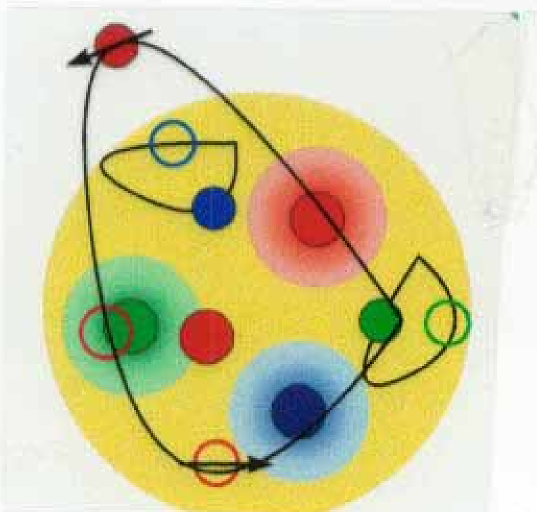
D. Drechsel

N*2002, 9-12 Oct 02

BAG DEFORMATION:
THE RATIO E2/M1 IN THE NΔ TRANSITION

- deformation
- models
- formalism
- real photons
- virtual photons
- summary



Deformation of Hadrons

- static multipole moments $\langle JJ | \mathcal{M}_{L0} | JJ \rangle$

spin J	multipole moments	comment
0	C0	$F_{\pi^+}(Q^2)$
½	C0, M1	$G_E^p(Q^2), G_M^p(Q^2)$
1	C0, M1, C2	e.g. ρ, ω
3/2	C0, M1, C2, M3	e.g. $\Delta(1232)$

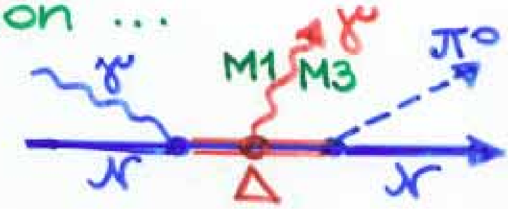
N.B.: only Coulomb (charge) and magnetic multipoles, no electric ones if time reversal invariance valid

- deformation of the Delta resonance

$$\langle \Delta_{1232}, J = J_z = \tfrac{3}{2} \,|\, \underbrace{\sum_i (3z_i^2 - \vec{r}_i^{\,2})}_{Q_{20} \text{ quadrupole operator}} \,|\, \Delta_{1232}, J = J_z = \tfrac{3}{2} \rangle$$

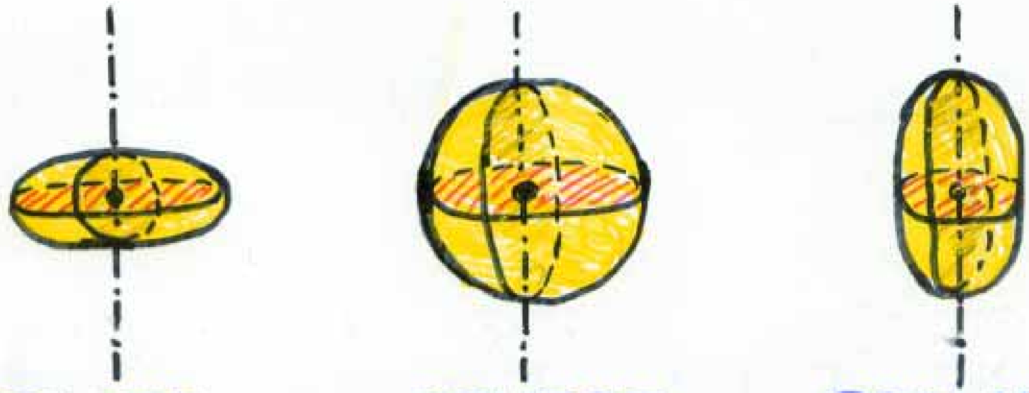
no Δ target ($T_{1/2} \sim 10^{-23}$ sec) to measure Q_{20}

no sensitive/practical reaction …



- transition matrix elements

$$\langle \Delta_{1232}, J^P = \tfrac{3}{2}^+ \,\|\, M1, E2, C2 \,\|\, N_{938}, J^P = \tfrac{1}{2}^+ \rangle$$



OBLATE $Q_{20}/R^2 < 0$ | SPHERE $Q_{20} = 0$ | PROLATE $Q_{20}/R^2 > 0$

$-0.1 \longleftarrow$ nuclei $\longrightarrow +0.3$ –superdef. ...

earth $\Rightarrow -0.0045$

deuteron $\Rightarrow 0.074$

$\Delta \Rightarrow -0.09$

$\mathcal{N} \Rightarrow 0$

Buchmann & Henley (2000)

$$Q_{20}(p \to \Delta^+) = \frac{1}{\sqrt{2}}\, r_n^2 \approx -0.08\ \mathrm{fm}^2$$

$$Q_{20}(\Delta^+) = r_n^2 < 0 \quad \text{oblate}$$

$$Q_{20}(p) = 0 \quad [\text{"intrinsic"}: > -r_n^2 > 0 \ \text{prolate}]$$

$$r_n^2 = -0.113\ \mathrm{fm}^2 \quad \text{"neutron radius"}$$

N. B.:

observable | intrinsic

$$Q_{20} = \frac{(2J-1)J}{(J+1)(2J+3)}\, Q_0$$

classical case: $J \to \infty \quad Q_{20} \Rightarrow Q_0$

quantum mechanics: $J = 0, \frac{1}{2} \Rightarrow Q_{20} = 0$

$J = 1 \Rightarrow Q_{20} = \frac{1}{10} Q_0$

$J = \frac{3}{2} \Rightarrow Q_{20} = \frac{1}{}$[illegible] Q

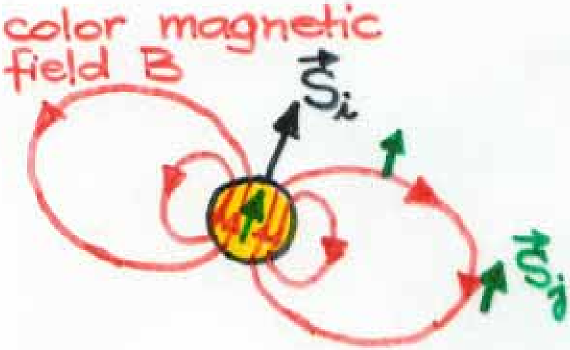
Isgur, Karl, Koniuk (1982)

TENSOR FORCE & CQM

$$H^{ij}_{hyp} = \frac{2}{3}\frac{\alpha_s}{m_i m_j}\left\{\frac{8\pi}{3}\vec{S}_i\cdot\vec{S}_j\,\delta^3(\vec{r}_{ij}) + \frac{1}{r^3_{ij}}\left[\frac{3\,\vec{S}_i\cdot\vec{r}_{ij}\;\vec{S}_j\cdot\vec{r}_{ij}}{r^2_{ij}} - \vec{S}_i\cdot\vec{S}_j\right]\right\}$$

short range repulsion for parallel spins

intermediate range tensor force



HYPERFINE INTERACTION

color magnetic dipole - magnetic dipole interaction between quarks in a baryon

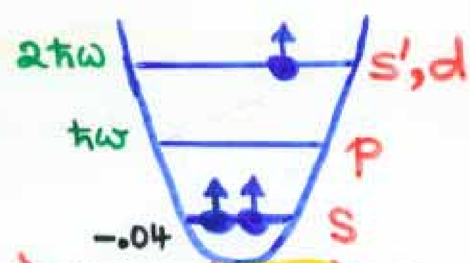
- contact term

attractive repulsive

$\rightarrow \alpha_s = \text{const} * \underbrace{(m_\Delta - m_\mathcal{N})}_{\delta}$

- tensor force

$\rightarrow$ configuration mixing



$$|\mathcal{N}\rangle = \overset{.93}{a_S}\,|^2S_\mathcal{S}\rangle + \overset{-.29}{a_{S'}}\,|^2S'_\mathcal{S}\rangle + \overset{-.23}{a_M}\,|^2S_\mathcal{M}\rangle + \overset{-.04}{a_D}\,|^4D_\mathcal{M}\rangle + \dots$$

$$|\Delta\rangle = \overset{.97}{b_S}\,|^4S_\mathcal{S}\rangle + \overset{.20}{b_{S'}}\,|^4S'_\mathcal{S}\rangle + \overset{-.10}{b_D}\,|^4D_\mathcal{S}\rangle + \overset{+.07}{b'_D}\,|^2D_\mathcal{M}\rangle + \dots$$

$\mathcal{S}$ = symmetry, $\mathcal{M}$ = mixed symmetry spin-flavor

- (intrinsic) quadrupole moments

$Q_0(\mathcal{N}) \sim a_D,\quad Q_0(\Delta) \sim b_D,$

D.D., M.M.Giannini (1984)

• ELECTRIC VS. COULOMB QUADRUPOLE

$\boxed{N \to \Delta}$

charge: $\rho_{\Delta N} \to G_{C2} = \dfrac{\overbrace{m(m_\Delta - m)\langle r^2\rangle_p}^{\delta}}{6\sqrt{5}} \left\{ b_D' a_S - b_S a_D + \dots \right\}$

current: $\vec{J}_{\Delta N} \to G_{E2} = \dfrac{1}{\sqrt{2}} \left\{ b_D' a_S + b_S a_D + \dots \right\}$ retardation & small comp.

$a_S^{(1)} \approx b_S^{(1)} = 1 + [\delta^2]$

- 1st order perturbation $a_D^{(1)} = -b_{D'}^{(1)} = -\frac{1}{8}\sqrt{\frac{3}{5}}\,\dfrac{m_\Delta - m}{\omega_0} = \dfrac{b_D^{(1)}}{\sqrt{2}}$

$\to G_{C2}^{(1)} = \frac{\sqrt{3}}{40}\left(\frac{\delta}{\omega_0}\right)^2 \approx .04 * \left(\frac{\delta}{\omega_0}\right)^2$

$G_{E2}^{(1)} = 0 + \text{h.o.t.}$

however, Siegert limit $(\vec{q} \to 0) \,\hat{=}$ no retardation, should yield $G_{E2}/G_{C2} = 1$. PROBLEM?

2nd order perturbation & up to $4\hbar\omega_0$ states $\to 55\%$
$2\hbar\omega_0$ states $\to 12\%$

G_{E2} sensitive to truncation & exchange currents

$\left.\begin{array}{l} G_{C2} \text{ more stable} \\ G_{M1} = \frac{2\sqrt{2}}{3}\mu_p = 2.83 \end{array}\right\} R = G_{C2}/G_{M1} \approx 2\%$

- quadrupole transition calculated via current operator often a 'random number", gauge invariance should be checked, in particular $G_{E2} \overset{?}{\to} G_{C2}$ if $\vec{q} \to 0$.

- for physical process, however, $G_{E2} \neq G_{C2}$, because for real photons $|\vec{q}| \approx m_\pi \neq 0$.
$\to (m_\Delta - m_N)$

$\boxed{\Delta \to \Delta}$

$Q_{20}(\Delta) = \dfrac{4\langle r^2\rangle_p}{\sqrt{\dots}} \left\{ b_S b_D + \dots \right\} \approx -\dfrac{3}{\dots}\,\dfrac{m_\Delta - m}{\dots}$

Problems, Solutions and New Problems

CQM

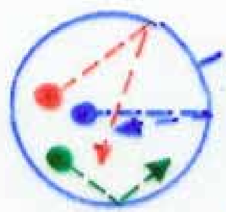
$G_{M1}(N \to \Delta) \approx 70\%$ of exp. value

$G_{C2}/G_{M1} \approx$ exp., but large dependence on parameters

v/c close to 1 $\Rightarrow$ relativ. bag model

MIT-bag

A. Chodos et al.,
T. A. De Grand et al.
(1975)



- fixed surface at $r = R$ (large!)
- interaction spin independent

m_q very small, often $m_q = 0$

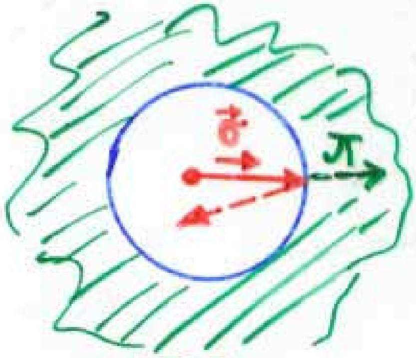
⊕ (current quark masses of $\mathcal{L}_{QCD} \approx 5-10$ MeV, while constituent quark masses $m_{const} \approx 350$ MeV)

⊕ relativity

⊖ relativistic shell model has spurious c.m. motion

→ projection techniques

⊖ approximate chiral invariance of $\mathcal{L}_{QCD}$ not fulfilled



axial charge $\sim \vec{\sigma} \cdot \vec{p}$ changes by scattering off the wall: pion cloud produced

→ Chiral Bag Models

CHIRAL BAG MODELS

S. Théberge, A.W. Thomas, G.A. Miller (1980)

G. Kälbermann, J.M. Eisenberg (1983)

K. Bermuth, D.D., L. Tiator (1988)

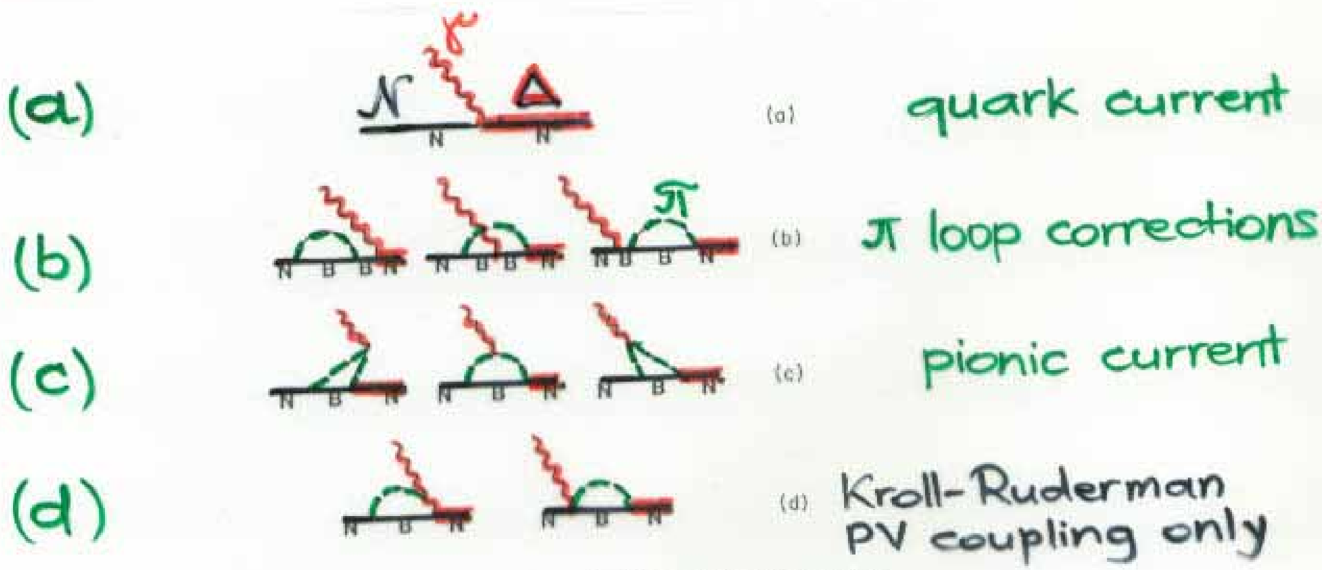
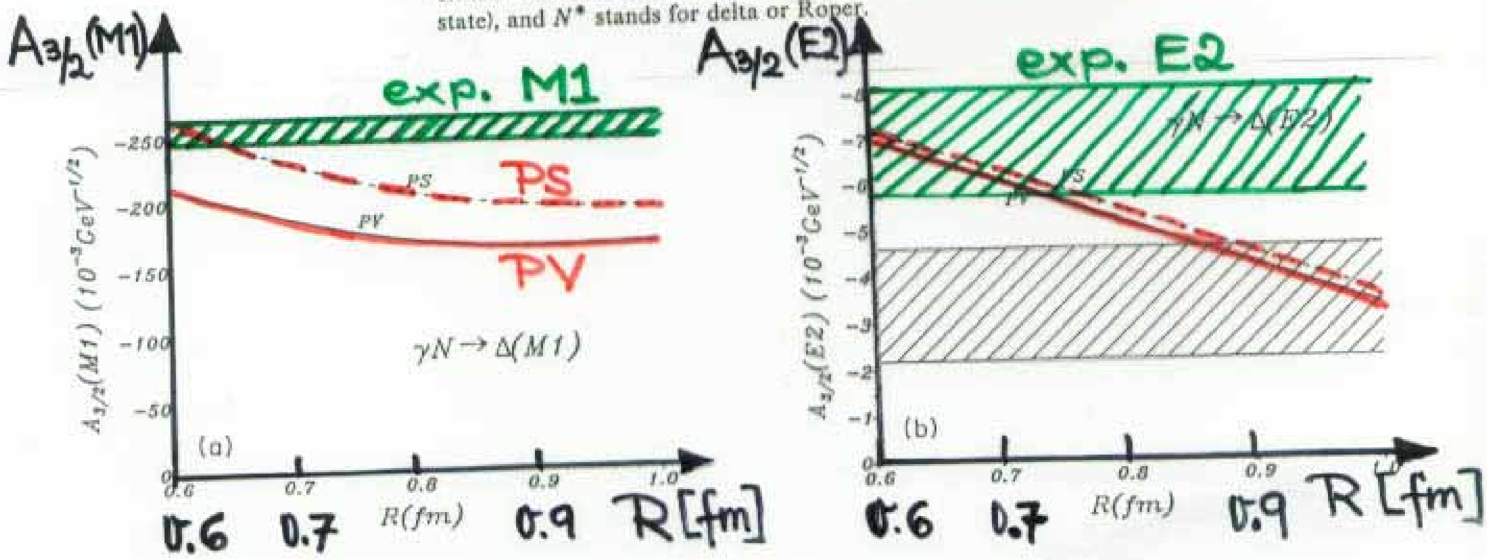


FIG. 1. Diagrams used in the calculation of the $M1$ and $E2$, C2 form factors. The intermediate states B and B' represent either nucleon, delta, Roper, or a quadrupole excitation (D state), and N^* stands for delta or Roper.



at quark bag radius $R = 0.6$ fm

good agreement with PDG (2002)

HOWEVER ...

