N^* Properties from the $1/N_c$ Expansion

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Outline

1) Why would anyone consider $N_c \neq 3$?

2) Does it give useful phenomenology?

3) What does it say about N^*'s?

The Rise of QCD

1947: Discovery of Strangeness (K, Λ)

1952: Discovery of Δ resonance

1961: Discovery of new mesons (ρ, ω, η, K^*)

Eightfold Way

1964: Quark Model

Greenberg conjecture (3 colors)

Problem: $\Delta^{++}(J_z = +\frac{3}{2}) = (u\uparrow)(u\uparrow)(u\uparrow)$ is a fermion with a wavefunction that appears symmetric under particle exchange

Solution: Each quark possesses color quantum number, and the wavefunction is antisymmetric under exchange including color

1965-1968: Discovery of many N^*'s

1968-1972: Deep Inelastic Scattering on Nucleons/Scaling

1969: Parton model

1973: QCD developed as a renormalizable gauge field theory with 3 color charges, incorporating quarks as its fundamental matter fields, and exhibiting asymptotic freedom to explain scaling and the parton model

- BUT – It is instantly recognized that QCD is not perturbative in its coupling at low energies: $\alpha_s = \mathcal{O}(1)$

Classical problems of hadron physics – masses, spatial wavefunctions, electromagnetic couplings – remain relatively untouched by ab initio QCD studies, with the exception of lattice simulations

1974: Gerard 't Hooft points out that allowing the number N_c of colors in QCD to be treated as a large parameter creates a perturbative expansion of QCD at all energy scales

[The $1/N_c$ expansion]

- To this day, no alternate way is known for treating the fundamental QCD field theory perturbatively at all scales

Why consider the large N_c limit of QCD?

1) A perturbative QCD expansion

Compare QED - why does it work so well?

-Because it has a perturbative expansion parameter, $\alpha_{EM} \sim \frac{1}{137}$

But $\alpha_{strong} = \mathcal{O}(1)$ at hadronic energy scales $\Rightarrow$ not perturbative!

Does QCD possess any perturbative parameter, even if generalized?

Only one is known: $\frac{1}{N_c}$ ('t Hooft [1974])

2) Simplification of physics in the large N_c limit

Certain classes of Feynman diagrams dominate over others in physical amplitudes

$\Rightarrow$ Leads to many interesting simplifications

eg: A color singlet $q\bar{q}$ pair always appears at leading order in $1/N_c$ as a single meson

3) Explanation of phenomena hard or impossible to explain in field theory

e.g. OZI suppression (the suppression of $q\bar{q}$ pair annihilation);

Dominance of diagrams with resonances over diagrams with multiparticle states

4) It seems to work, even for $N_c = 3$

But isn't $1/N_c = 1/3$ too large an expansion parameter?

Answer A: Witten's "wisecrack" (S. Coleman)

If $1/N_c = 1/3$ is too large an expansion parameter,

consider

$$\alpha_{EM} = \frac{e^2}{4\pi} \sim \frac{1}{137}$$

$$\Rightarrow \underline{\underline{e \sim 0.3}}$$

Why don't we say the same thing about QED?

Turning the argument around, suppose expansion parameter in practice turns out to be $1/N_c^2$, say

Answer B: Pragmatism

The ultimate justification of any approach is whether it successfully explains physical observations and can be tested in new situations

How can increasing the number of degrees of freedom (N_c) simplify the strong interaction problem?

On one hand, detailed interactions of any particular quark or gluon much more complicated if we solve theory exactly

Analogy 1: Particle moving in complicated potential

Analogy 2: Many-body problem in mechanics

However, if increasing the number of degrees of freedom permits one to consider only collective couplings in a systematic way, then the form of dynamics simplifies

Analogy 1: Decomposition of potential into simple dominant piece and subleading perturbations

Analogy 2: Statistical mechanics

Nature of (valence) hadrons in large N_c QCD

ASSUMPTION: For arbitrary N_c, assume confinement still occurs

Otherwise, the large N_c universe is nothing like ours

Mesons in arbitrary N_c

Suppose flavor wavefunction of meson is $\bar{Q}q$

Then normalized wavefunction including color indices α, β is

$$\frac{1}{\sqrt{N_c}} \bar{Q}_\alpha q^\beta \delta^\alpha{}_\beta \qquad \text{(Puts } \bar{Q}, q \text{ in color singlet)}$$

The important point:

Even with N_c colors, mesons still have quantum numbers of system with one quark and one antiquark

Baryons in arbitrary N_c

Built in analogy to $N_c = 3$:

