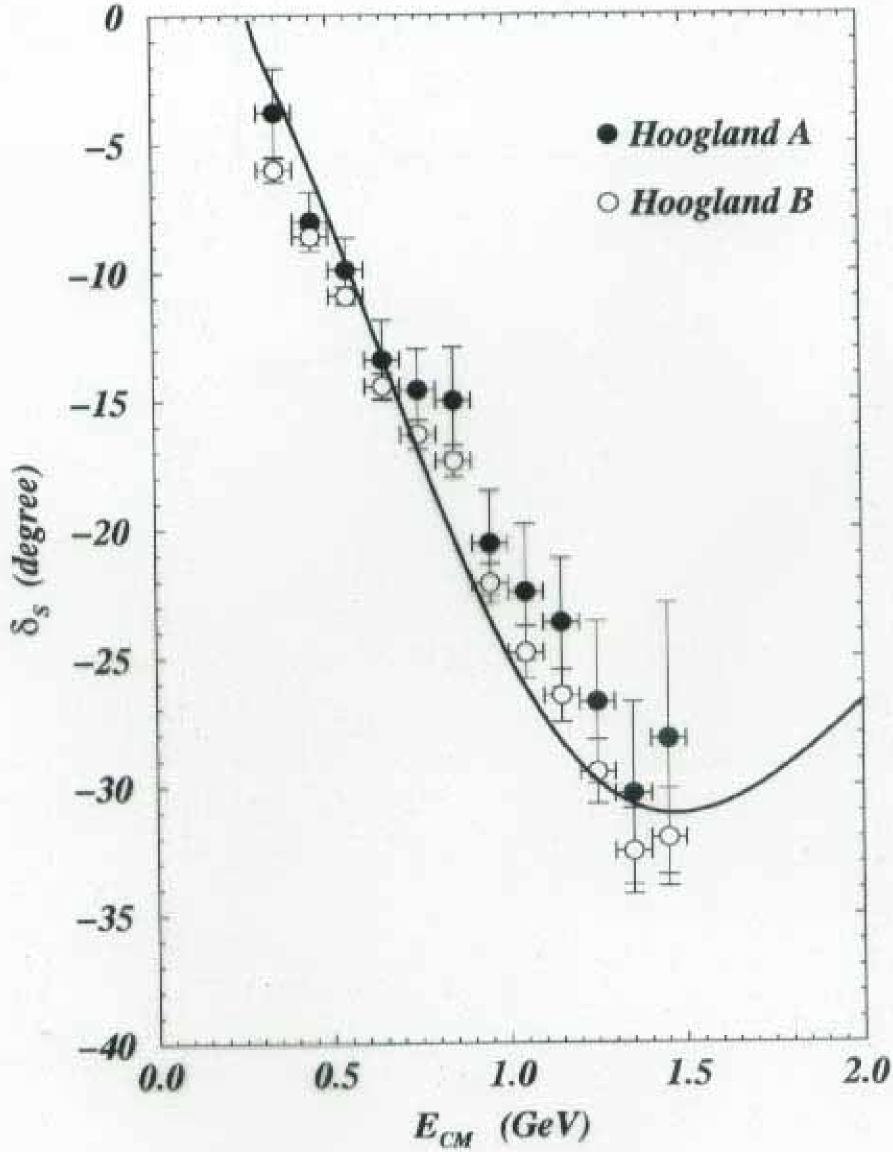
$$= I_{\text{color}} I_{\text{spin}} I_{\text{space}}$$

$$I_{\text{space}} = \int d\boldsymbol{\kappa}\, d\boldsymbol{\kappa}' \Phi_A(2\boldsymbol{\kappa} - \mathbf{A}) \Phi_B(2\boldsymbol{\kappa} - \mathbf{A} - 2\mathbf{C}) V(\boldsymbol{\kappa} - \boldsymbol{\kappa}')$$

$$\times \Phi_C(2\boldsymbol{\kappa}' - \mathbf{C}) \Phi_D(2\boldsymbol{\kappa} - \mathbf{C} - 2\mathbf{A})$$