D.H. Lu, A.W. Thomas, A.G. Williams (1997)
Y.B. Dong, K. Shimizu, A. Faessler (2001)

Lu, Dong • spurious c.m. motion ~ 5% effect
Lu • values at S matrix pole → complex amplitudes

	a	b	c	total	E2/M1
Bermuth et al.	-33	-102	-132	-267	-2.8%
Lu et al.	-28	-94	-166	-288	0.1%
Dong et al.	-113	-31	-136	-280	-2.0%
PDG (2002)				-255 ±8	-2.5% ±0.5%

TABLE: $A_{3/2}$ amplitude in units $10^{-3}\,\mathrm{GeV}^{-1/2}$, PS coupling, graphs a, b, c and total. E2/M1 ratio. Calculations for $R = 0.6$ fm.

B & L agree in terms a & b and disagree with D
B & D agree in term c and disagree with L
B & D "agree" in E2/M1 and disagree with L
B & L & D agree on

CONCLUSION:

- pion cloud is necessary to bring $A_{3/2}$ amplitude to experimental value, experimental value of $G_{M1}(N \to \Delta)$ is reached at $R \approx 0.65$ fm (quark bag radius)
- Bermuth et al. actually calculated C2/M1 = −2.8% and included intermediate D-wave resonances.

model predictions for E/M ratio

Model	E2/M1 [%]	Authors
nonrel. CQM	0	
nonrel.QM with Color-Hyperfine-Interaction	-0.32	S. S. Gershtein, G. V. Dzhikiya, *Sov. J. Nucl. Phys. 34(1981)870*
	-0.7	N. Isgur, G. Karl, R.Koniuk, *Phys. Rev. D 25(1982)2394*
CQM with CHI	-2	D. Drechsel, M. M. Giannini, *Phys. Lett. B 143(1984)329*
nonrel. QM with CHI and pion exchange	-1	M. Weyrauch, H. J.Weber, *Phys. Lett. B 171(1986)13*
Chiral Bag-Model	-0.92	G. Kälbermann, J. M. Eisen-berg, *Phys. Rev. D 28(1983)71*
Cloudy Bag-Model	-1.5	K. Bermuth, D. Drechsel, L. Tiator, *Phys. Rev. D 37(1988)89*
rel. QM	-0.2 to -0.1	J. Bienkowska, Z. Dziembowski, H. J. Weber, *Phys. Rev. Lett. 59(1987)624*
modified Skyrme Model	-5 to -2	A. Wirzba, W. Weise, *Phys. Lett. B 188(1987)6*
two-body-exchange-currents	-2.5	A.J. Buchmann, E. Hernandez, U.Meyer, A. Faessler, *Phys. Rev. C 58(1998)2478*

LATTICE CALCULATIONS

D.B. Leinweber et al. (1993)

C. Alexandrou et al. (2002) Nikosia - Geneva - Wuppertal - Athens - MIT

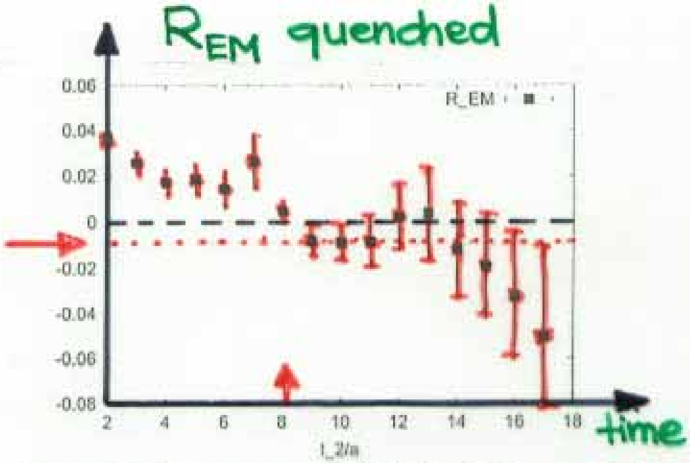


Figure 2. R_{EM} at $\kappa = 0.1558$ for 100 quenched confs. The photon is injected at $t_1/a = 8$.

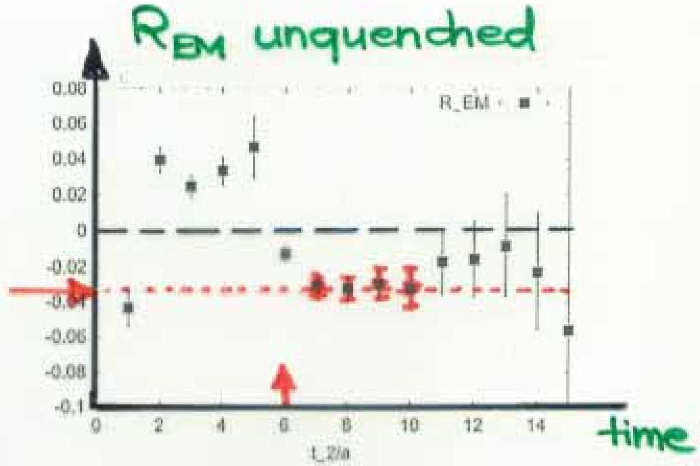


Figure 3. R_{EM} ratio for 100 SESAM confs. at $\kappa = 0.1570$. The photon is injected at $t_1/a = 6$.

est possible momentum transfer for this lattice, $q^2 \sim 0.53\text{GeV}^2$ (taking $a^{-1} = 1.85$ GeV from the chiral extrapolation of the nucleon mass).

κ_{sea}	m_π/m_ρ	$R_{EM}(\%)$	$R_{SM}(\%)$
0.1560	0.83	-2.24 ± 0.46	
0.1565	0.81	-2.25 ± 0.55	
0.1570	0.76	-3.40 ± 0.61	-3.2 ± 2.1
0.1575	0.69	-2.98 ± 0.90	

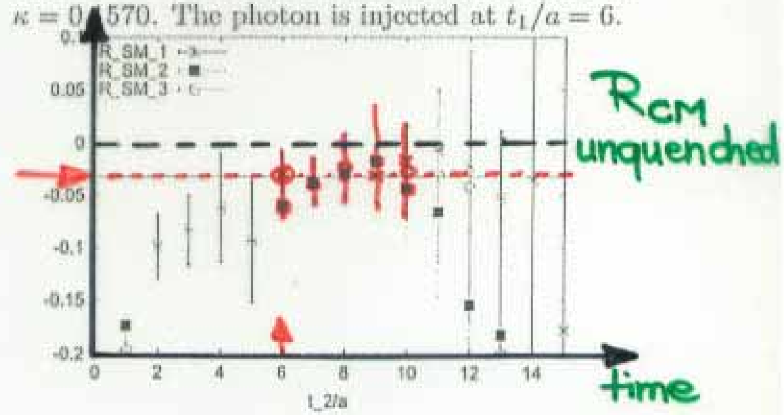


Figure 4. R_{SM} plateaus for 100 $\kappa = 0.1570$ SESAM lattices. The three equivalent definitions are consistent within the errors. $t_1/a = 6$.

- 3-point correlation fcts. of current between interpol. N & Δ fields
- value of $R_{EM} = E2/M1$ and $R_{CM} = C2/M1$ observed for 100 quenched or unquenched configurations, as function of time
- photon injected at a certain time ↑
- plateau interpreted as R_{EM} or R_{CM} →

SUMMARY:

R_{EM} (quenched) = (-0.9 ± 0.8) % $\quad Q^2 = 0.14$ GeV^2

R_{EM} (unquenched) = (-3.40 ± 0.61) %

R_{CM} (unquenched) = (-3.2 ± 2.1) %

$\Big\} \; Q^2 = 0.53$ GeV^2

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D. B. Leinweber et al. (1993)

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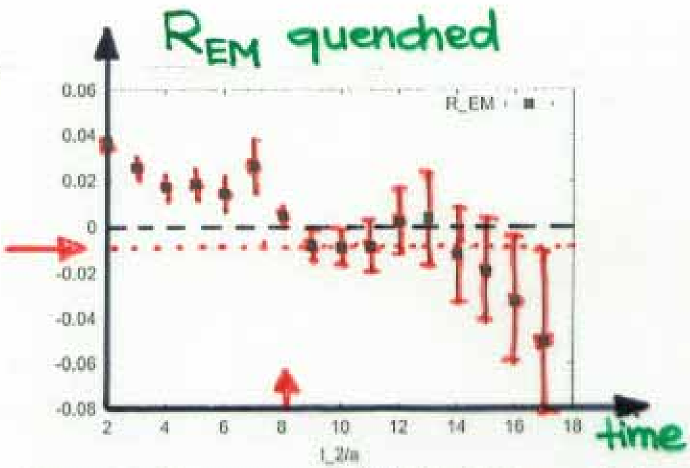

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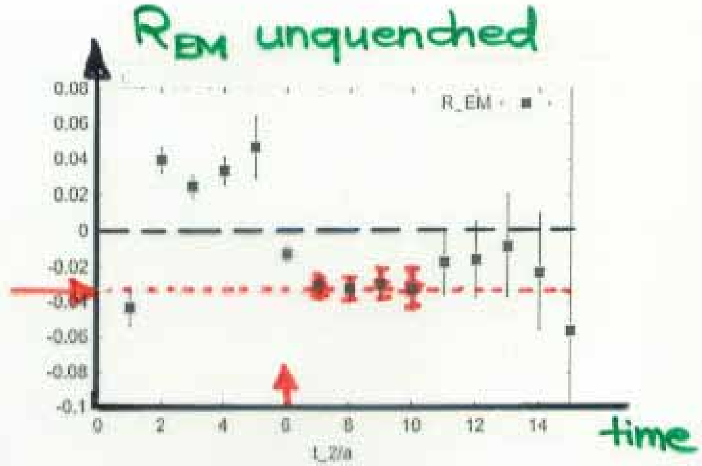

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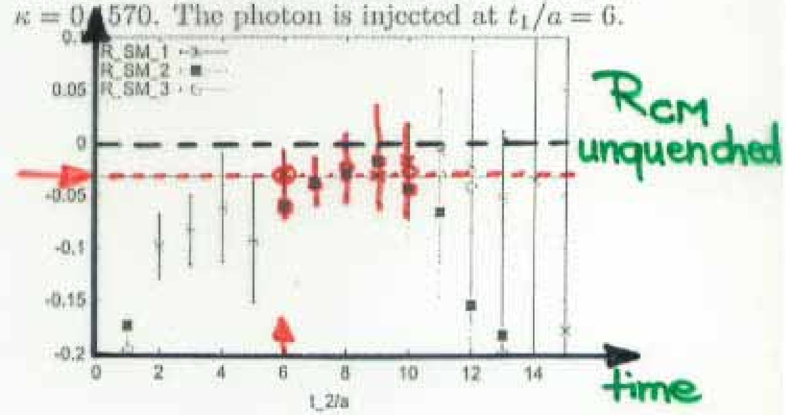

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$\Big\}$ $Q^2 = 0.53$ GeV^2

CHIRAL PERTURBATION THEORY

G.C. Gellas, T.R. Hemmert, C.N. Ktorides, G.I. Poulis (1999)

- ε^3 "SSE", 3 LEC's fitted to G_{M1}, G_{E2}, G_{C2}
- f.f. complex @ S matrix pole

Re G_{C2}

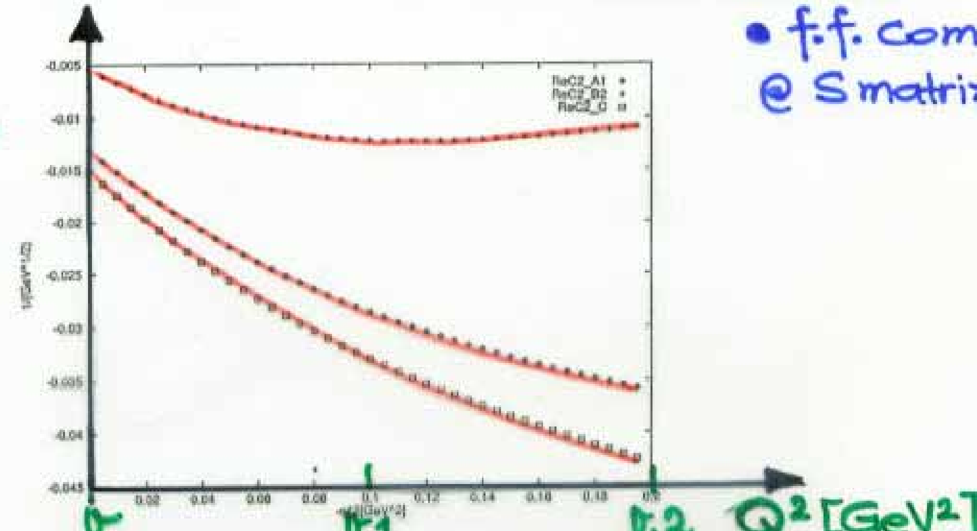


FIG. 7. The real part of the form factor $C2$ with C_2, C_3 given by the sets A_1, B_2, C.

$R_{CM} = \mathrm{Re}\left(\frac{G_{C2}}{G_{M1}}\right)$

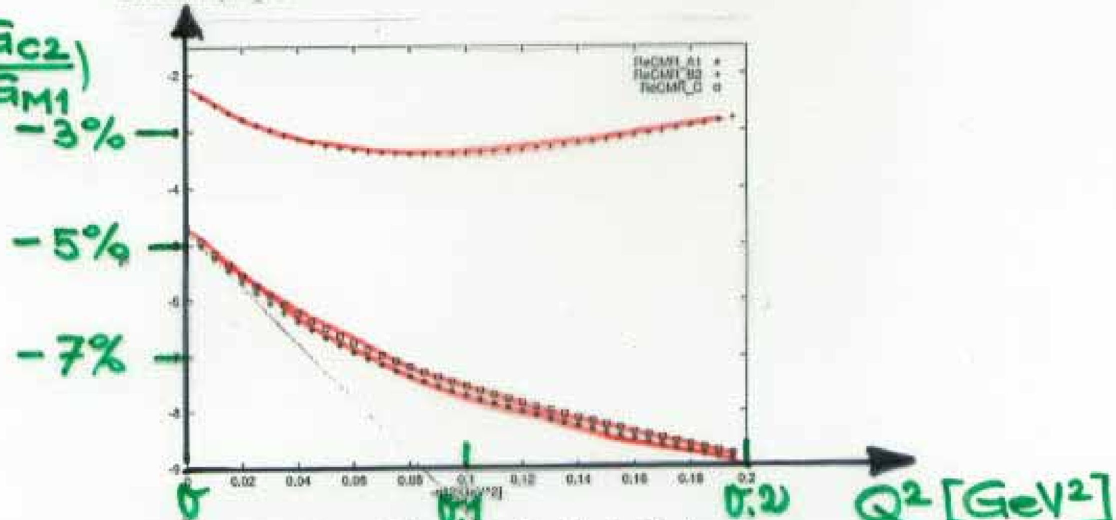
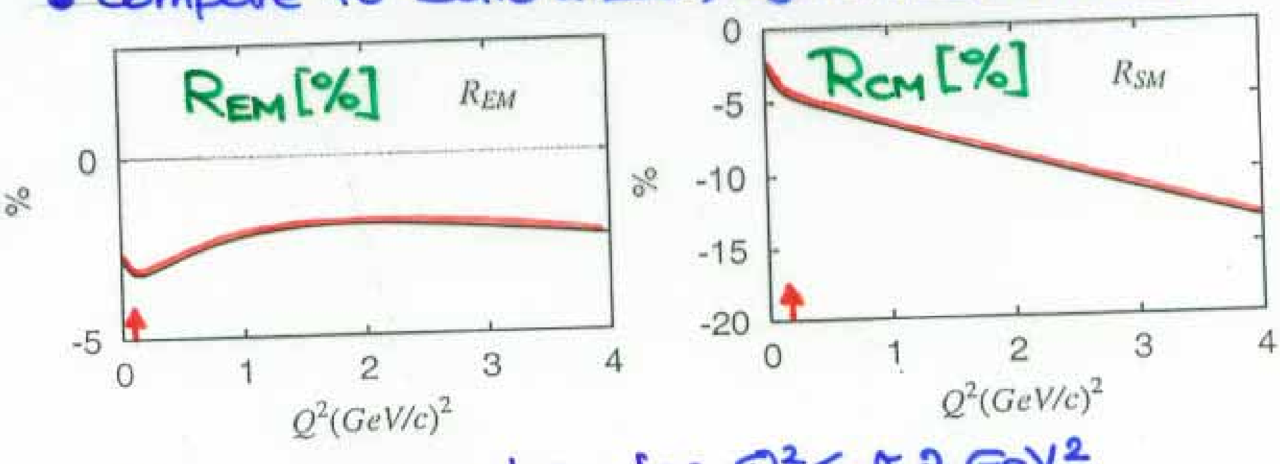


FIG. 11. The real part of the CMR ratio with C_2, C_3 given by the sets A_1, B_2, C.