1) N_c quarks still in completely <u>symmetric</u> spin-flavor wavefunction (also symmetric in spatial wavefunction indices for ground state baryons)

2) Color wavefunction completely antisymmetric to satisfy Pauli principle for fermions

Suppose flavor wavefunction of baryon is $q^1 q^2 \cdots q^{N_c}$

Then normalized wavefunction including color indices $\alpha_1, \alpha_2, \ldots, \alpha_{N_c}$ is

$$\frac{1}{\sqrt{N_c!}} q^1{}_{\alpha_1} q^2{}_{\alpha_2} \cdots q^{N_c}{}_{\alpha_{N_c}} \varepsilon^{\alpha_1 \alpha_2 \cdots \alpha_{N_c}}$$

3) For baryons to be fermions, since quarks have spin $1/2$, N_c must be odd

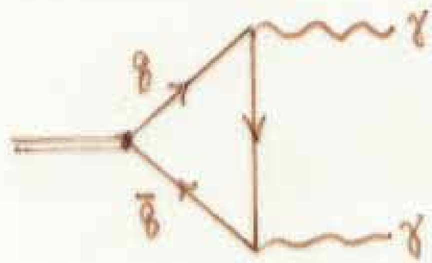
How do we know $N_c = 3$?

1) The Greenberg conjecture and spin-statistics of baryons

2) Results from the asymptotic freedom regime:

- $R \equiv \sigma(e^+e^- \to \text{hadrons}(q\bar{q})) / \sigma(e^+e^- \to \mu^+\mu^-)$
  essentially counts number of particle degrees of freedom
  If color is ignored, get a result a factor $3 = N_c$ smaller than experiment
- The running of α_s between 1.8 GeV (m_τ) and 91 GeV (M_Z)
- Many, many high-momentum QCD processes

3) $\pi^0 \to \gamma\gamma$:



Each color of quark can appear in the loop
$\Rightarrow$ Amplitude $\propto N_c^{1/2}$

Width calculation agrees beautifully with experiment only if $N_c = 3$

4) Chiral anomaly cancellation of Standard Model

- "Anomalies" are a kind of interaction (loosely speaking) appearing in the quantum but not classical theory
- In a gauge theory, they must all cancel out, or the theory cannot be renormalized
- The anomalies in the Standard Model (QCD + QED + weak interactions) depend on fermion charges, and cancel only if $N_c = 3$

Does physics "allow" $N_c \neq 3$?

- Greenberg conjecture ⇒ Satisfied by N_c quarks in baryon

- Ratio R, high-momentum QCD processes, $\pi^0 \to \gamma\gamma$

  change numerically but not qualitatively

- Chiral anomaly cancellation

  Still works if $Q_u = \frac{N_c+1}{2N_c}$, $Q_d = Q_s = -\frac{N_c+1}{2N_c}$

  An unexpected bonus!

  With these charges, hadrons turn out to have same charges as in $N_c = 3$

  $Q_p = +1$, $Q_n = 0$, etc.

The essential predictive power of the large N_c limit just comes from

trap doors and mirrors

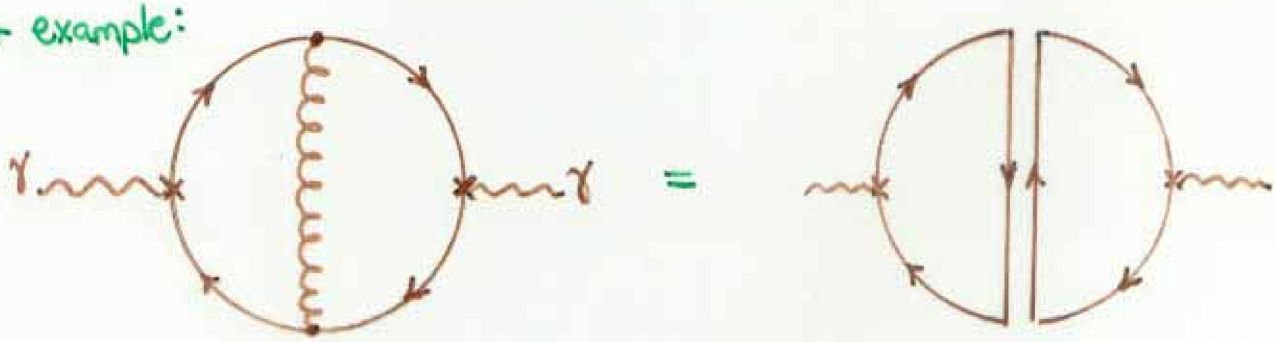
(topology) (group theory)

