EXPLORING BARYON

χ-RAL MULTIPLETS

N^* 2002

- INTRODUCTION
- ADLER-WEISBERGER SUM RULE(S)
- GROUND STATE χ-RAL MULTIPLET
- QUARK MODEL INTERPRETATION
- GENERALIZATIONS*
- CONCLUSION

INTRODUCTION

$$\mathcal{L}_{QCD} = \bar{q}_L \not{D} q_L + \bar{q}_R \not{D} q_R + \frac{1}{2} F^2 + \mathcal{O}\left(\frac{1}{\Lambda}\right)$$

$$q_L \to L\, q_L \qquad\qquad q_R \to R\, q_R$$

$$SU(2)_L \otimes SU(2)_R$$

$$\Downarrow \qquad \langle \bar{q}_L q_R \rangle, \ldots \neq 0$$

$$SU(2)_I$$

$$\boxed{\text{CANNOT CLASSIFY } N^*\text{'s USING } SU(2)_L \times SU(2)_R}$$

NOT QUITE...

QCD

N^*'s ?? LATTICE , $1/N_c$

+ MODELS

+ $SU(2) \times SU(2)$

M_π ← χ-ral perturbation theory (χ-pt)

$$\mathcal{L}_{QCD}$$

THIS TALK HAS <u>NOTHING</u> TO DO WITH

χ-RAL SYMMETRY RESTORATION !!

THE ADLER-WEISBERGER SUM RULE

$$g_A^2 = 1 - \frac{2F_\pi^2}{\pi}\int_0^\infty \frac{d\omega}{\omega}\left[\sigma^{\pi^- p}(\omega) - \sigma^{\pi^+ p}(\omega)\right]$$

INGREDIENTS:

(1) χ-pt

(2) $A^{(-)} = A^{\pi^- p} - A^{\pi^+ p} \xrightarrow[\omega \to \infty]{} 0$

SATURATE w/ N's ($I = \frac{1}{2}$) AND Δ's ($I = \frac{3}{2}$)

$$g_A^2 = 1 - \sum_N I_N + \sum_\Delta I_\Delta + \text{CONTINUUM}$$

$$I_R \propto \Gamma(R \to \pi p)$$

P. D. G. *** AND **** N^*'s

	$\mathcal{R}$	$\mathcal{I}_{\mathcal{R}}$		$\mathcal{R}$	$\mathcal{I}_{\mathcal{R}}$
$P_{11}\,(\frac{1}{2}^+)$	$N(940)$	--	$P_{11}\,(\frac{1}{2}^+)$	$N(1710)$	0.01
$P_{33}\,(\frac{3}{2}^+)$	$\Delta(1232)$	1.02	$P_{13}\,(\frac{3}{2}^+)$	$N(1720)$	0.02
$P_{11}\,(\frac{1}{2}^+)$	$N(1440)$	0.23	$F_{35}\,(\frac{5}{2}^+)$	$\Delta(1905)$	0.02
$D_{13}\,(\frac{3}{2}^-)$	$N(1520)$	0.09	$P_{31}\,(\frac{1}{2}^+)$	$\Delta(1910)$	0.01
$S_{11}\,(\frac{1}{2}^-)$	$N(1535)$	0.04	$P_{33}\,(\frac{3}{2}^+)$	$\Delta(1920)$	0.01
$P_{33}\,(\frac{3}{2}^+)$	$\Delta(1600)$	0.06	$D_{35}\,(\frac{5}{2}^-)$	$\Delta(1930)$	0.03
$S_{31}\,(\frac{1}{2}^-)$	$\Delta(1620)$	0.02	$F_{37}\,(\frac{7}{2}^+)$	$\Delta(1950)$	0.08
$S_{11}\,(\frac{1}{2}^-)$	$N(1650)$	0.04	$G_{17}\,(\frac{7}{2}^-)$	$N(2190)$	0.03
$D_{15}\,(\frac{5}{2}^-)$	$N(1675)$	0.08	$H_{19}\,(\frac{9}{2}^+)$	$N(2220)$	0.03
$F_{15}\,(\frac{5}{2}^+)$	$N(1680)$	0.10	$G_{19}\,(\frac{9}{2}^-)$	$N(2250)$	0.02
$D_{13}\,(\frac{3}{2}^-)$	$N(1700)$	0.01	$H_{3,11}\,(\frac{11}{2}^+)$	$\Delta(2420)$	0.02
$D_{33}\,(\frac{3}{2}^-)$	$\Delta(1700)$	0.03	$I_{1,11}\,(\frac{11}{2}^-)$	$N(2600)$	0.02