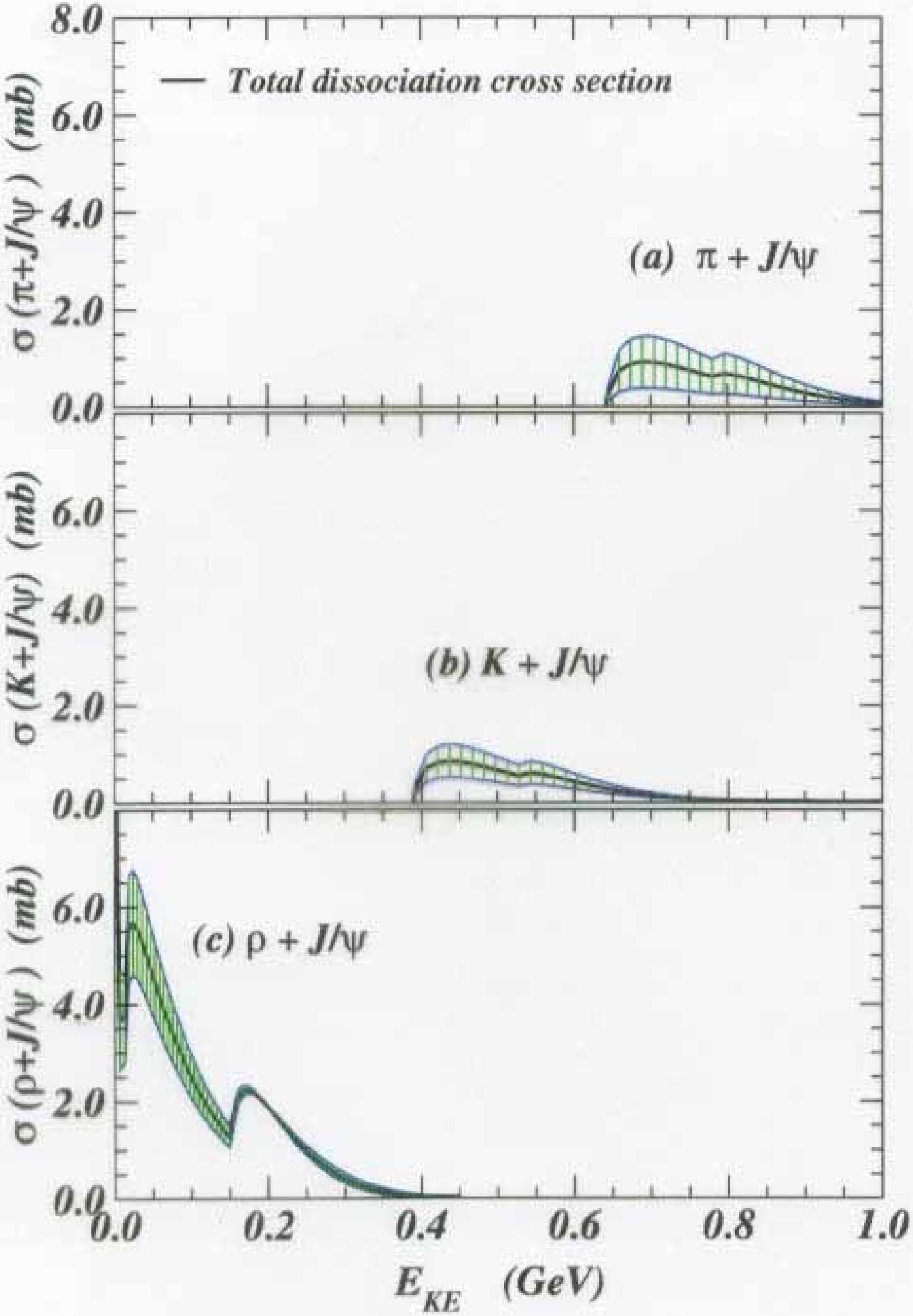
where $V(\boldsymbol{\kappa}' - \boldsymbol{\kappa})$ is the Fourier transform of $V(\mathbf{r})$

Comparison of the Barnes and Swanson Model with experimental data



Good agreement of the theoretical $I = 2$, $\pi\pi$ phase shift (solid curve) with experimental data.