Double-line notation

The $SU(N_c)$ gauge group of large N_c QCD allows the flow of color to be depicted graphically:

quark  antiquark  gluon 

For example:

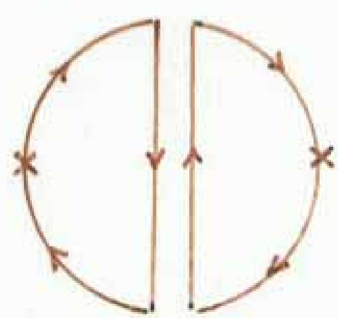


Key point: A color loop allows for all N_c colors

't Hooft's accomplishment: In order that Green functions have smooth, finite limits as $N_c \to \infty$, must take $\boxed{g_s \propto \frac{1}{\sqrt{N_c}}}$

For example,  and  should be the same order in N_c

1 color loop: N_c^1



2 color loops, 2 quark-gluon vertices
$\Rightarrow N_c^2 \cdot (1/\sqrt{N_c})^2 = N_c^1$

The general proof that $g_s \propto 1/\sqrt{N_c}$ is most easily done by turning double-line diagrams into solid figures, and then using topology (Euler character theorem)

"I have a truly remarkable proof which the margin is too small to contain"

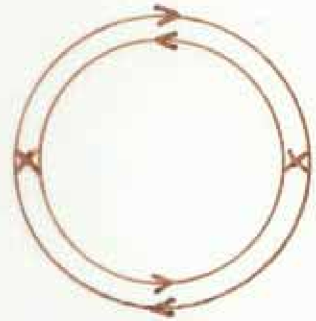
Fun with one particular result:

Compare internal quark loops to internal gluon loops

$\propto N_c^1$

vs.

=



$\propto N_c^2$

Quark loops are always subleading by a factor N_c

THEREFORE:

Popping extra quarks out of the vacuum is suppressed in $1/N_c$

THEREFORE:

Having $q\bar{q}$ mesons decay into more $q\bar{q}$ mesons is suppressed in $1/N_c$

THEREFORE: **True for $N_c = 3$?**

- Mesons in large N_c don't like to decay ✓
- They are stable, long-lived resonances ✓
- When a meson does decay, it produces as few "daughter" mesons as possible, even when phase space favors more (resonance dominated) ✓
- When a meson does decay, quarks prefer to propagate from the initial to final state, rather than annihilate (OZI rule) ✓

Other results can be proved almost as easily:

- Mesons have masses of $\Theta(N_c^0)$ (they propagate freely) and decay constants of $\Theta(\sqrt{N_c})$
- Interactions between mesons are suppressed by powers of $1/N_c$
- Hybrid (non-valence quark) mesons exist in large N_c

What large N_c really does:

The infinite number of QCD Feynman diagrams is divided into classes based on the N_c factor in each
Less suppression in $1/N_c$ powers $\Longleftrightarrow$ Greater physical significance

A simple picture for the large N_c baryon

Recall from previously : Baryons in large N_c have quantum numbers of N_c quarks

How can we understand the physics of a system with $N_c \to \infty$ quarks?

Witten [1979] : A mean-field Hartree-Fock picture

To lowest order in $1/N_c$, each quark moves in the static field caused by the other (N_c-1) quarks acting together

$\Rightarrow$ Each quark satisfies the same (quantum mechanical) wave equation

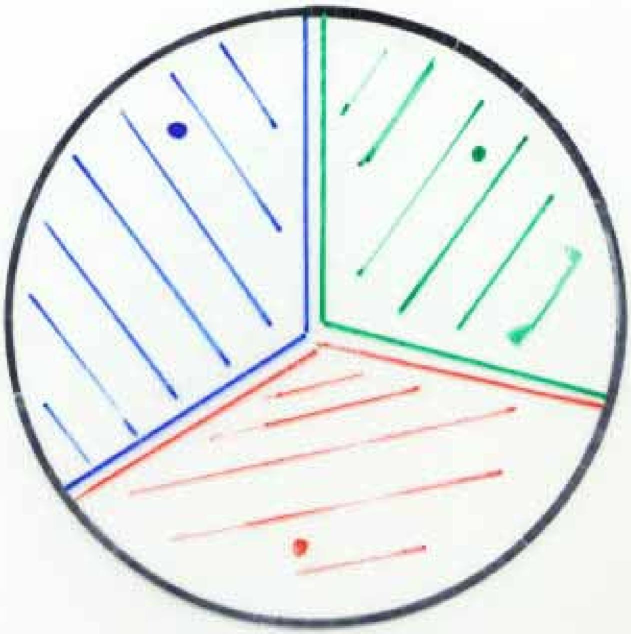
[For very heavy quarks would reduce to Schrödinger equation]

$\Rightarrow$ Each quark in the baryon ground state has the same wavefunction $\phi(\vec{r})$, which is $\propto N_c^0$

For the last statement to be sensible, the potential due to (N_c-1) quarks must be $\propto N_c^0$, or equivalently baryon diagrams must never give interaction energies more than $\propto N_c^1$

Is this true? How does large N_c counting work for baryons?