$$g_A^2 = 1 - \sum_N I_N + \sum_\Delta I_\Delta$$

$\sum_N I_N = 0.72$ $\quad\quad$ $\sum_\Delta I_\Delta = 1.3$ $\quad\quad$ $g_A = 1.26$

LOWEST STATES DOMINATE !!!

KEEPING Δ AND $N'(1440)$:

$$1 = g_A^2 + g_A'^2 - \frac{4}{9} C_{\Delta N}^2$$

$g_A \sim \langle p | Q_A | n \rangle$

$g_A' \sim \langle p | Q_A | n' \rangle$

$C_{\Delta N} \sim \langle p | Q_A | \Delta \rangle$

$SU(2)_L \times SU(2)_R$

$[Q_A, Q_A] = T$

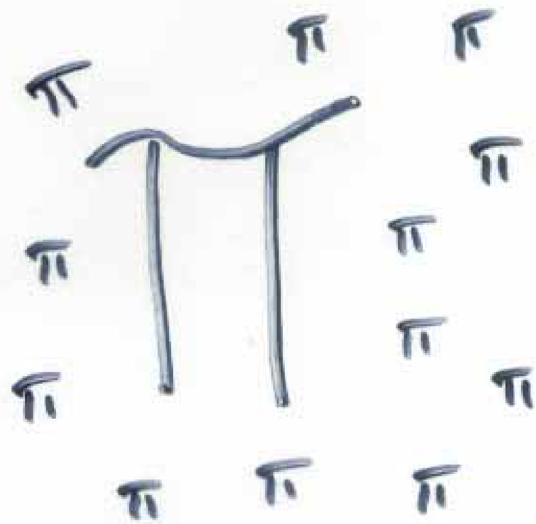
$[Q_A, T] = Q_A$

$[T, T] = T$

$$\langle p | \; [Q_A, Q_A] \; | p \rangle = \langle p | T | p \rangle$$

$|0\rangle\langle 0|$!

PROBLEM:

$Q_A |0\rangle =$ 

∞ MOMENTUM FRAME

N^*

$p \to \infty$

$$Q_A \, |0\rangle_\infty = 0$$

HELICITY λ IS CONSERVED

BUT SOMEWHAT BIZARRE...

COLLINEAR FRAMES

$$A^{(-)} \xrightarrow[\omega \to \infty]{} \langle p | \left([Q_A, Q_A] - T \right)_\lambda | p \rangle + \mathcal{O}\left(\frac{1}{\omega}\right)$$

$Q_A |0\rangle_{cc} = 0$ λ CONSERVED

$$\langle p | [Q_A, Q_A] | p \rangle = \langle p | T | p \rangle$$

$Q_A |0\rangle = 0$ $\Downarrow$ N, N', Δ

$$\underline{1 = g_A^2 + g_A'^2 - \frac{4}{9} C_{\Delta N}^2}$$

COMPLETE SET OF AW $\equiv$ SU(2) x SU(2) FOR EACH λ

MASS-SQUARED SPLITTINGS ??