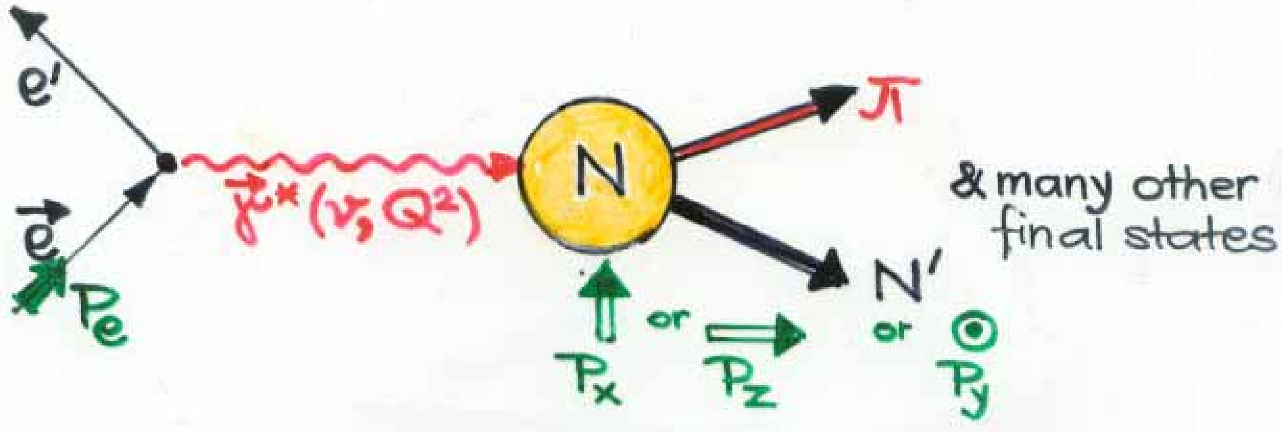
- compare to Sato & Lee, dynamical model



steep negative slope for $Q^2 \lesssim 0.2$ GeV^2 as in ChPT

E2 and C2 Q^2 dependence relative to value

VIRTUAL PHOTONS



$$\frac{d^5\sigma}{d\Omega_{e'}\,dE_{e'}\,d\Omega_\pi} = \Gamma_V \frac{d\sigma^V}{d\Omega_\pi}$$

$$\frac{d\sigma^V}{d\Omega_\pi} = \frac{d\sigma_T}{d\Omega_\pi} + \varepsilon\frac{d\sigma_L}{d\Omega_\pi} + \varepsilon\frac{d\sigma_{TT}}{d\Omega_\pi}\cos 2\phi_\pi + \sqrt{2\varepsilon(\varepsilon+1)}\,\frac{d\sigma_{LT}}{d\Omega_\pi}\cos\phi_\pi$$

+ 14 other structures involving $\vec{P}_e$ and $\vec{P}_N$

- super "Rosenbluth plot" by varying $\varepsilon, \phi_\pi, \vec{P}_e, \vec{P}_N$
- $\frac{d\sigma_i}{d\Omega_\pi} = f_i(\nu, Q^2, \theta_\pi)$ → angular distribution yields multipoles $\mathcal{M}^I_{\ell\pm}(\nu, Q^2)$
- special cases :

REAL PHOTON $Q^2 = 0$, only Transverse polarization

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^0}{d\Omega}\left\{1 - P_\gamma\,\Sigma\,\cos 2\phi\right\}$$

photon asymmetry

in plane → 1
⊥ plane → −1

INCLUSIVE ELECTROPRODUCTION, $\int d\Omega_\pi$

$$\sigma^V = \sigma_T + \varepsilon\,\sigma_L + P_e P_x\sqrt{2\varepsilon(1-\varepsilon)}\,\sigma'_{LT} + P_e P_z\sqrt{1-\varepsilon^2}\,\sigma'_{TT}$$

$\sigma_i(\nu, Q^2)$ ⟷ quark structure functions $\{F_1, F_2, g_1, g_2\}$

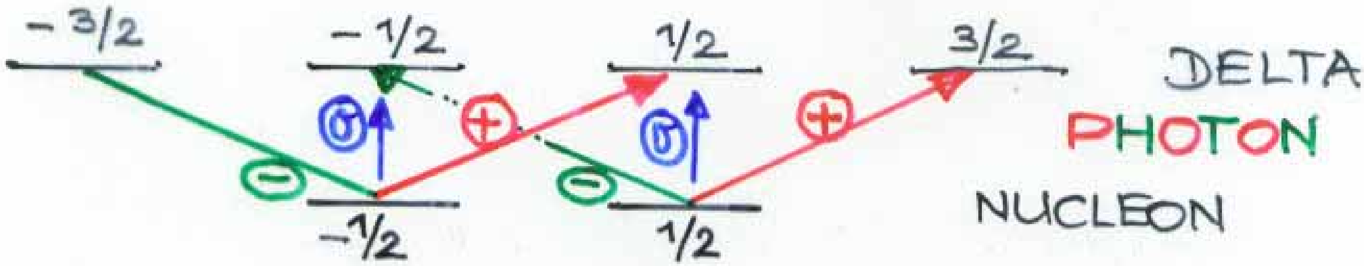
- **HELICITY AMPLITUDES** $\{A_{1/2}, A_{3/2}, S_{1/2}\}$

$$\vec{J}\cdot\hat{k} = (\vec{L}+\vec{S})\cdot\hat{k} = \vec{S}\cdot\hat{k}$$

because $\vec{L}\cdot\hat{k} = (\vec{r}\times\vec{k})\cdot\hat{k} = 0$

$\vec{S}\cdot\hat{k}$ helicity $h = \begin{cases} \pm 1 & \text{real photon} \\ \pm 1 \ \& \ 0 & \text{virtual photon} \\ \pm\frac{1}{2} & \text{proton} \end{cases}$

- invariant under rotations
- equivalent with 'S_z' for collinear process if "z-axis" in direction of motion



- under parity transformation

$$h = \vec{S}\cdot\hat{k} \rightarrow \vec{S}\cdot(-\hat{k}) = -h$$

$$\langle \Delta(S_z') | \gamma(\lambda) | N(S_z) \rangle \stackrel{!}{=} \langle \Delta(-S_z') | \gamma(-\lambda) | N(-S_z) \rangle$$

$\rightarrow$ only 2 independent "helicity amplitudes" for <u>real</u> photon

$$A_{3/2} = \langle \Delta(\tfrac{3}{2}) | \mathcal{H}^{(+)} | N(\tfrac{1}{2}) \rangle$$

! NOT POSSIBLE ON SINGLE QUARK

$$A_{1/2} = \langle \Delta(\tfrac{1}{2}) | \mathcal{H}^{(+)} | N(-\tfrac{1}{2}) \rangle$$

for <u>virtual</u> photon

• MULTIPOLES $\{E_{\ell\pm}, M_{\ell\pm}, S_{\ell\pm}\}$

[i] $\gamma^* \mathcal{N}$: electric (**E**), magnetic (**M**), charge (**S**);
L includes relative orbital angular momentum + photon spin

$$\rightarrow J = L \pm \tfrac{1}{2}$$
$$\mathcal{P} = \begin{cases} (-)^{L} & \text{for } E, S \\ (-)^{L+1} & \text{for } M \end{cases}$$

[f] $\pi \mathcal{N}$: ℓ relative orbital angular momentum

$$\rightarrow J = \ell \pm \tfrac{1}{2}$$
$$\mathcal{P} = (-)^{\ell+1}$$

<u>examples:</u>

$\boxed{M_{1+}}$: magnetic, $\ell = 1$, $J = \ell + \frac{1}{2} = \frac{3}{2}$

from [f] $\rightarrow \mathcal{P} = (-)^{\ell+1} = +1$

resonance $\frac{3}{2}^{+}$

from [i] $\rightarrow L = J \pm \frac{1}{2} = \begin{cases} \cancel{2} \\ 1 \end{cases}$

$$\rightarrow \mathcal{P} = (-)^{L+1} = \begin{cases} \cancel{-1} & \rightarrow L = 2 \\ +1 & \rightarrow L = 1 \ ✓ \end{cases}$$

M1 - magnetic dipole radiation

CROSS SECTIONS
HELICITY AMPLITUDES
E/M/S MULTIPOLES

def. $\sigma_T = \frac{1}{2}(\sigma_{3/2} + \sigma_{1/2})$, $\sigma'_{TT} = \frac{1}{2}(\sigma_{3/2} - \sigma_{1/2})$

$$\sqrt{\sigma_{1/2}} \sim A_{1/2} \sim \{(\ell+2)E_{\ell+} + \ell M_{\ell+}\} \quad \text{or} \quad \{(\ell-1)E_{\ell-} - (\ell+1)M_{\ell-}\}$$

$$\sqrt{\sigma_{3/2}} \sim A_{3/2} \sim \sqrt{\ell(\ell+2)}\{E_{\ell+} - M_{\ell+}\} \quad \text{or} \quad \sqrt{(\ell-1)(\ell+1)}\{E_{\ell-} + M_{\ell-}\}$$

$$\sqrt{\sigma_L} \sim S_{1/2} \sim (\ell+1)^3 S_{\ell+} \quad \text{or} \quad \ell^3 S_{\ell-}$$

$$\sigma'_{LT} \sim S^*_{1/2} A_{1/2} \sim \ldots$$ (relative sign!)

for the Δ resonance:

kin. factor — $\Im m\, M_{1+}(W = m_\Delta)$

$$A_{1/2} = -\frac{a}{\sqrt{6}}(\bar{M}_{1+} + 3\bar{E}_{1+})$$

$$A_{3/2} = -\frac{a}{\sqrt{2}}(\bar{M}_{1+} - \bar{E}_{1+})$$

$$S_{1/2} = -\frac{2a}{\sqrt{3}}\bar{S}_{1+}$$

kin. factor — ! — !

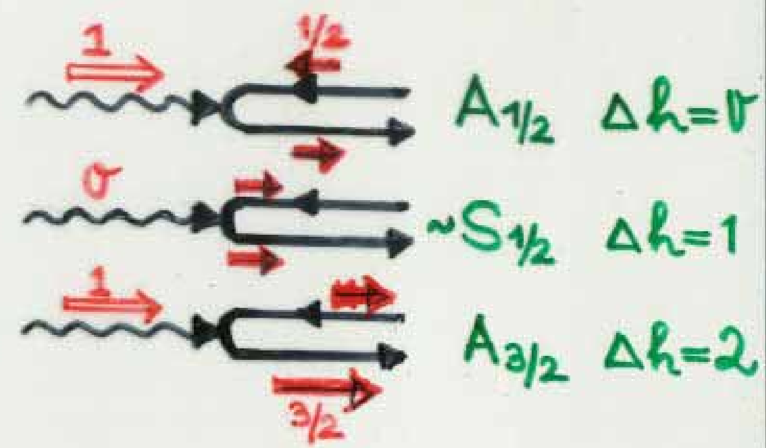
$$G_{M1} = b\bar{M}_{1+}, \quad G_{E2} = -b\bar{E}_{1+}, \quad G_{C2} = -b\bar{S}_{1+}$$

$$\boxed{\begin{aligned} R_{EM} &= -\frac{G_{E2}}{G_{M1}} = \frac{A_{1/2} - \frac{1}{\sqrt{3}}A_{3/2}}{A_{1/2} + \sqrt{3}A_{3/2}} = \frac{\bar{E}_{1+}}{\bar{M}_{1+}} \\ R_{SM} &= -\frac{G_{C2}}{G_{M1}} = \frac{\sqrt{2}S_{1/2}}{A_{1/2} + \sqrt{3}A_{3/2}} \end{aligned}}$$

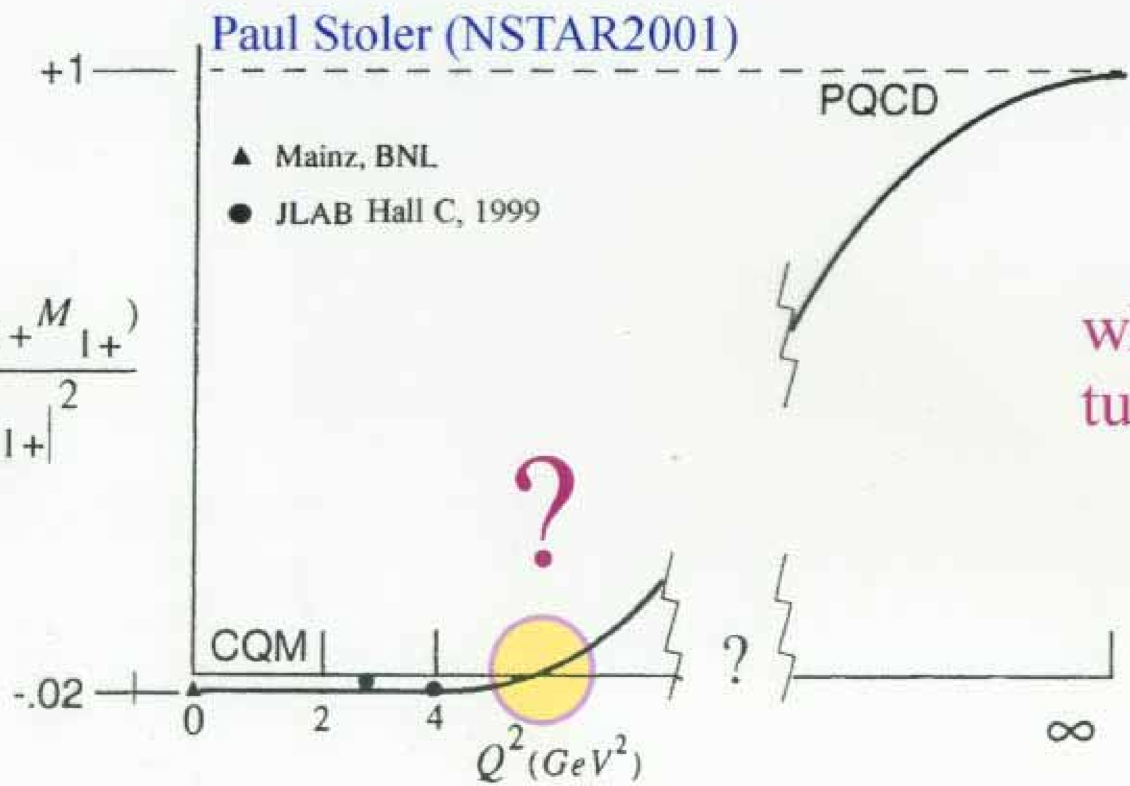
- $A_{3/2}$ not possible on individual quark, needs correlations $\rightsquigarrow$ $R_{EM} \to 100\%$ if $Q^2 \to \infty$

$$A_{3/2} \xrightarrow{Q^2 \to \infty} \frac{1}{Q^2} A_{1/2}$$

$$R_{EM} = \frac{A_{1/2} - \frac{1}{\sqrt{3}} A_{3/2}}{A_{1/2} + \sqrt{3} A_{3/2}} \xrightarrow{Q^2 \to \infty} +1$$