J/ψ Dissociation Cross Sections at $T = 0$



Dissociation cross sections of J/ψ in collision with π, K and ρ at T =0 as a function of the kinetic energy E_{KE}

Advantages

1. The model posseses strong phenomenological basis as the interaction is obtained from the meson mass spectrum. It gives good agreement with experimental scattering data for π-π, K-π, and K-N reactions.

   C. Y. Wong, E. S. Swanson, and T. Barnes, PRC 62, 045201, 2000
   T. Barnes, E. S. Swanson, and J. Weinstein, PRD 46, 4868 (1992)
   T. Barnes and E. S. Swanson, PRC 49, 1166 (1994)

2. The model provides theoretical framework to study temperature dependence of dissociation cross sections.

Disadvantages

1. Non-relativisitc wave functions.

   Relativistic formulation initiated

   C.Y. Wong and H. Crater, Phys. Rev. **63**, 044907 (2001)

2. Chiral symmetry not part of the dynamics

Relativistic Two-body Bound State Problem

- Fully four-dimensional Bethe-Salpeter equation for a bound state is difficult to solve because of the difficulty dealing with relative time. It is necessary to reduce it to have a three-dimension reduction.

- One consistent way to reduce it is by using Dirac's constraint dynamics.

- Constraint dynamics is equivalent to solving the Bethe-Salpeter equation subject to a constraint.

  [Dirac, Todorov, Crater, van Alstine, Sazdjian, ...]

Truncation of the Bethe-Salpeter Equation

The Bethe-Salpeter equation has the symbolic form

$$\phi = K G_1 G_2 \phi$$

ϕ is the Bethe-Salpeter wave function, K the Bethe-Salpeter kernel, and $\{G_i\}$ the constituent Green function

$$G_i(p_i) = \frac{1}{p_i^2 - m_i^2 - i0}.$$

Consider two-equal mass constituents, the standard three-dimensional truncation

$$G_1 G_2 \to \pi i \delta(P \cdot p) \frac{f(p_\perp)}{p_1^2 - m_1^2 + p_2^2 - m_2^2 + i0},$$

$$\text{where } \; p = (p_1 - p_2)/2, \quad P = p_1 + p_2,$$

$$p_\perp = p - (p \cdot P) P / P^2,$$

and the replacement of the Bethe-Salpeter wave function

$$\frac{f(p_\perp)}{p_1^2 - m_1^2 + p_2^2 - m_2^2 + i0} \phi \to \psi.$$

The choice $f = 1 \quad \rightarrow \quad$ Dirac's constraint dynamics. It lead to the following truncated Bethe-Salpeter equation

$$\left[p_1^2 - m_1^2 + p_2^2 - m_2^2 + K'(x_{12\perp})\right] \psi(x_1 x_2) = 0$$

$$\left[(p_1 - p_2) \cdot (p_1 + p_2)\right] \psi(x_1 x_2) = 0$$

This set of coupled equation can be recombined to form a set of two equations as a constraint on the mass shell of each particle

$$\left[p_i^2 - m_i^2 + \Phi_{12}(x_{12\perp})\right] \psi(x_1 x_2) = 0 \quad \text{for i} = 1, 2.$$

The above set of equation is then the equation of constraint dynamics.