$$A^{(+)} \xrightarrow[\omega \to \infty]{} \langle p | \left([Q_A, [Q_A, \hat{M}^2]] \right)_\lambda | p \rangle + \mathcal{O}\left(\frac{1}{\omega}\right)$$

$$\hat{M}^2 = \hat{M}_s^2 + \sum_R \hat{M}_R^2$$

$$[\hat{M}_s^2, Q_A] = 0 \qquad [\hat{M}_R^2, Q_A] \neq 0$$

"BASIS" IN WHICH ALL SYMMETRY BREAKING IS IN THE MASS-SQUARED MATRIX

THE GROUND STATE MULTIPLET

$$SU(2)_L \otimes SU(2)_R$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$I = 0, \tfrac{1}{2}, 1, \tfrac{3}{2} \qquad\qquad I = 0, \tfrac{1}{2}, 1, \tfrac{3}{2}$$

$$\boxed{(0,\tfrac{1}{2}),\ (\tfrac{1}{2},0),\ (0,\tfrac{3}{2}),\ (\tfrac{3}{2},0),\ \underset{\tfrac{1}{2}\oplus\tfrac{3}{2}}{\underset{\shortparallel}{(\tfrac{1}{2},1)}},\ (1,\tfrac{1}{2})}$$

TENSOR ANALYSIS:

$$\langle 0,\tfrac{1}{2} |\, Q_A \,| 0,\tfrac{1}{2} \rangle = -1$$

$$_{\frac{1}{2}}\langle \tfrac{1}{2},1 |\, Q_A \,| \tfrac{1}{2},1 \rangle_{\frac{1}{2}} = -5/3$$

$$\vdots$$

SIGMA MODELS:

$(0\,\tfrac{1}{2})$ AND $(\tfrac{1}{2}\,0)$ ONLY

$$|p, \tfrac{1}{2}\rangle = |0, \tfrac{1}{2}\rangle$$

$$g_A = -1$$

LINEAR σ MODEL
$N_L \sim (\tfrac{1}{2}\,0)$
$N_R \sim (0\,\tfrac{1}{2})$

$$|p, \tfrac{1}{2}\rangle = \sum_\lambda a_\lambda |0, \tfrac{1}{2}\rangle^\lambda + \sum_k b_k |\tfrac{1}{2}, 0\rangle^k$$

$$|g_A| = \left| \frac{\sum |a|^2 - \sum |b|^2}{\sum |a|^2 + \sum |b|^2} \right| \leq 1$$

NEED $(1, \tfrac{1}{2})$, $(\tfrac{1}{2}, 1)$ ADMIXTURE!!

SIMPLEST NON TRIVIAL REPRESENTATION ...

$$|p, \tfrac{1}{2}\rangle = |\tfrac{1}{2}, 1\rangle_{\frac{1}{2}}$$

$$|\Delta, \tfrac{1}{2}\rangle = |\tfrac{1}{2}, 1\rangle_{3/2}$$

$g_A = \frac{5}{3}$ $\qquad$ $C_{\Delta N} = 2$ $\qquad$ $M_\Delta^2 = M_p^2$

EQUIVALENT TO N.R. QUARK MODEL !!

TO AVOID DEGENERACY NEED:

$(\tfrac{1}{2}, 1)\ ,\ (0, \tfrac{1}{2})$ $\qquad$ OR

$(\tfrac{1}{2}, 1)\ ,\ (1, \tfrac{1}{2})$ $\qquad$ MIXTURE

$$\langle \tfrac{1}{2}, 1|\, \hat{M}^2_{(\frac{1}{2},\frac{1}{2})}\, |0, \tfrac{1}{2}\rangle \ ,\ \langle \tfrac{1}{2}, 1|\, \hat{M}^2_{(\frac{1}{2},\frac{1}{2})}\, |1, \tfrac{1}{2}\rangle$$