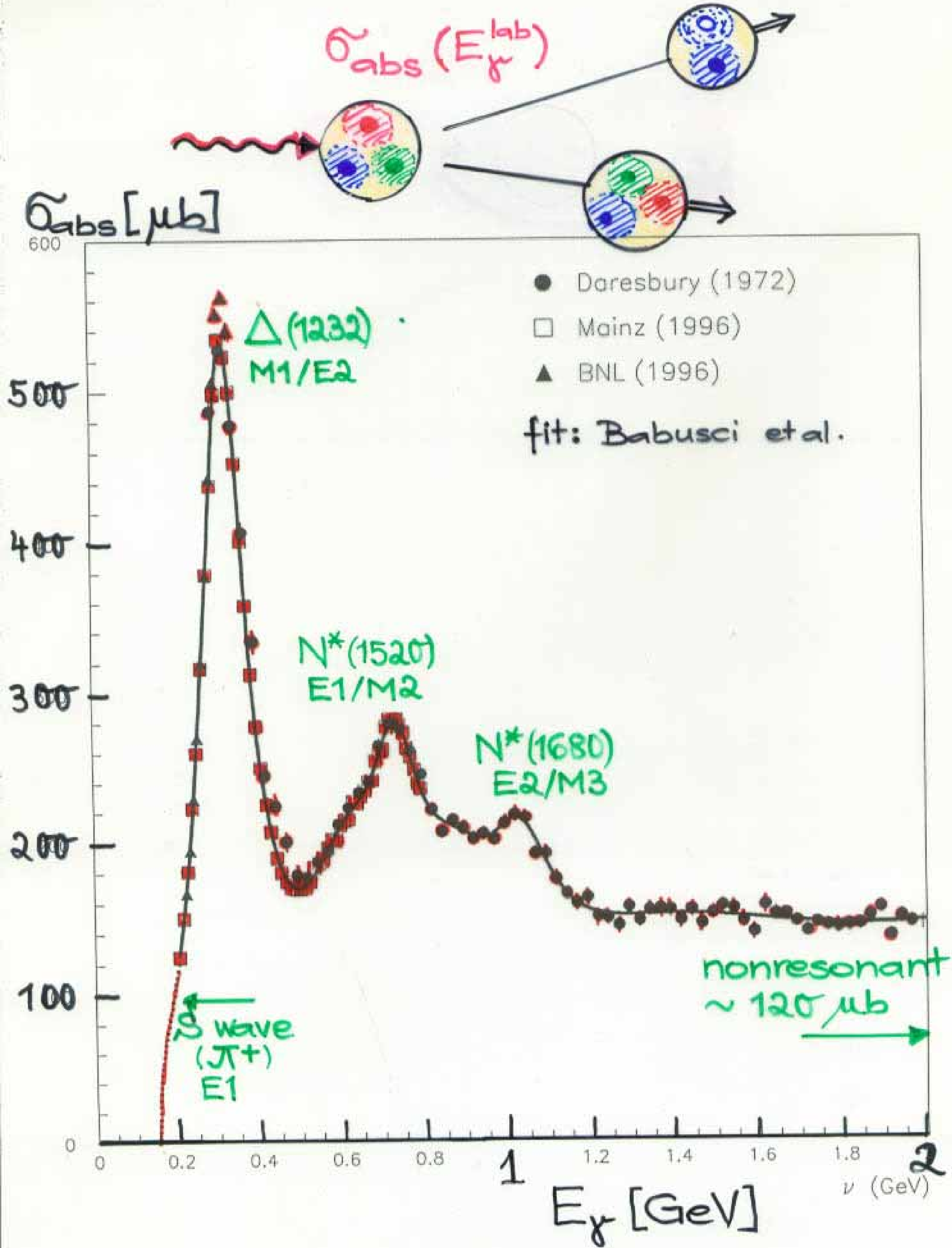


$$R_{EM} = \frac{Re(E_{1+} M_{1+}^{*})}{|M_{1+}|^2}$$



where is the turning (crossing) point?

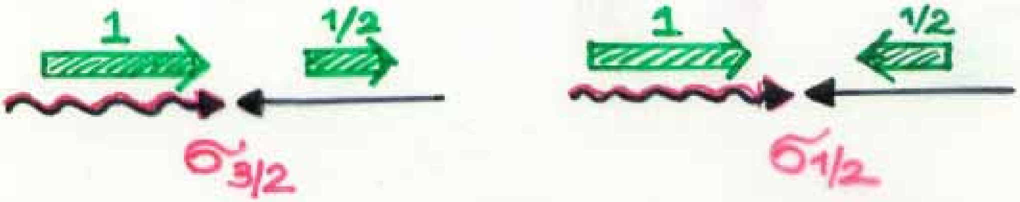
TOTAL PHOTOABSORPTION

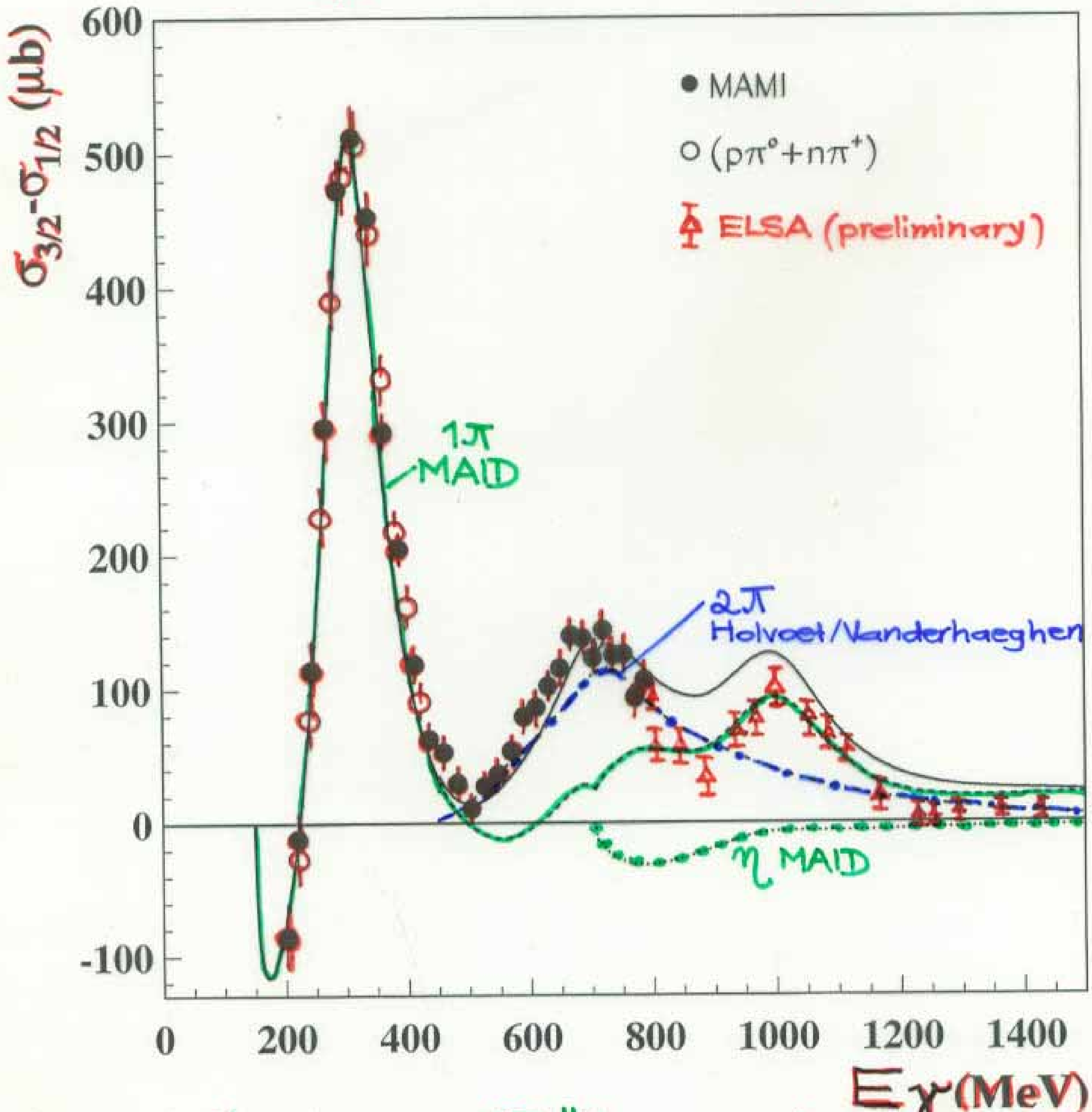


HELICITY DIFFERENCE

$\sigma_{3/2} - \sigma_{1/2}$

N.B.: $\sigma_{abs} = \frac{\sigma_{3/2} + \sigma_{1/2}}{2}$





S-wave threshold

Δ(1232)

2π thr.

η thr

N*(1520)

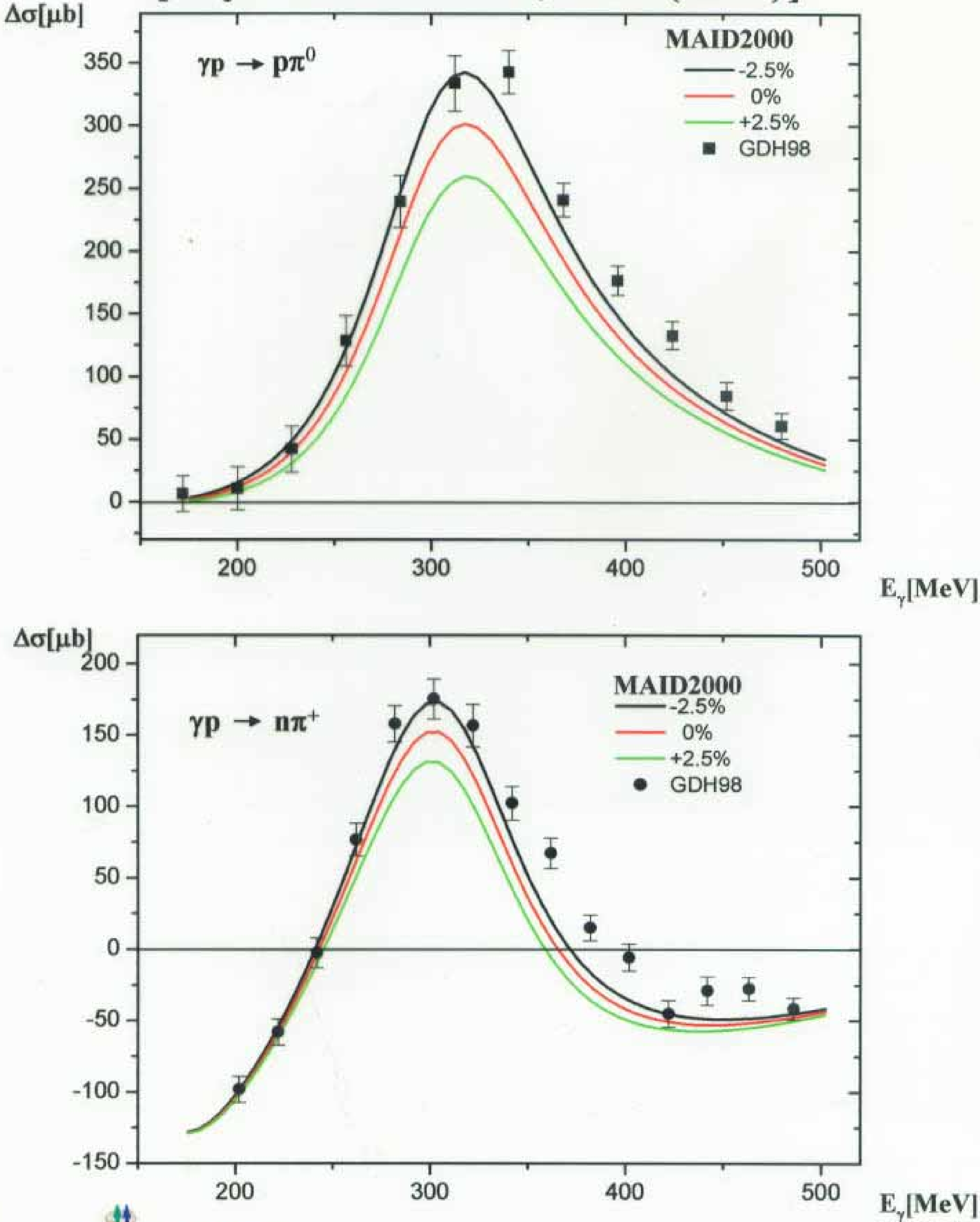
3rd res. reg.

N*(1680)

cross-over to neg. values

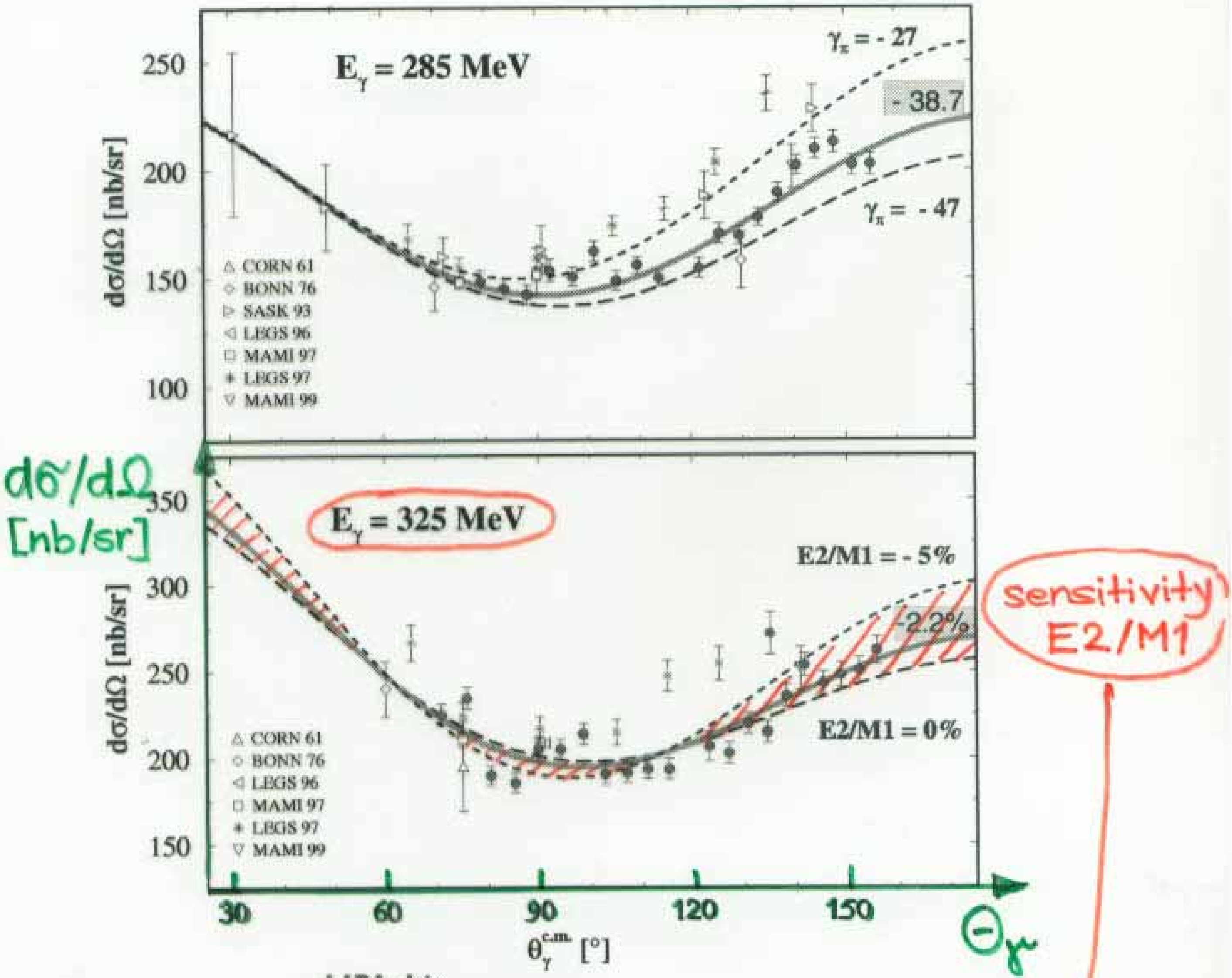
Single π-prod. –> Δ(1232) –>E2/M1
INCLUSIVE /GDH EXP'T

[Phys. Rev. Lett. 84, 5950 (2000)]



COMPTON SCATTERING

Spin polarizability and E2/M1 ratio



LARA data
Brookhaven data
nonsubtracted dispersion theory

γ_π(SAID-parameterization)	=	$-37.1 \times 10^{-4} fm^4$
γ_π(MAID-parameterization)	=	$-40.9 \times 10^{-4} fm^4$
E2/M1(SAID-parameterization)	=	−1.7%
E2/M1(MAID-parameterization)	=	−2.0%