The relativistic N-body Hamiltonian is

$$\mathcal{H} = \sum_{i=1}^{N} \frac{1}{2m_i}(p_i^2 - m_i^2) - \sum_{i=1}^{N} \sum_{j\; j>i}^{N} \frac{\Phi_{ij}}{2\mu_{ij}}$$
$$= \sum_{i=1}^{N} \frac{1}{2m_i}(p_i^2 - m_i^2) - \sum_{i=1}^{N} \sum_{j\; j>i}^{N} V_{ij}$$

The dynamics of the relativistic N-body system is determined by the search for the state $|\psi>$ such that

$$\mathcal{H}|\psi> = \{\sum_{i=1}^{N} \frac{1}{2m_i}(p_i^2 - m_i^2) - \sum_{i=1}^{N} \sum_{j>i}^{N} V_{ij}\}|\psi> = 0.$$

Advantages

- The quadratic form of the momentum operators p_i in the N-body Hamiltonian equation makes it easy to manipulate the momentum terms to obtain the center-of-mass momentum and other relative momenta.
- The potential term in the equation appears in a way similar to that in which it appears in the non-relativistic case.

Relativistic Reaction of Two Mesons

Wong & Crater, Phys. Rev. C63, 044907 (2001)

Each particle is subject to a constraint:

$$H_i = p_i^2 - m_i^2 + \sum_{j\neq i} \Phi_{ij}(x_{ij}), \qquad i = 1,2,3,4$$

Total Hamiltonian is

$$H = \frac{p_1^2 - m_1^2}{2m_1} + \frac{p_2^2 - m_2^2}{2m_2} + \frac{p_3^2 - m_3^2}{2m_3} + \frac{p_4^2 - m_4^2}{2m_4} + V_{12} + V_{13} + V_{14} + V_{23} + V_{24} + V_{34}$$

where $V_{ij} = \Phi_{ij}(x_{ij\perp})/2\mu_{ij}$. There are two ways to split this Hamiltonian

$$H = H_0 + V_{\rm int}$$

$$H_0(\text{prior}) = \frac{p_1^2 - m_1^2}{2m_1} + \frac{p_2^2 - m_2^2}{2m_2} + V_{12} + \frac{p_3^2 - m_3^2}{2m_3} + \frac{p_4^2 - m_4^2}{2m_4} + V_{34}$$

$$V_{\rm int}(\text{prior}) = V_{13} + V_{14} + V_{23} + V_{24}$$

$$H_0(\text{post}) = \frac{p_1^2 - m_1^2}{2m_1} + \frac{p_4^2 - m_4^2}{2m_2} + V_{14} + \frac{p_3^2 - m_3^2}{2m_3} + \frac{p_2^2 - m_2^2}{2m_4} + V_{23}$$