$$\neq 0$$

$(0,\frac{1}{2}) \oplus (\frac{1}{2},1)$: N, N', Δ

	TH1	TH2	EXP	PROCESS
$M_{N'}$	1386	1476	1440 ± 30	$N\pi \to N\pi, \ldots$
g_A	1.26 (input)	1.34 (input)	1.26	$N \to N\pi$
g'_A	0.33	0.33	0.71 ± 0.20	$N' \to N\pi$
g''_A	1.41	1.33	$--$	$N' \to N'\pi$
$\mathcal{C}_{\Delta N}$	1.25	1.43	1.51 ± 0.10	$\Delta \to N\pi$
$\mathcal{C}_{\Delta N'}$	1.56	1.40	1.38 ± 0.50	$N' \to \Delta\pi$

$$|N, \tfrac{1}{2}\rangle = \sin\theta\,|0, \tfrac{1}{2}\rangle + \cos\theta\,|\tfrac{1}{2}, 1\rangle_{\frac{1}{2}}$$
$$|N', \tfrac{1}{2}\rangle = -\cos\theta\,|0, \tfrac{1}{2}\rangle + \sin\theta\,|\tfrac{1}{2}, 1\rangle_{\frac{1}{2}}$$
$$|\Delta, \tfrac{1}{2}\rangle = |\tfrac{1}{2}, 1\rangle_{\frac{3}{2}}$$

$\Rightarrow$

$$g_A = 1 + \tfrac{2}{3}\cos^2\theta$$

$\theta \sim 45°$

$$g'_A = \tfrac{2}{3}\sin\theta\cos\theta$$
$$g''_A = 1 + \tfrac{2}{3}\sin^2\theta$$
$$\mathcal{C}_{\Delta N} = 2\cos\theta$$

$\mathring{g}_A \simeq 1.34$

$$\mathcal{C}_{\Delta N'} = 2\sin\theta$$

$\Rightarrow$

$$\cos^2\theta\, M_N^2 + \sin^2\theta\, M_{N'}^2 = M_\Delta^2$$

AW SUM RULES !!

$(0,\frac{1}{2}) \oplus (\frac{1}{2},1)$: $\underline{N, N', \Delta}$

	TH1	TH2	EXP	PROCESS
$M_{N'}$	$\underline{1386}$	$\underline{1476}$	1440 ± 30	$N\pi \to N\pi, \ldots$
g_A	1.26 (input)	1.34 (input)	1.26	$N \to N\pi$
g'_A	0.33	0.33	0.71 ± 0.20	$N' \to N\pi$
g''_A	1.41	1.33	--	$N' \to N'\pi$
$\mathcal{C}_{\Delta N}$	1.25	1.43	1.51 ± 0.10	$\Delta \to N\pi$
$\mathcal{C}_{\Delta N'}$	1.56	1.40	1.38 ± 0.50	$N' \to \Delta\pi$

$$|N, \tfrac{1}{2}\rangle = \sin\theta\,|0, \tfrac{1}{2}\rangle + \cos\theta\,|\tfrac{1}{2}, 1\rangle_{\frac{1}{2}}$$
$$|N', \tfrac{1}{2}\rangle = -\cos\theta\,|0, \tfrac{1}{2}\rangle + \sin\theta\,|\tfrac{1}{2}, 1\rangle_{\frac{1}{2}}$$
$$|\Delta, \tfrac{1}{2}\rangle = |\tfrac{1}{2}, 1\rangle_{\frac{3}{2}}$$

$|N\rangle = \sin\theta\,|\{4\}\rangle + \cos\theta\,|\{20\}\rangle$

$|N'\rangle = -\cos\theta\,|\{4\}\rangle + \sin\theta\,|\{20\}\rangle$

$|\Delta\rangle = |\{20\}\rangle$

$\Downarrow$ $\Downarrow$ a.M.