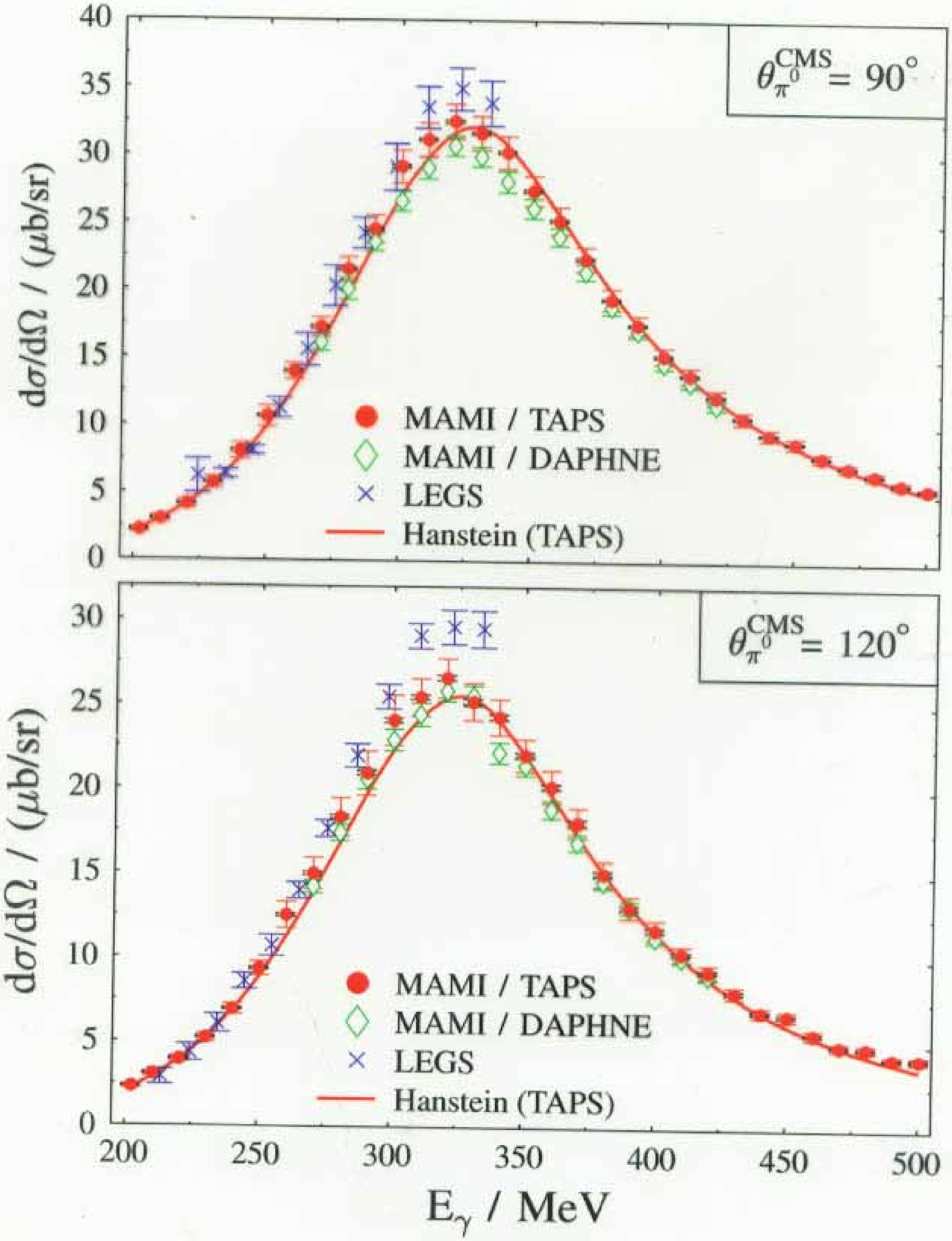
G. Galler et al., Physics Letters B 503 (2001) 245; S. Wolf et al., Eur. Phys. J. A 12 (2001) 231; M. Camen et al., Phys. Rev. C 65 (2002) 032202(R)

PION PHOTOPRODUCTION

New Results for $\frac{d\sigma}{d\Omega}$

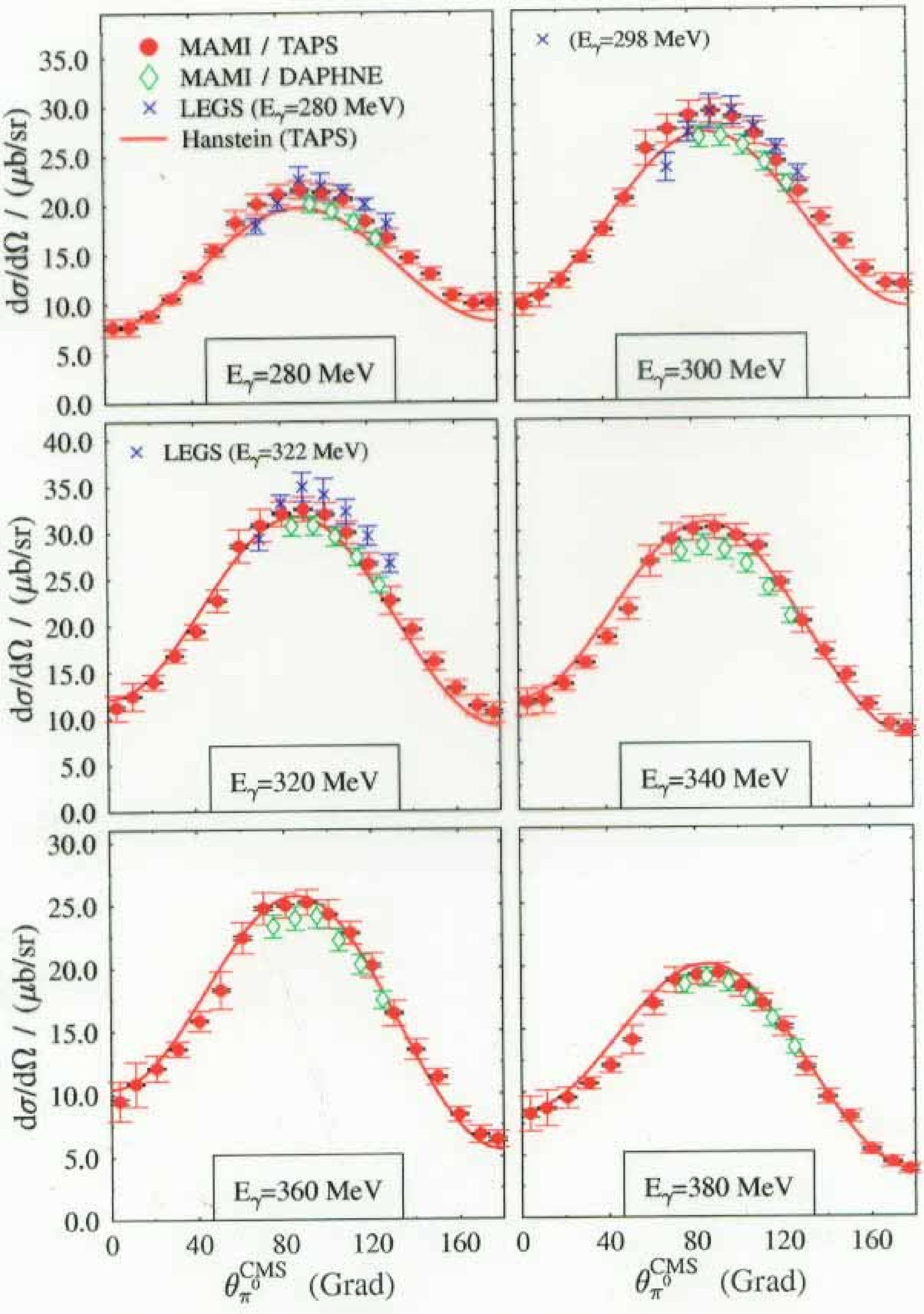
PhD Thesis Roman Leukel, 2001, EPJ (2002)

$p(\vec{\gamma}, \pi^0)p$



Ergebnisse

Differentieller Wirkungsquerschnitt: $\frac{d\sigma}{d\Omega}$



New Results for $R_{EM} = E2/M1$

Electric Quadrupole ($E2$) vs. Magnetic Dipole ($M1$) in the $N \to \Delta(1232)$–Transition

π^0 Photo Production with linearly polarised Photons:

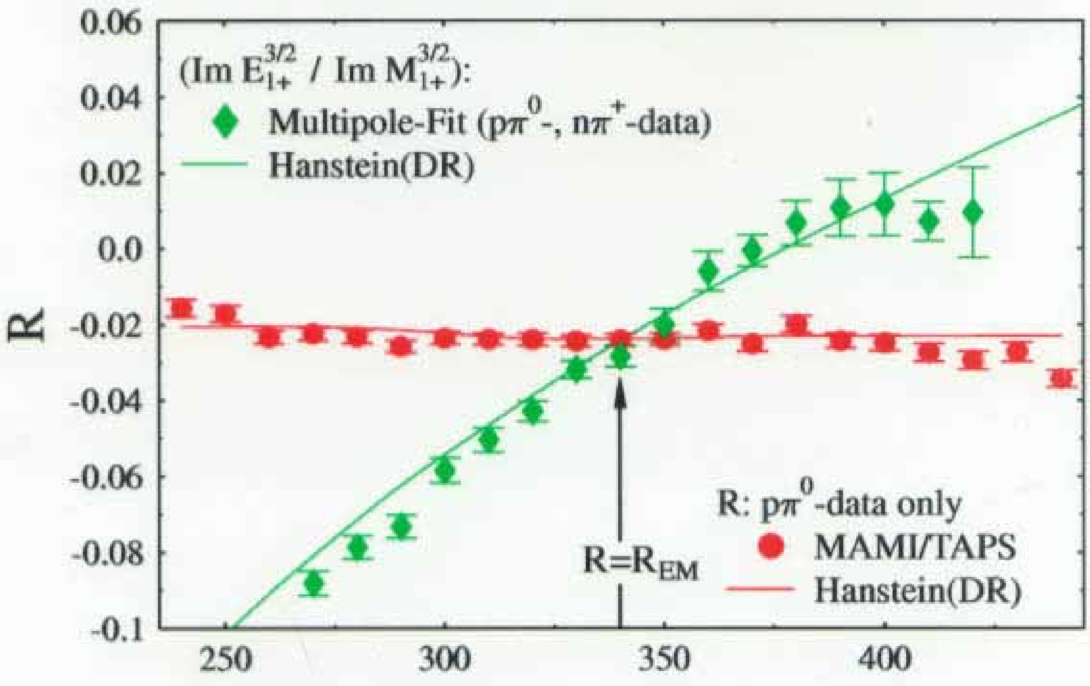
$$\boxed{\frac{d\sigma^T}{d\Omega}(\theta_\pi^*, \varphi, \Pi_T) = \frac{d\sigma^0}{d\Omega}(\theta_\pi^*)\{1 - \Pi_T\,\Sigma(\theta_\pi^*)\,\cos 2\varphi\}}$$

At $E_\gamma = 340\ MeV$ (Resonance Point):
s– and p–Wave Approximation

$$\frac{d\sigma^{0,\|,\perp}}{d\Omega}(\theta_{\pi^0}^*) = \frac{q}{k}\left(A_{0,\|,\perp} + B_{0,\|,\perp}\cos\theta_{\pi^0}^* + C_{0,\|,\perp}\cos^2\theta_{\pi^0}^*\right)$$

$$\boxed{R = \frac{C_\|}{12A_\|} = \frac{1}{12}\frac{\frac{C}{A} + \Sigma(\theta_\pi^* = 90°)}{1 - \Sigma(\theta_\pi^* = 90°)} \approx R_{EM} = \frac{E2}{M1}}$$

MAMI/TAPS-Experiment (PhD Thesis R. Leukel, 2001)



$$\Rightarrow \boxed{R_{EM} = (-2.40 \pm 0.16_{(st.)} \pm 0.24_{(sy.)})\ \%}$$

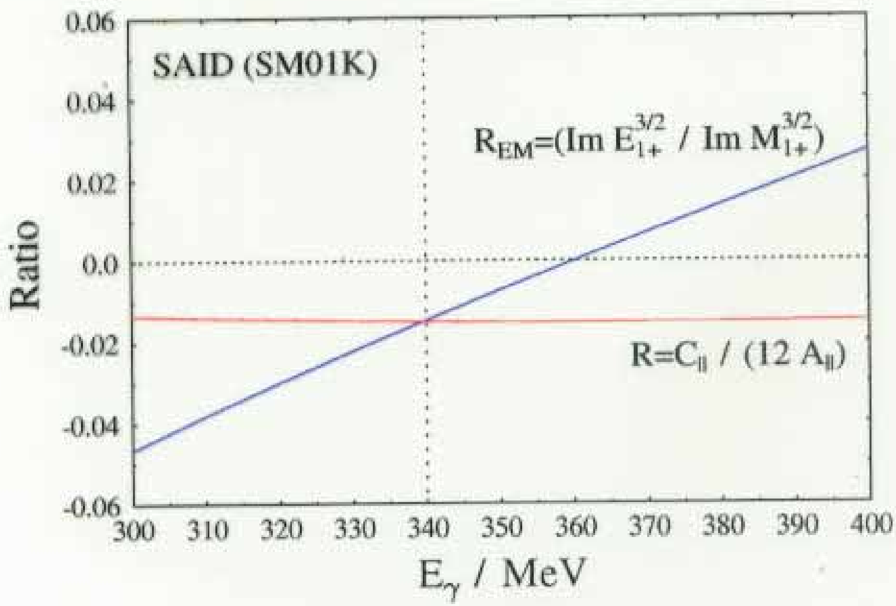
MAMI/DAPHNE : $R_{EM} = (-2.5 \pm 0.1_{(st.)} \pm 0.2_{(sy.)})\ \%$

(R. Beck et al., Physical Review, C61 (2000) 035204)

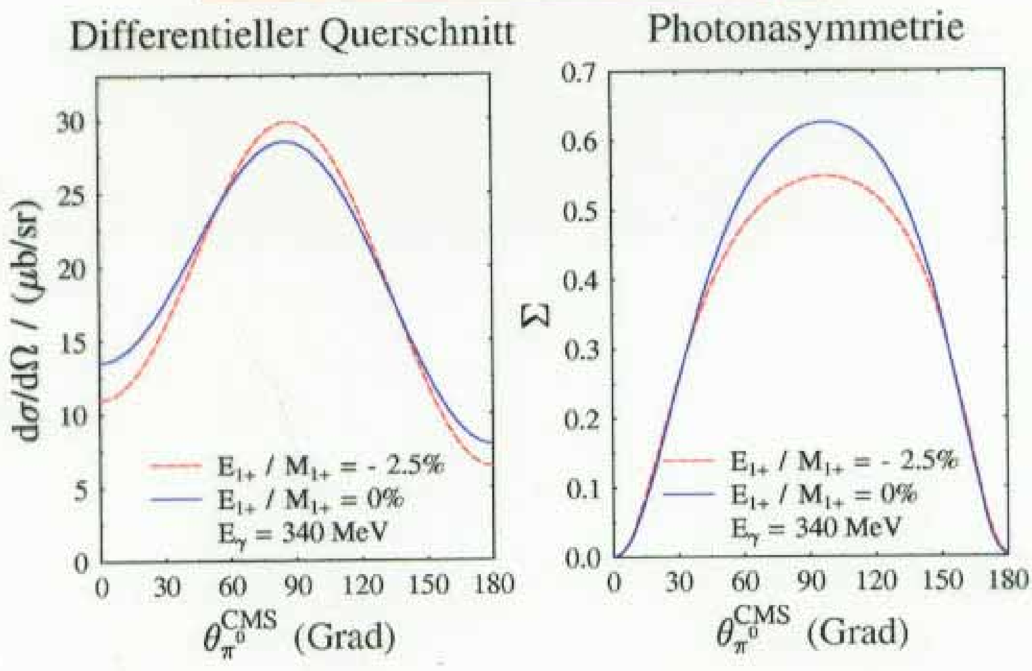
Motivation

Am Resonanzpunkt bei $E_\gamma = 340\ MeV$ gilt:

$$\boxed{R = \frac{C_{||}}{12A_{||}} = \frac{1}{12}\frac{\frac{C}{A} + \Sigma(\theta_\pi^* = 90°)}{1 - \Sigma(\theta_\pi^* = 90°)} = R_{EM} \pm 0.1 R_{EM}}$$

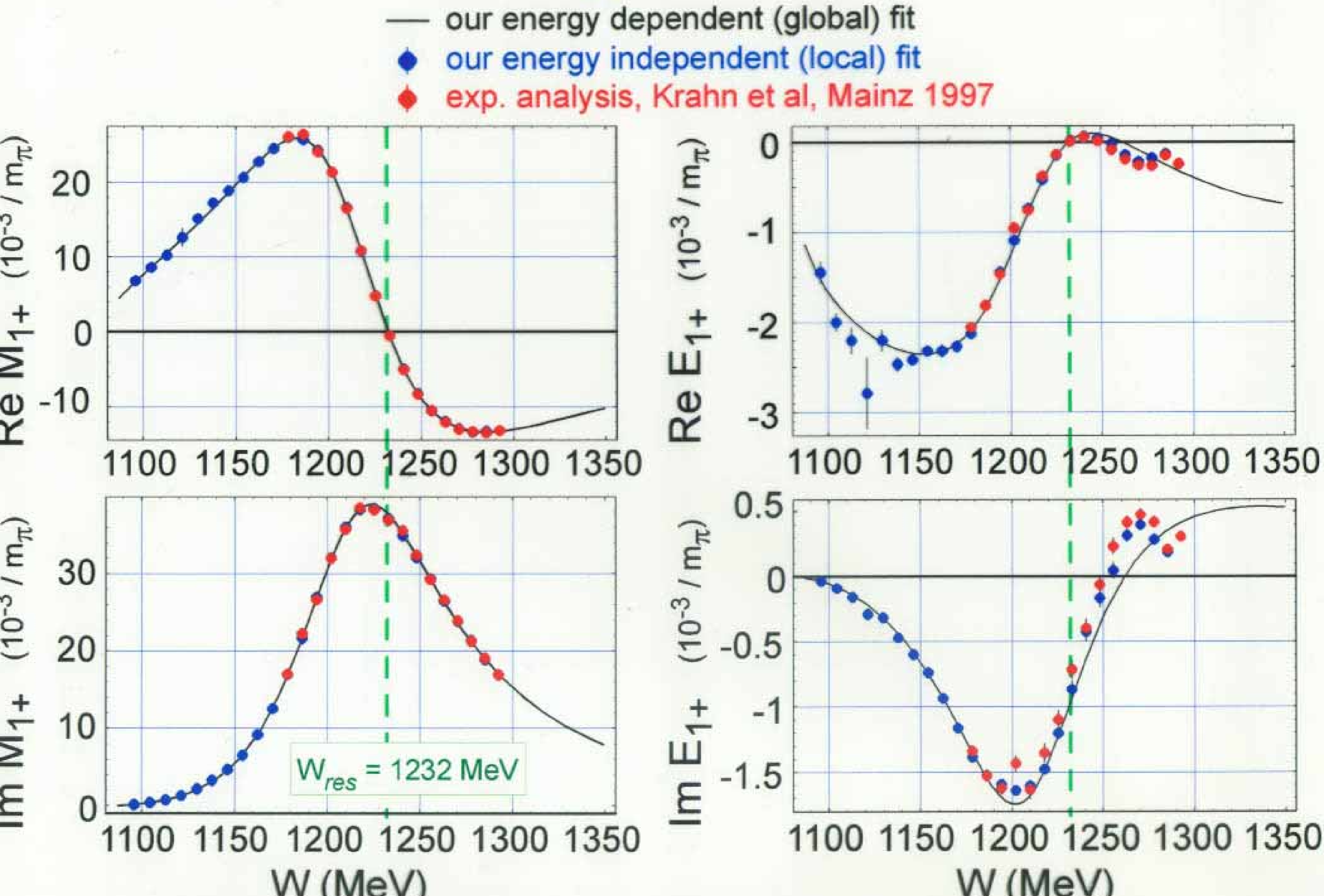


Empfindlichkeit auf E_{1+}:



Maßgebend: volle Winkelinformation für $\frac{d\sigma}{d\Omega}(\theta_\pi^*)$ und Photonasymmetrie unter $\theta_\pi^* = 90°$!