Physical meaning of "quark representation"



For more details, see A.J. Buchmann + RFL (2000)

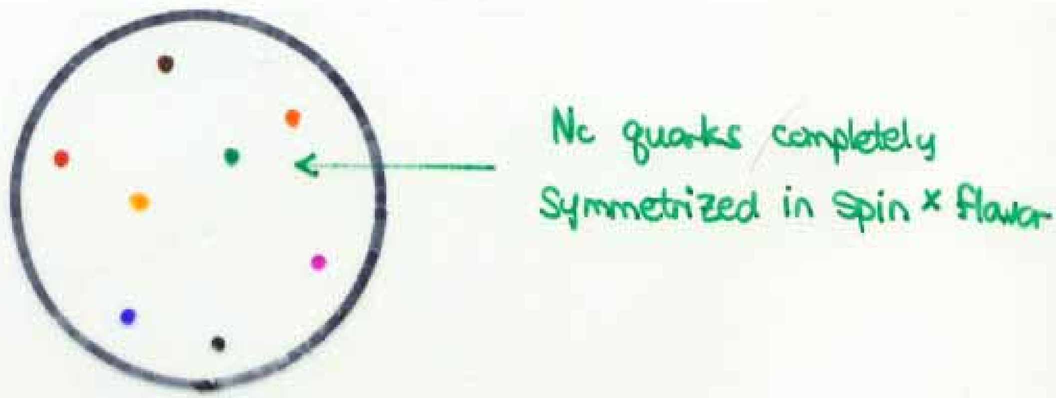
A "quark" is really an interpolating field with the same quantum numbers as the current quark

If one assumes that physical baryons are members of some symmetry multiplet with given quantum nubers, then there is an unambiguous way to chop it up into pieces transforming under the fundamental representation of $SU(N_c)$

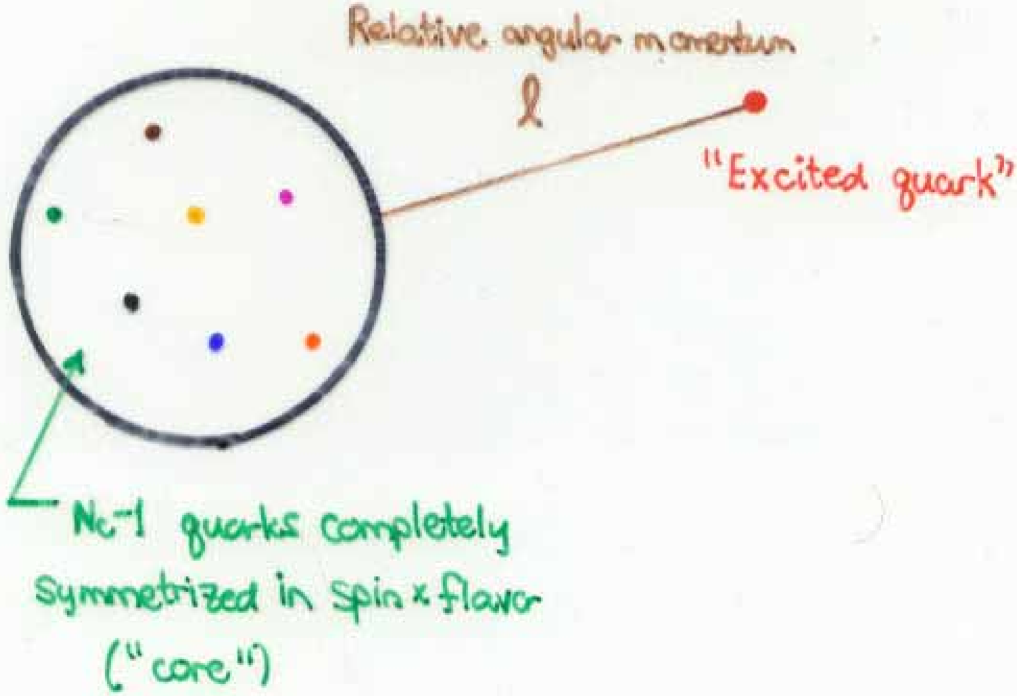
i.e., it has same quantum numbers as a current quark, and in general consists of many Fock components:

$$``q" = q + qg + qq\bar{q} + qgg + \cdots$$

Ground-state baryons in large N_c



Orbitally-excited baryons in large N_c



The $1/N_c$ Expansion and SU(6)