$\theta \sim 45°$

$$g_A = 1 + \tfrac{2}{3}\cos^2\theta$$
$$g'_A = \tfrac{2}{3}\sin\theta\cos\theta$$
$$g''_A = 1 + \tfrac{2}{3}\sin^2\theta$$
$$\mathcal{C}_{\Delta N} = 2\cos\theta$$
$$\mathcal{C}_{\Delta N'} = 2\sin\theta$$

$\underline{|\{4\}, l=1\rangle}$??

$\mathring{g}_A \doteq 1.34$

$\Downarrow$

$$\cos^2\theta\, M_N^2 + \sin^2\theta\, M_{N'}^2 = M_\Delta^2$$

AW SUM RULES !!

$(\frac{1}{2},1)\oplus(1,\frac{1}{2})$: N, N', Δ, Δ'

	TH1	TH2	EXP	PROCESS
$M_{N'}$	1810 ± 67	1810 ± 67	1440 ± 30	$N\pi \to N\pi, \ldots$
$M_{\Delta'}$	(input)	(input)	1625 ± 75	$N\pi \to N\pi, \ldots$
g_A	1.26 (input)	1.34 (input)	1.26	$N \to N\pi$
g'_A	1.09	0.99	0.71 ± 0.20	$N' \to N\pi$
g''_A	1.26	1.34	--	$N' \to N'\pi$
$\mathcal{C}_{\Delta N}$			1.51 ± 0.10	$\Delta \to N\pi$
$\mathcal{C}_{\Delta N'}$	NO SOLUTION		1.38 ± 0.50	$N' \to \Delta\pi$
$\mathcal{C}_{\Delta' N}$			0.38 ± 0.19	$\Delta' \to N\pi$
$\mathcal{C}_{\Delta' N'}$			5.6 ± 4.8	$\Delta' \to N'\pi$

$$|N, \tfrac{1}{2}\rangle = \sin\phi\, |\tfrac{1}{2}, 1\rangle'_{\frac{1}{2}} + \cos\phi\, |1, \tfrac{1}{2}\rangle_{\frac{1}{2}}$$
$$|N', \tfrac{1}{2}\rangle = -\cos\phi\, |\tfrac{1}{2}, 1\rangle'_{\frac{1}{2}} + \sin\phi\, |1, \tfrac{1}{2}\rangle_{\frac{1}{2}}$$
$$|\Delta, \tfrac{1}{2}\rangle = \sin\delta\, |\tfrac{1}{2}, 1\rangle'_{\frac{3}{2}} + \cos\delta\, |1, \tfrac{1}{2}\rangle_{\frac{3}{2}}$$
$$|\Delta', \tfrac{1}{2}\rangle = -\cos\delta\, |\tfrac{1}{2}, 1\rangle'_{\frac{3}{2}} + \sin\delta\, |1, \tfrac{1}{2}\rangle_{\frac{3}{2}}$$

$|N\rangle = \sin\varphi\, |\{20\}\rangle + \cos\varphi\, |\{20'\}\rangle$

$|N'\rangle = -\cos\varphi\, |\{20\}\rangle + \sin\varphi\, |\{20'\}\rangle$

$|\Delta\rangle = \sin\delta\, |\{20\}\rangle + \cos\delta\, |\{20'\}\rangle$

$|\Delta'\rangle = -\cos\delta\, |\{20\}\rangle + \sin\delta\, |\{20'\}\rangle$

a.m.

$$g_A = \tfrac{5}{3}\cos 2\phi$$
$$g'_A = \tfrac{5}{3}\sin 2\phi$$
$$g''_A = -\tfrac{5}{3}\cos 2\phi$$
$$\mathcal{C}_{\Delta N} = -\mathcal{C}_{\Delta' N'} = -2\cos(\phi+\delta)$$
$$\mathcal{C}_{\Delta N'} = \mathcal{C}_{\Delta' N} = -2\sin(\phi+\delta)$$

$\phi \sim 20°$

$\delta \sim i\,35°$!

$$M_N^2 + M_{N'}^2 = M_\Delta^2 + M_{\Delta'}^2$$
$$\left(M_{\Delta'}^2 - M_\Delta^2\right)\cos 2\delta = \left(M_{N'}^2 - M_N^2\right)\cos 2\phi.$$

NATURE SEEMS TO FAVOR

$$(\tfrac{1}{2}, 1) \oplus (0, \tfrac{1}{2}) \oplus \ldots$$

BUT ...