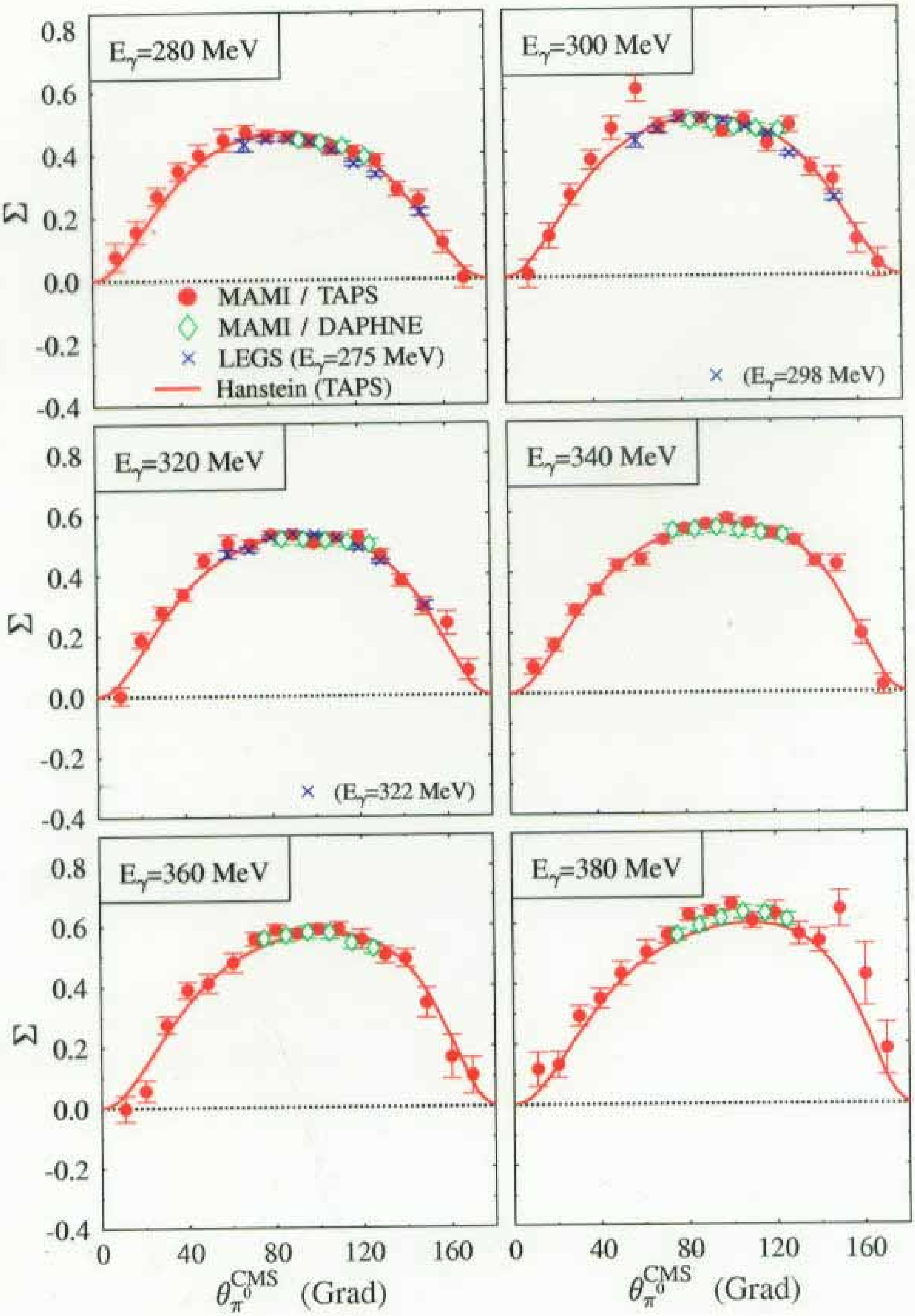
P_{33} waves



our energy dependent (global) fit
our energy independent (local) fit
exp. analysis, Krahn et al, Mainz 1997
Re M_{1+} $(10^{-3} / m_\pi)$
Re E_{1+} $(10^{-3} / m_\pi)$
Im M_{1+} $(10^{-3} / m_\pi)$
Im E_{1+} $(10^{-3} / m_\pi)$
W_{res} = 1232 MeV
W (MeV)
W (MeV)

Ergebnisse

Photonasymmetrie: Σ



Baryon Resonances Analysis Group (BRAG)

Models and Approaches for Partial Wave Analysis

- **ELA (RPI) by Davidson, Mukhopadhyay**
  effective lagrangian
  Born terms + vector meson (ω, ρ) exchange
  + s, u–channel Δ excitation
  in covariant Rarita–Schwinger formalism
- **SAID (GWU) by Workman, Arndt, Strakovsky**
  phenomenological analysis
  Born terms + 4-6 free parameters in each partial wave
- **MAID (Mainz) by Drechsel, Kamalov, Tiator**
  unitary isobar model
  Born terms + ω, ρ
  + s–channel Δ, N^* excitation
  in nonrel. Breit–Wigner ansatz
- **DM (NTU) by Yang, Kamalov**
  dynamical model for $\pi N \to \pi N$ and $\gamma N \to \pi N$
  background generated dynamically by Born + ω, ρ
  resonance contributions similar to MAID
- **DR (Mainz) by Hanstein, Drechsel, Tiator**
  fixed-t dispersion relations
  method by Omnes based on Watson's theorem
  S,P and D_{13} partial waves
- **DR (Yerevan) by Aznauryan**
  fixed-t dispersion relations
  similar to HDT
  in addition input from SAID for non-P_{33} imaginary parts

Results of the low-energy benchmark fits ($E_\gamma < 500 MeV$)

	pars.	χ^2	M1	E2	E2/M1 [%]
ELA (RPI)	9	4.1	286	-7.2	-2.55
SAID (GWU)	31	2.8	281	-7.2	-2.57
MAID (Mainz)	5	4.6	275	-5.3	-1.93
DM (NTU)	4	3.6	280	-6.2	-2.24
DR (Mainz)	8	3.7	281	-6.6	-2.35
DR (Yerevan)	14	3.1	278	-6.3	-2.28
			281.3 ± 4.5	-6.6 ± 0.8	-2.38 ± 0.27

Conclusion

- spread in the determination of M1, E2 and E/M ratio is most important
- gives so far the best estimate of model dependence in partial wave analysis
- absolute numbers depend on the dataset
- need to determine the 'best' dataset

PION ELECTROPRODUCTION

$p(e,e'\pi^0)p$

G. Laveissiere et al. (JLab Hall A Collaboration), private communication

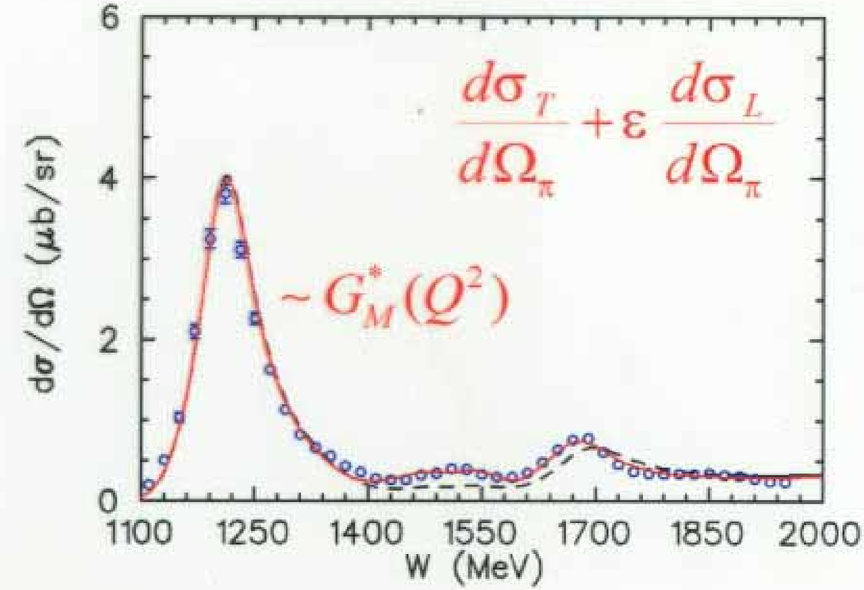
$Q^2 = 1.0\ \mathrm{GeV}^2$

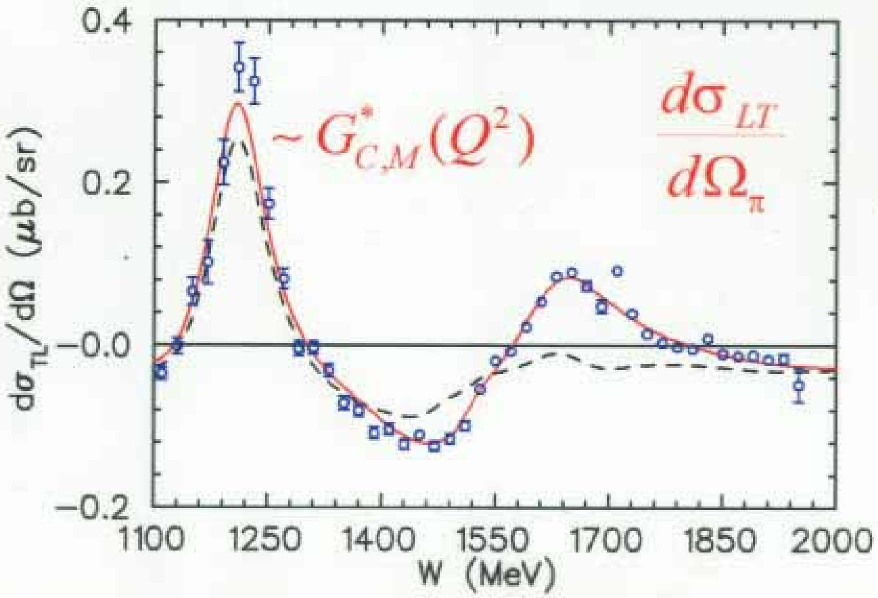
$\epsilon = 0.90$

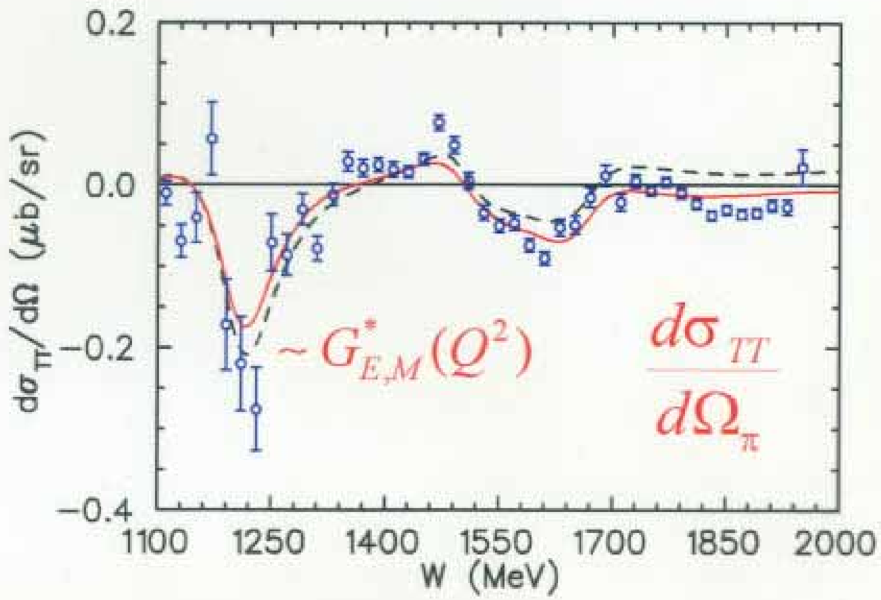
$\theta = 167^0$

—— new MAID fit

- - - - MAID 2000







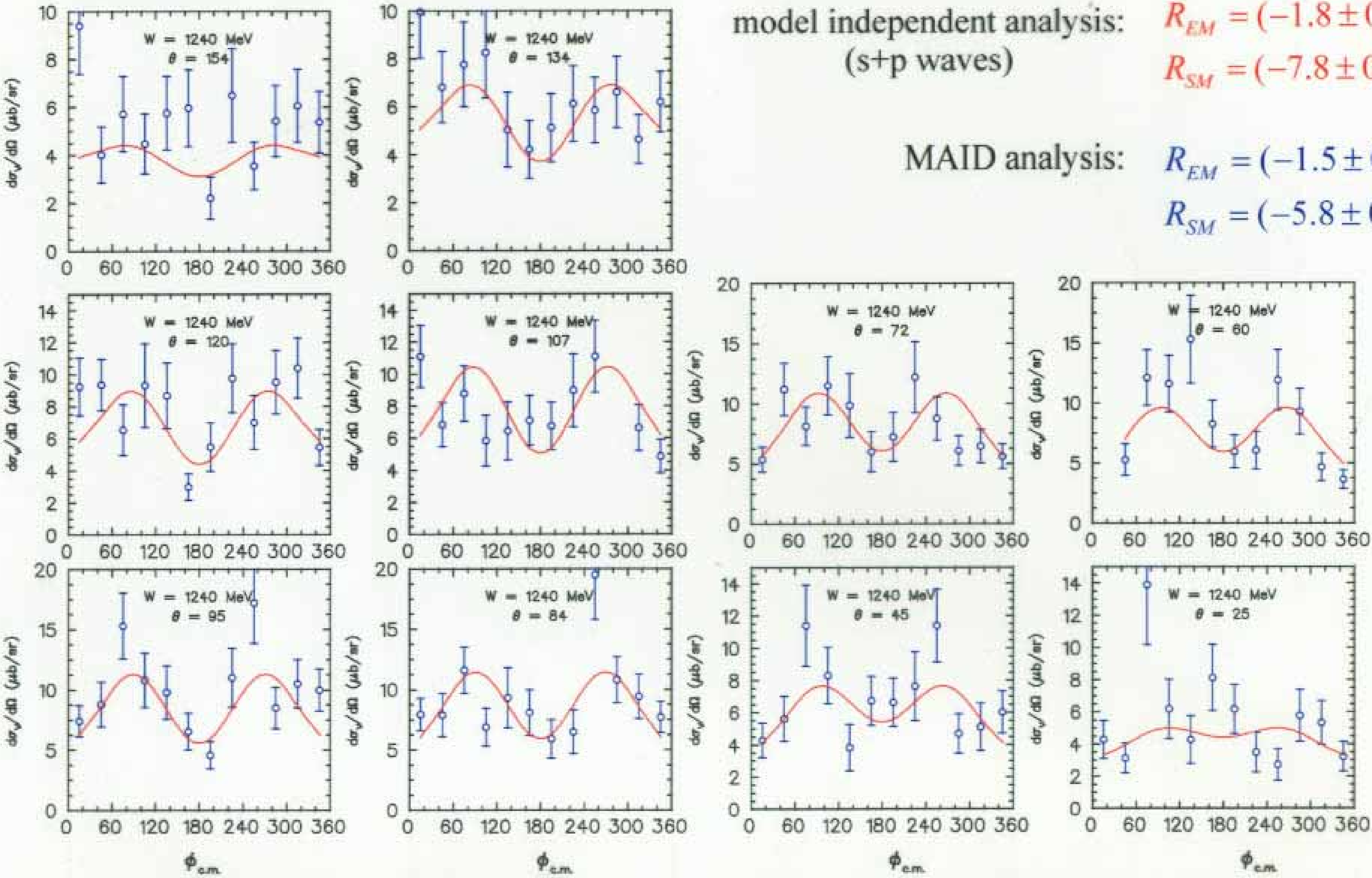
$p(e,e'\pi^0)p$ at JLAB/CLAS

Joo et al, CLAS collaboration
Phys. Rev. Lett. 88 (2002) 122001

$Q^2 = 0.90\,GeV^2, \quad \varepsilon = 0.58$

model independent analysis: (s+p waves)
$R_{EM} = (-1.8 \pm 0.5)\%$
$R_{SM} = (-7.8 \pm 0.6)\%$