$$V_{\rm int}(\text{post}) = V_{12} + V_{13} + V_{24} + V_{34}$$

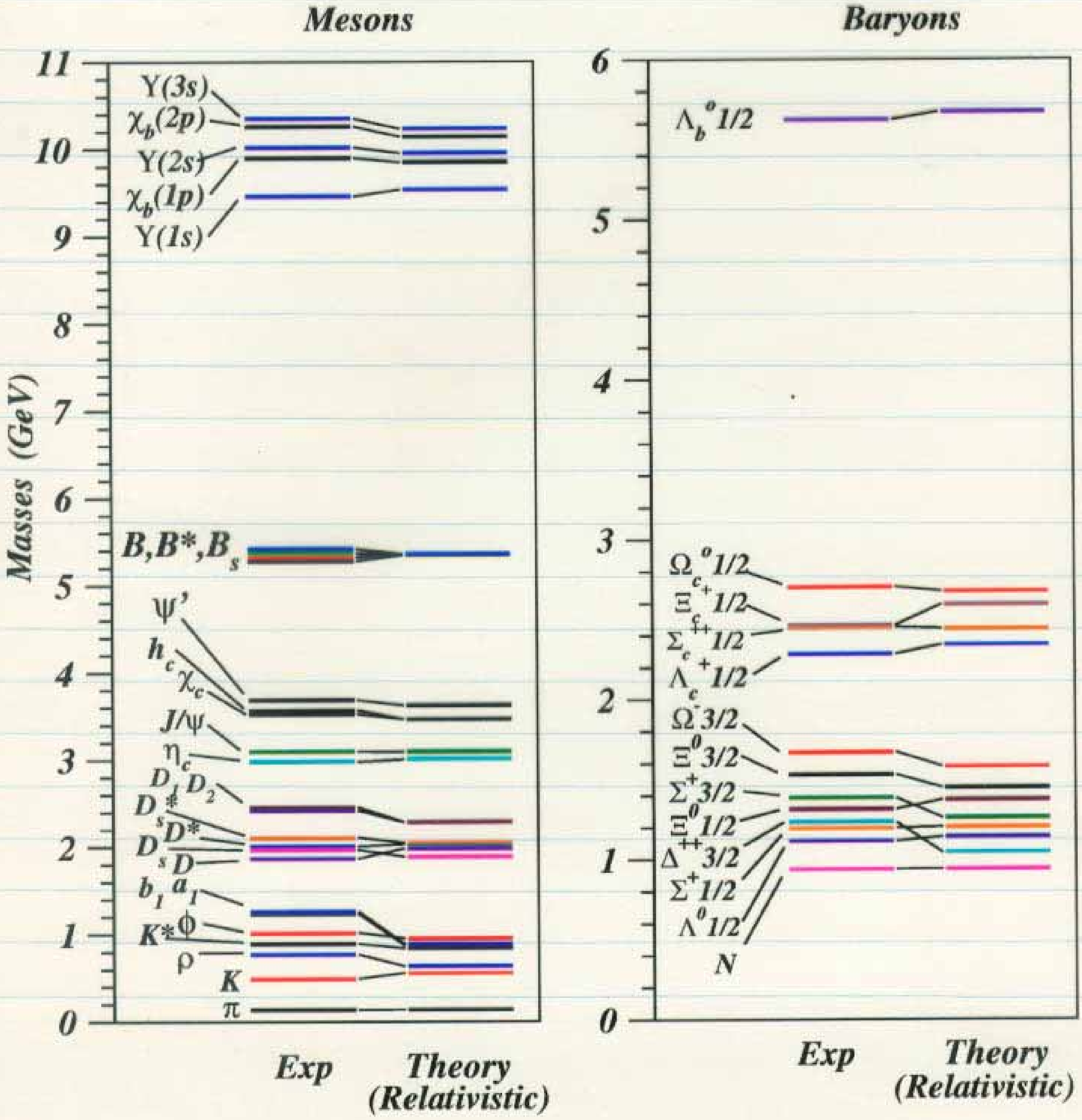
It is easy to prove that

$$\langle C(14)D(32)|V_{\rm int}(\text{prior})|A(12)B(34)\rangle$$

$$\langle C(14)D(32)|H_0(\text{prior}) + V_{\rm int}(\text{prior})|A(12)B(34)\rangle$$

$$= \langle C(14)D(32)|H_0(\text{post}) + V_{\rm int}(\text{post})|A(12)B(34)\rangle$$

$$= \langle C(14)D(32)|V_{\rm int}(\text{post})|A(12)B(34)\rangle$$


Mesons
Baryons
Masses (GeV)
Υ(3s)
χb(2p)
Υ(2s)
χb(1p)
Υ(1s)
B,B*,Bs
ψ'
hc
χc
J/ψ
ηc
D1 D2
Ds*
D*
Ds
D
b1 a1
ϕ
K*
ρ
K
π
Λb° 1/2
Ωc° 1/2
Ξc+ 1/2
Σc++ 1/2
Λc+ 1/2
Ω- 3/2
Ξ0 3/2
Σ+ 3/2
Ξ0 1/2
Δ++ 3/2
Σ+ 1/2
Λ0 1/2
N
Exp
Theory (Relativistic)