QUARK MODEL INTERPRETATION

SPIN-FLAVOR $\quad SU(4)$

$$4 \otimes 4 \otimes 4 = \bar{4} \oplus 20 \oplus 20' \oplus 20''$$

$$\begin{array}{cccc} \bar{4} & 20 & 20' & 20'' \\ A & S & M & M \end{array}$$

$$\left(SU(6): \quad 6 \otimes 6 \otimes 6 = 20 \oplus 56 \oplus 70 \oplus 70' \right)$$

$20 \Leftrightarrow (\frac{1}{2}, 1)$

$g_A = 5/3 \qquad C_{\Delta N} = 2$

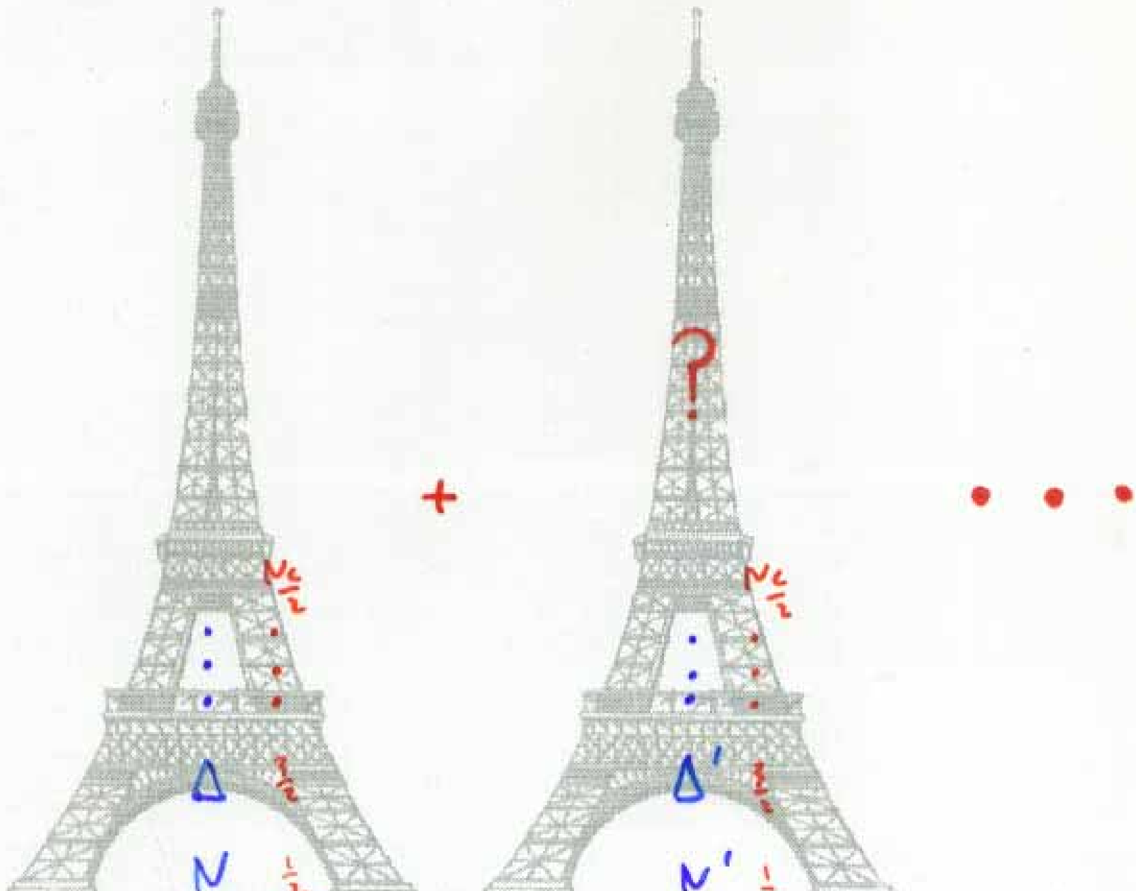
$\bar{4} \Leftrightarrow (0, \frac{1}{2})$