MAID analysis:
$R_{EM} = (-1.5 \pm 0.6)\%$
$R_{SM} = (-5.8 \pm 0.7)\%$



$p(e,e'\pi^0)p$ at ELSA (Bonn)

Ralf Gothe, Tina Bantes,
private communication, June 2002

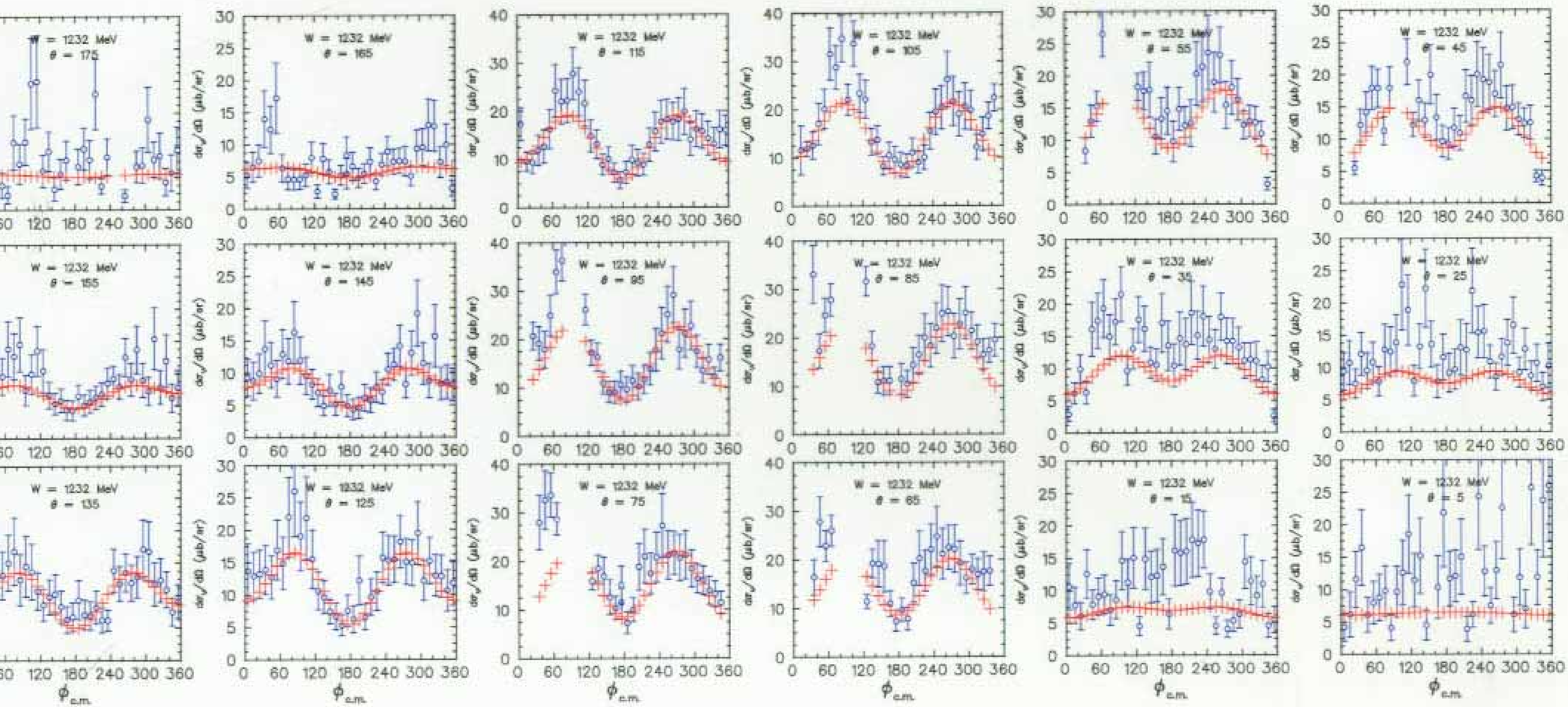
$$Q^2 = 0.63\,GeV^2, \quad \varepsilon \approx 0.88$$
$$W_{cm} = 1153 - 1312\,MeV$$

model independent analysis:
(s+p waves)

$R_{EM} = (-2.24 \pm 0.73)\,\%$
$R_{SM} = (-6.92 \pm 0.65)\,\%$

MAID analysis:

$R_{EM} = (-1.6 \pm 0.2)\,\%$
$R_{SM} = (-5.3 \pm 0.2)\,\%$



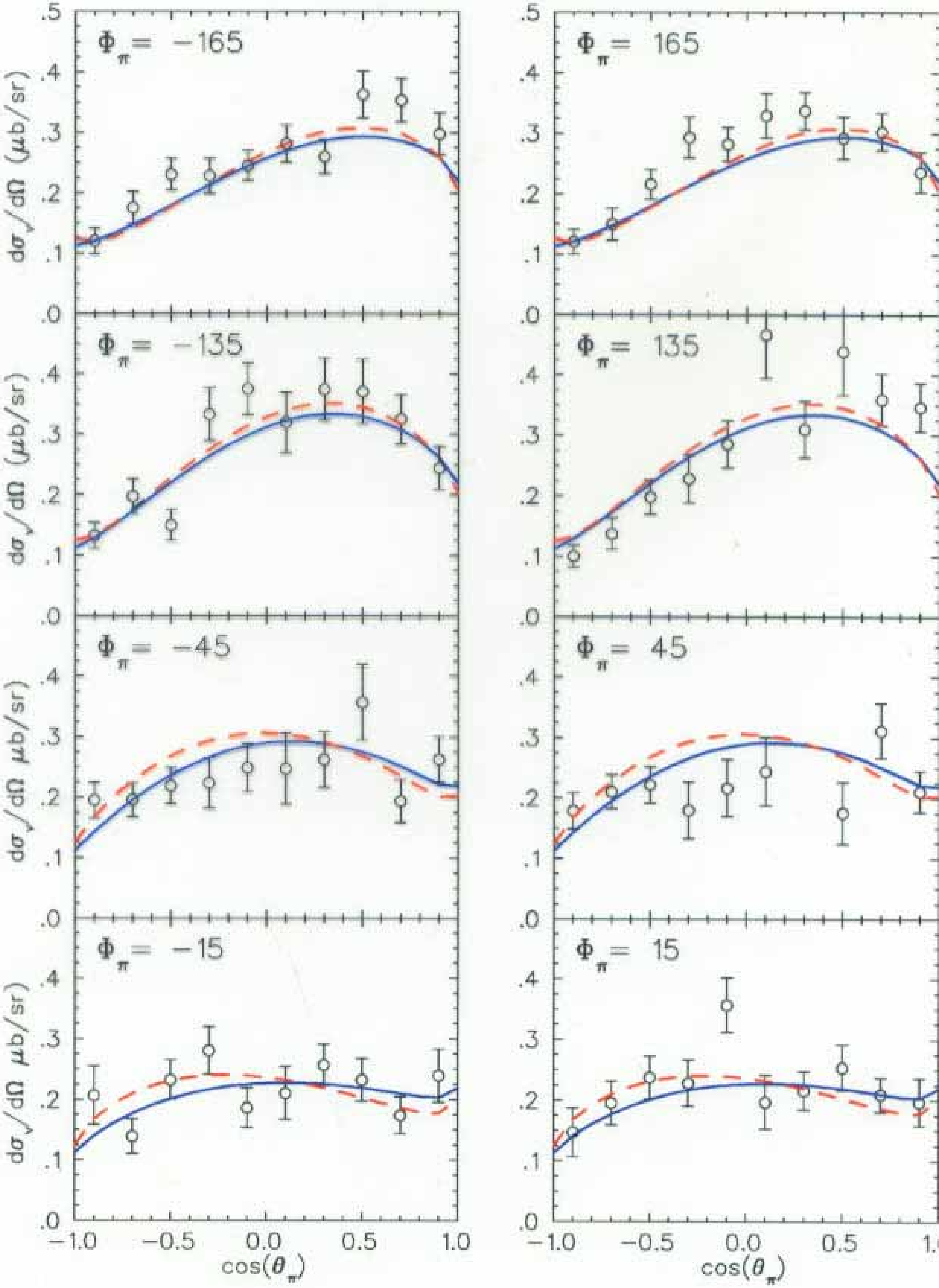
Frolov et al, PRL 82 (1999) 45: Cebaf Hall C

W = 1235 MeV

$Q^2 = 4.0\ GeV^2$

MAID2000

Dynamical Model

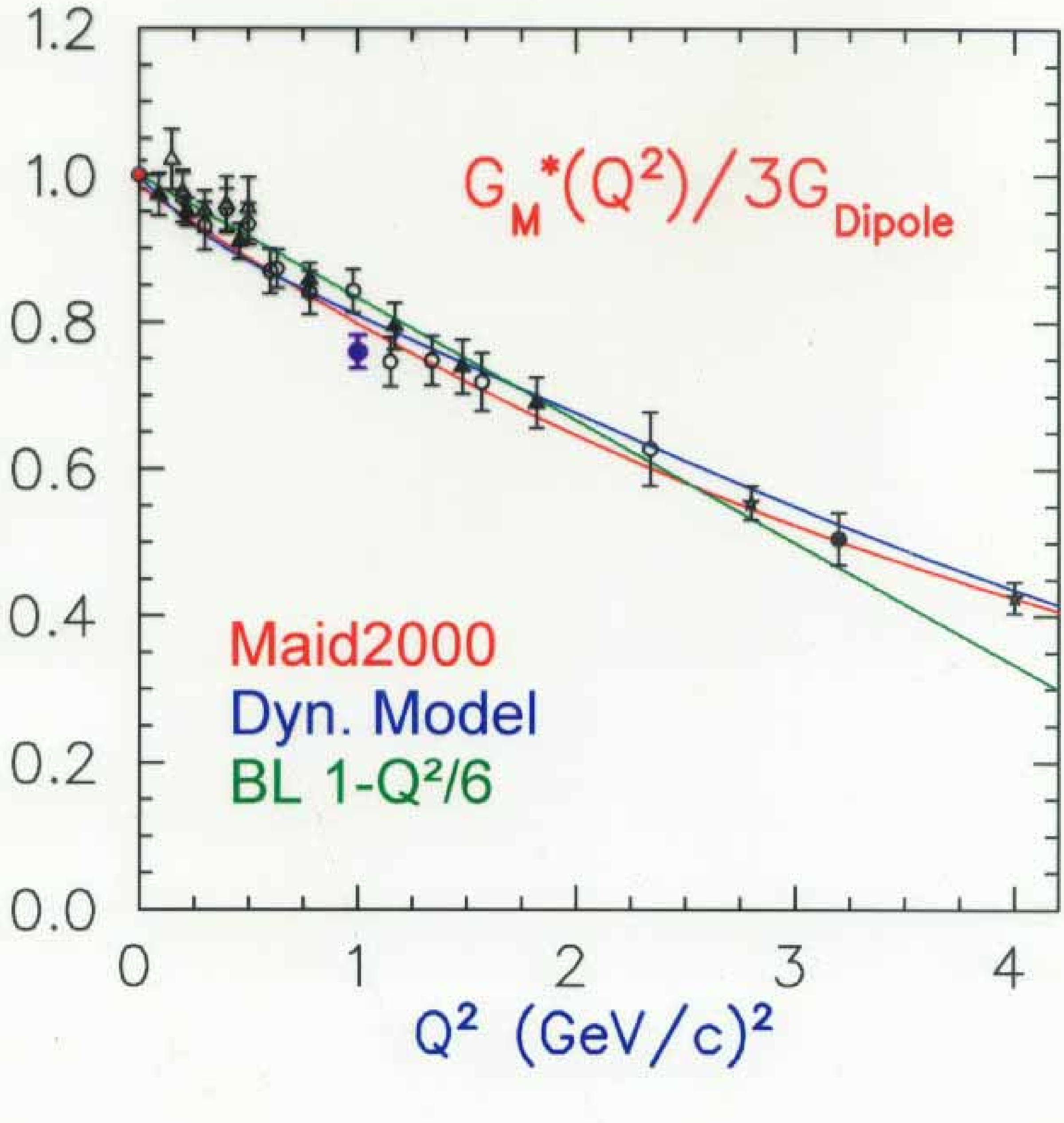


N -> Δ Transition Form Factors

magnetic form factor M1

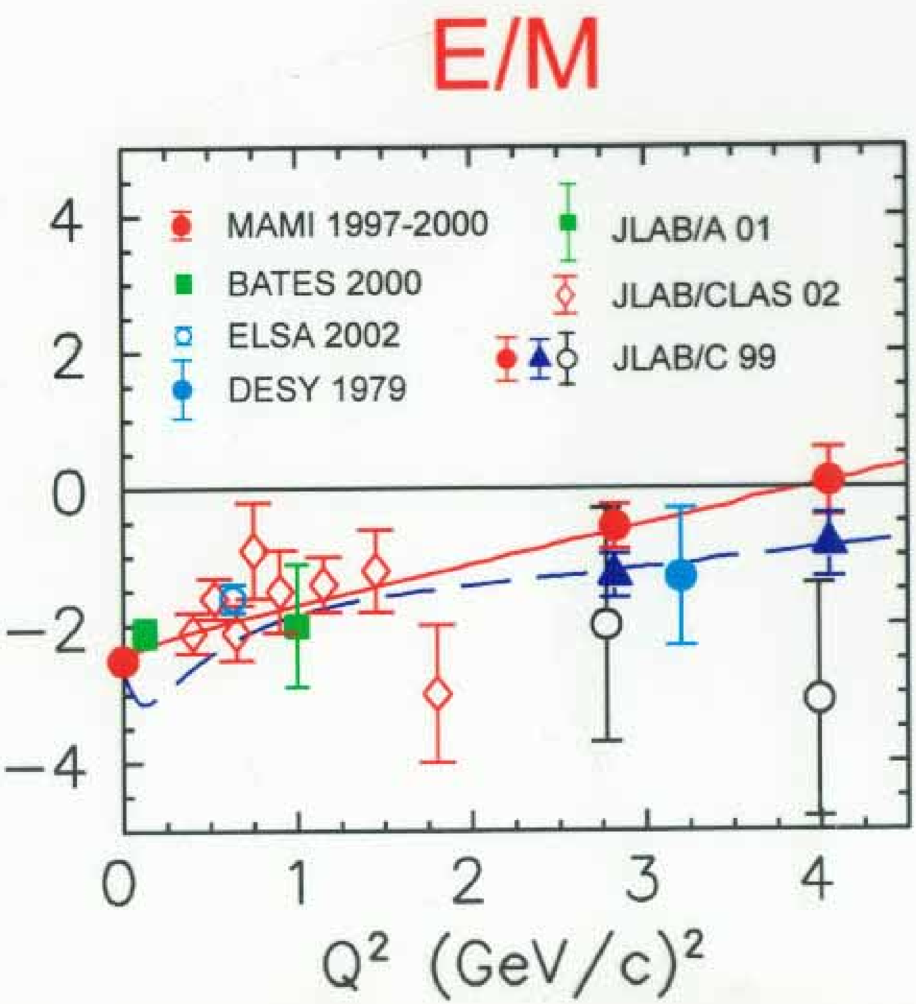
$$G_M^*(0) = \sqrt{\frac{m_N}{m_\Delta}}\,\mu_{N\Delta} = 1.006 \pm 0.010$$

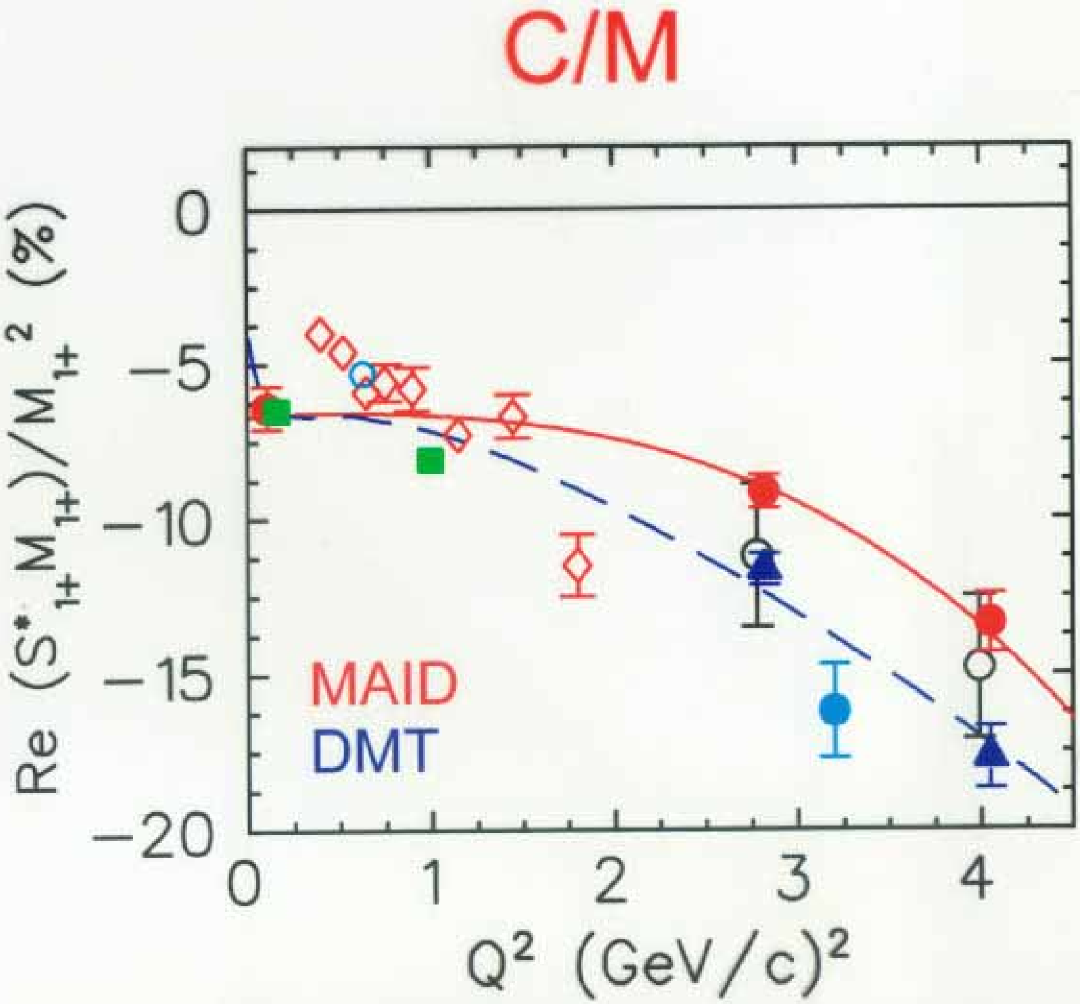
$$\mu_{N\Delta} = 3.46 \pm 0.03$$



N -> Δ Transition Form Factors

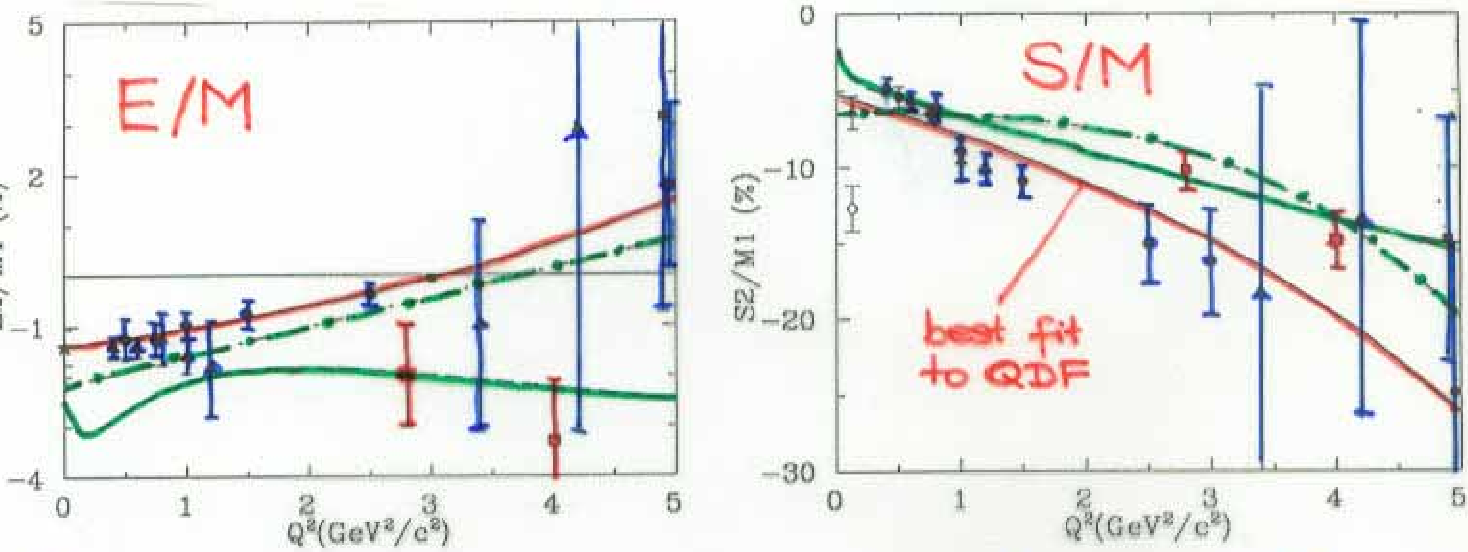
E/M and C/M ratios





R.A. Arndt, I.I. Strakovsky, R.L. Workman

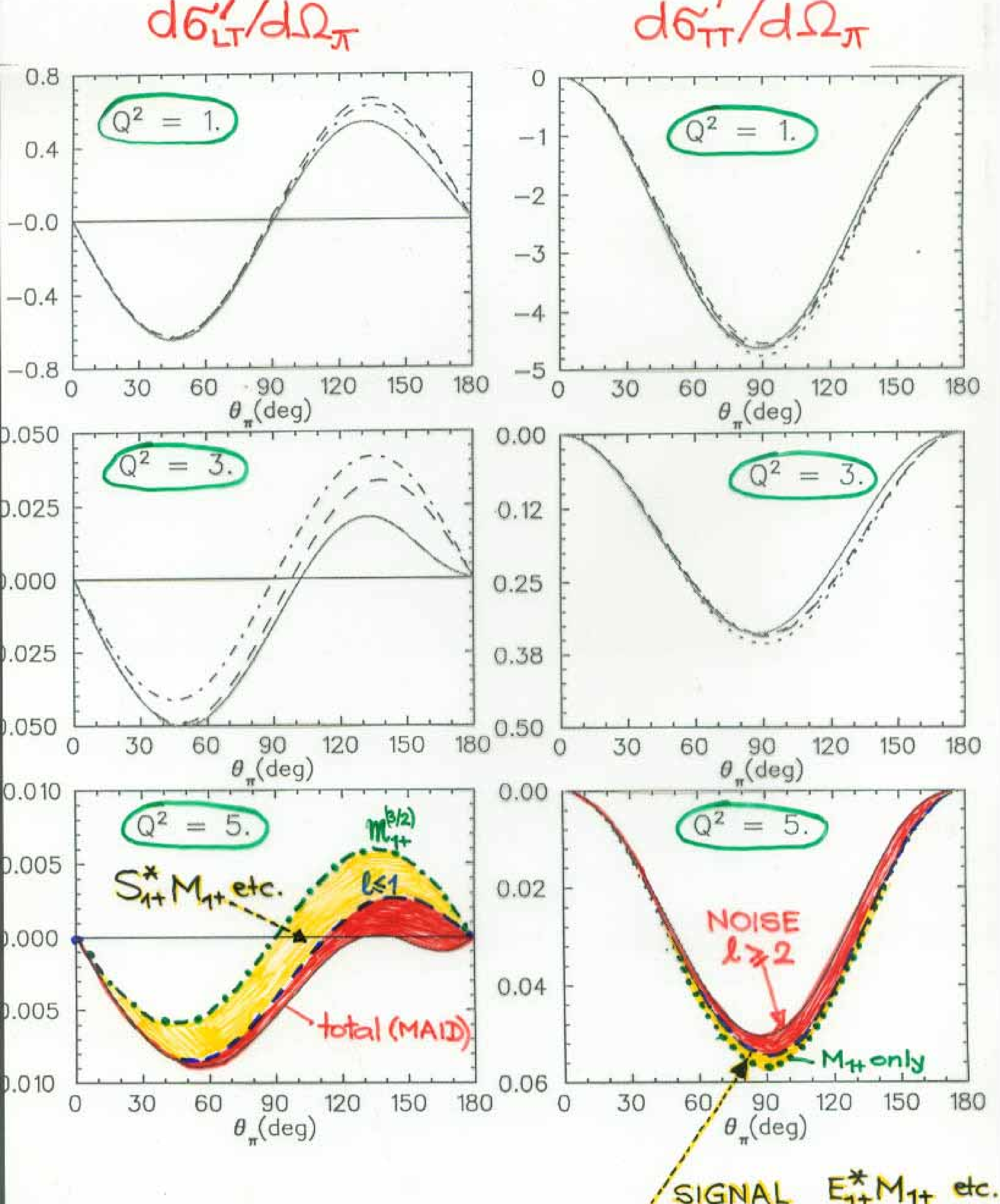
nucl-th / 0110001



Q^2-dependent = QDF, single-Q^2 = SQS

Frolov data (Hall C)

- E/M cross-over @ $Q^2 \approx 3$?
  absolute values small $\lesssim 2\%$
- S/M absolute value grows with Q^2
  no sign for leveling off to const.



- Importance of higher multipoles increases with Q^2

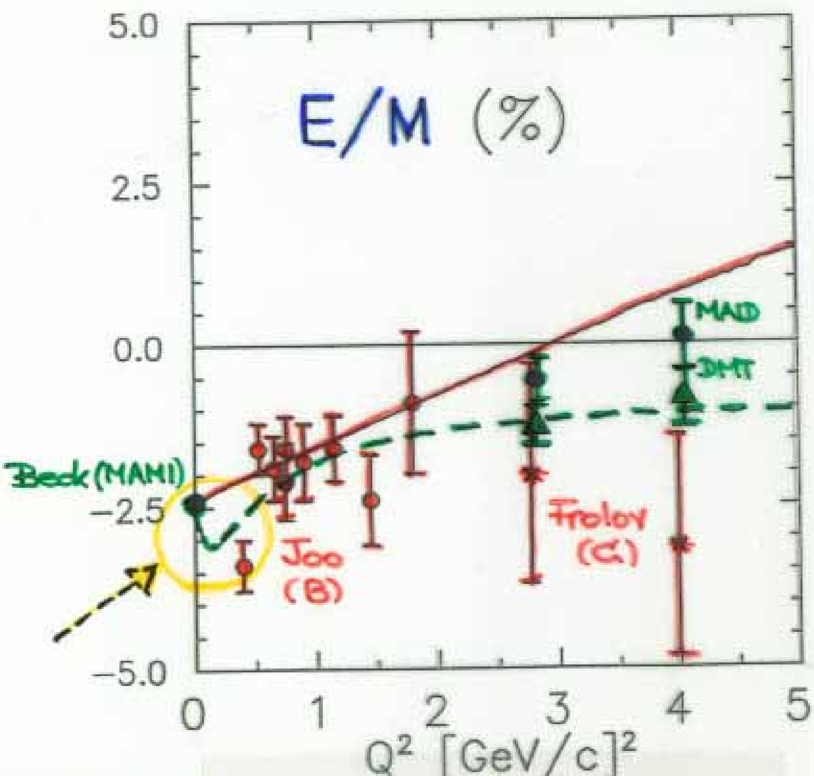
S. Kamalov (2002)

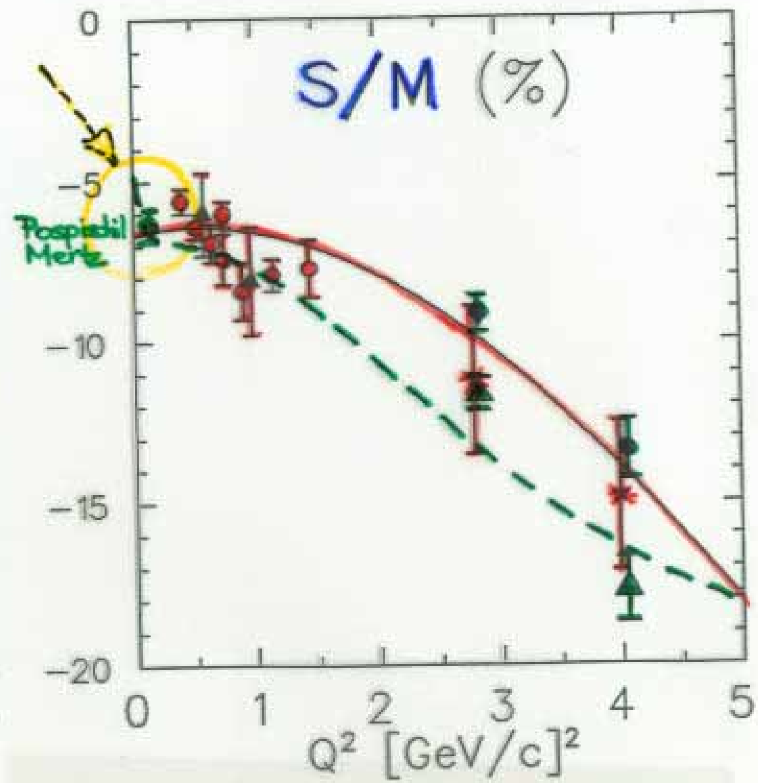
NEW GLOBAL FIT (10/2002)

1680 data points, Jlab A&B&C, Bonn, ...

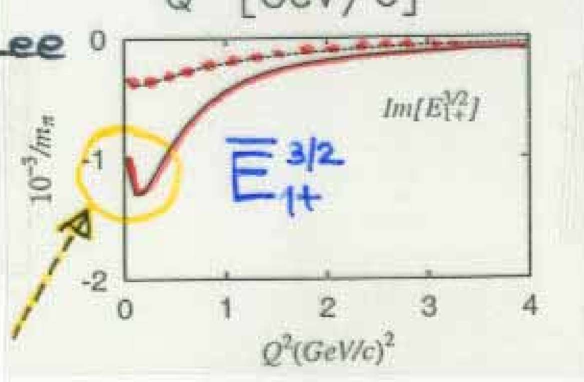
MAID 2000 $\chi^2 = 1.68$ | DMT dyn.mod. $\chi^2 = 1.86$

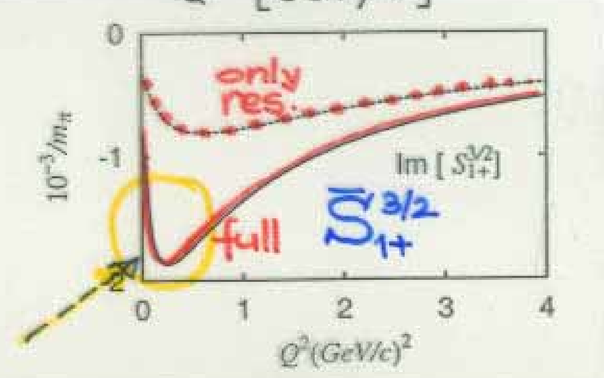
$$f(Q^2) = e^{-\alpha Q^2}(1+\beta Q^2 + \gamma Q^4)$$





Sato & T.-S. H. Lee
C63 (2001)
am. model





$dR_{EM,SM}/dQ^2$ AT $Q^2 = 0$

EVIDENCE FOR PION LOOPS

	R'_{EM}	R'_{SM}
DMT	−5.2	−17.4
Sato & Lee	−5 (?)	−20 (?)
ChPT ε^3	−16 (??)	−51 (??)

TABLE: Slopes of R_{EM} and R_{SM} at $Q^2 = 0$, in units of GeV^{-2}

? taken from figure

?? complex ratio, take ratio of (large) $\Im m$ parts

CONCLUSIONS

- ChPT : Q^2 dependence of E2 and C2 relative to value at $Q^2 = 0$ not suppressed by $1/m_N$, while such suppression takes place in case of M1.
- steep negative slope for $Q^2 \lesssim 0.1$ GeV^2, in both ChPT and dynamical models
- dynamical models predict minimum of $\bar{E}_{1+}$ and $\bar{S}_{1+}$ near $Q^2 = 0.15$ GeV^2
- lowest measured $Q^2 \approx 0.1$ GeV^2
  lower Q^2 possible ?
- Siegert limit at $Q^2 \approx -(m_\Delta - m_N)^2$: E2 = C2, predicted value $R'_{EM}/R'_{SM} \approx 1/3$ necessary to achieve that.

SUMMARY

- E2/M1 ratio for Δ excitation with real photons is well measured and analyzed
  $R_{EM} = (-2.5 \pm 0.5)\,\%$ (PDG 2002)
  model error $< 15\,\%$ (BRAG benchmark)
- quantitative & reliable model calculation is still missing. No prediction from ChPT, lattice calculations are improving but not yet very convincing.
- $|R_{SM}| > |R_{EM}|$, more stable value in model calculations
- low Q^2: slope of R_{EM}, R_{SM} not suppressed by $1/m_\pi$ $\rightarrow$ large fluctuations expected for $Q^2 \lesssim 0.15$ GeV^2. Experiment?
- Effects of pion cloud clearly important: M1, R_{SM}, R_{EM}, R'_{SM}, R'_{EM}.
- New precision data on Q^2 dependence, JLab A/B/C, ELSA, MAMI, MIT/Bates. Analysis need be improved, model error analysis for benchmark necessary.
- Possible cross-over of R_{EM} to positive values near $Q^2 \approx 4$ GeV^2, but asymptotia is far away.
- $|R_{SM}|$ still increasing, no evidence of const. yet