$g_A = -1$

LARGE N_c ??

$$\left(\tfrac{1}{4}(N_c-1),\ \tfrac{1}{4}(N_c+1)\right) \xRightarrow[N_c=3]{} \left(\tfrac{1}{2},1\right)$$

$SU(2) \times SU(2)$



$\Psi_{\mu\ldots\nu}$ $\qquad\qquad$ $\Psi'_{\mu\ldots\nu}$

$\Downarrow$ $\qquad N_c=3 \qquad$ $\Downarrow$

56 $\qquad\qquad$ $56'$

$SU(6)$

GENERALIZATIONS

- $SU(2) \times SU(2)$ $\quad\quad (\tfrac{1}{2},1) \oplus (0,\tfrac{1}{2})$

  $\Downarrow \quad\quad\quad\quad \Downarrow$

  $SU(3) \times SU(3)$ $\quad\quad (\underset{\sim}{3},\underset{\sim}{6}) \oplus (\underset{\sim}{\bar{3}},\underset{\sim}{3})$

  $M_q \to L M_q L^+$

- BARYONS W/ 1 HEAVY QUARK

  $\Lambda_a, \Sigma_a, \Sigma_a^* \in (0,0) \oplus (0,1) \oplus (0,1)$

- χ-PT FOR GROUND STATE MULTIPLET

  N, Δ AND N' $\quad\quad M_{N'} - M_N \sim M_K$

- ELECTROMAGNETICS*

  IN PROGRESS

CONCLUSION

- FOR EACH λ BARYONS FALL INTO REPS OF $SU(N_F) \times SU(N_F)$ (NOTHING TO DO WITH PARITY DOUBLING!!).

- IN ALL CASES, SIMPLEST NONTRIVIAL REP REPRODUCES SPIN-FLAVOR SU(4) (OR SU(6)) RESULTS FOR AXIAL COUPLINGS.

- GROUND STATE REP CONSISTENT WITH OBSERVATION SEEMS AT ODDS W/ NAIVE Q.M. AND IS SMALL!!

- INTERESTING ROLE FOR THE ROPER!!

CAN JLAB MAP THE BARYON χ-RAL MULTIPLETS ??

$N\,(940)$
$\Delta\,(1232)$
$N\,(1440)$

$(0,\tfrac{1}{2}) \oplus (\tfrac{1}{2},1) \oplus \ldots$

$N\,(1520)$
$N\,(1535)$
$\Delta\,(1600)$
$\Delta\,(1620)$
$N\,(1650)$
$N\,(1675)$
$N\,(1680)$
$N\,(1700)$
$\Delta\,(1700)$
$N\,(1710)$
$N\,(1720)$
$\Delta\,(1905)$
$\Delta\,(1910)$
$\Delta\,(1920)$
$\Delta\,(1930)$
$\Delta\,(1950)$
$N\,(2190)$
$N\,(2220)$
$N\,(2250)$
$\Delta\,(2420)$

??

LATTICE QCD INPUT ??

$N\,(2600)$

REFERENCES

DASHEN, GELL-MANN 65

HARARI 66A, 66B

LIPKIN, RUBINSTEIN, MESHKOV 66

GERSTEIN, LEE 66

GILMAN, HARARI 67

WEINBERG 68, 90, 94

GILMAN, KUGLER, MESHKOV 72, 73