With 3 quark flavors, all operators in the $1/N_c$ expansion that break SU(6) spin-flavor symmetry are <u>suppressed</u> by powers of $1/N_c$

Thus, 1960's-vintage SU(6) arises as a <u>rigorous consequence</u> of large N_c, perturbatively broken by powers of $1/N_c$

[Dashen and Manohar, PLB 315, 425 (1993)]

$\Rightarrow$ The ground-state multiplet is the **56**, containing $N, \Delta, \Sigma, \Omega, \ldots$

The first excited multiplet is the **70**, containing the negative parity N^*'s : $N(1535), \Lambda(1405), \ldots$

Large-N_c flavor reps for $F=3$

← "Physical" baryons

$\frac{1}{2}(N_c+1)$

1 1

1 2 1

1 2 2 1

1 2 2 2 1

1 2 2 2 2 1

1 2 2 2 2 2 1

1 2 2 2 2 2 2 1

1 2 2 2 2 2 2 2 1

1 2 2 2 2 2 2 2 2 1

1 1 1 1 1 1 1 1 1

$J=\frac{1}{2}$

← "Physical" baryons

$\frac{1}{2}(N_c-1)$

1 1 1 1

1 2 2 2 1

1 2 3 3 2 1

1 2 3 4 3 2 1

1 2 3 4 4 3 2 1

1 2 3 4 4 4 3 2 1

1 2 3 4 4 4 4 3 2 1

1 2 3 3 3 3 3 2 1

1 2 2 2 2 2 2 1

1 1 1 1 1 1 1

$J=\frac{3}{2}$

Operator analysis for baryons

- It is possible to write down an "effective" Hamiltonian for any multiplet of large N_c baryons in which each operator, classified by spin and flavor properties, has a well-defined $1/N_c$ power suppression

  Carone, Georgi, Osofsky; Dashen, Jenkins, Manohar; Luty; Luty and March-Russell [1994]

- It is possible to make this expansion unique, removing all redundant operators systematically

  Dashen, Jenkins, Manohar [1995];
  Carlson, Carone, Goity, RFL [1999]

- Since baryon multiplets are finite, so is the set of distinct spin-flavor operators with nonvanishing matrix elements

  Jenkins, RFL [1995]

$$\mathcal{H} = \sum_i \frac{c_i}{N_c^{n_i}} \mathcal{O}_i$$

- The analysis is essentially a complicated version of the Wigner-Eckart theorem

  $\mathcal{O}_i$: operators with calculable matrix elements ("Clebsch-Gordan coeff's")

  c_i : "reduced matrix elements" include all dynamics, some known (e.g., symmetry breaking due to strangeness typically $\varepsilon \approx 25\%$), some unknown: What remains is due to uncalculated strong dynamics, and should be of order unity (Naturalness)

Example of a mass relation

Consider operator $\hat{\mathcal{O}} = \frac{1}{N_c}\{T^8, T^8\}$

- has relative size $1/N_c^2$ compared to common baryon mass of $\mathcal{O}(N_c)$
- because of two explicit flavor=8 indices, breaks SU(3) at second order

$\Rightarrow$ Gives coefficient $\varepsilon^2 \sim 0.25^2 \sim 0.06$ in matrix elements due to SU(3) breaking

$\Rightarrow$ Should give $\mathcal{O}(\varepsilon^2/N_c^2)$ result

Of all operators in this basis, only $\hat{\mathcal{O}}$ contributes to mass combination

$$35(2N-\Sigma-3\Lambda+2\Xi)-4(4\Delta-5\Sigma^*-2\Xi^*+3\Omega)$$

Compare to common mass of baryons by dividing by $\frac{1}{2}$ of same expression with all signs changed to $+$ (remove absolute normalization):

$$\frac{1}{2}\left[35(2N+\Sigma+3\Lambda+2\Xi)+4(4\Delta+5\Sigma^*+2\Xi^*+3\Omega)\right]$$

obtain experimental result $\underline{\underline{0.37 \pm 0.01\%}}$

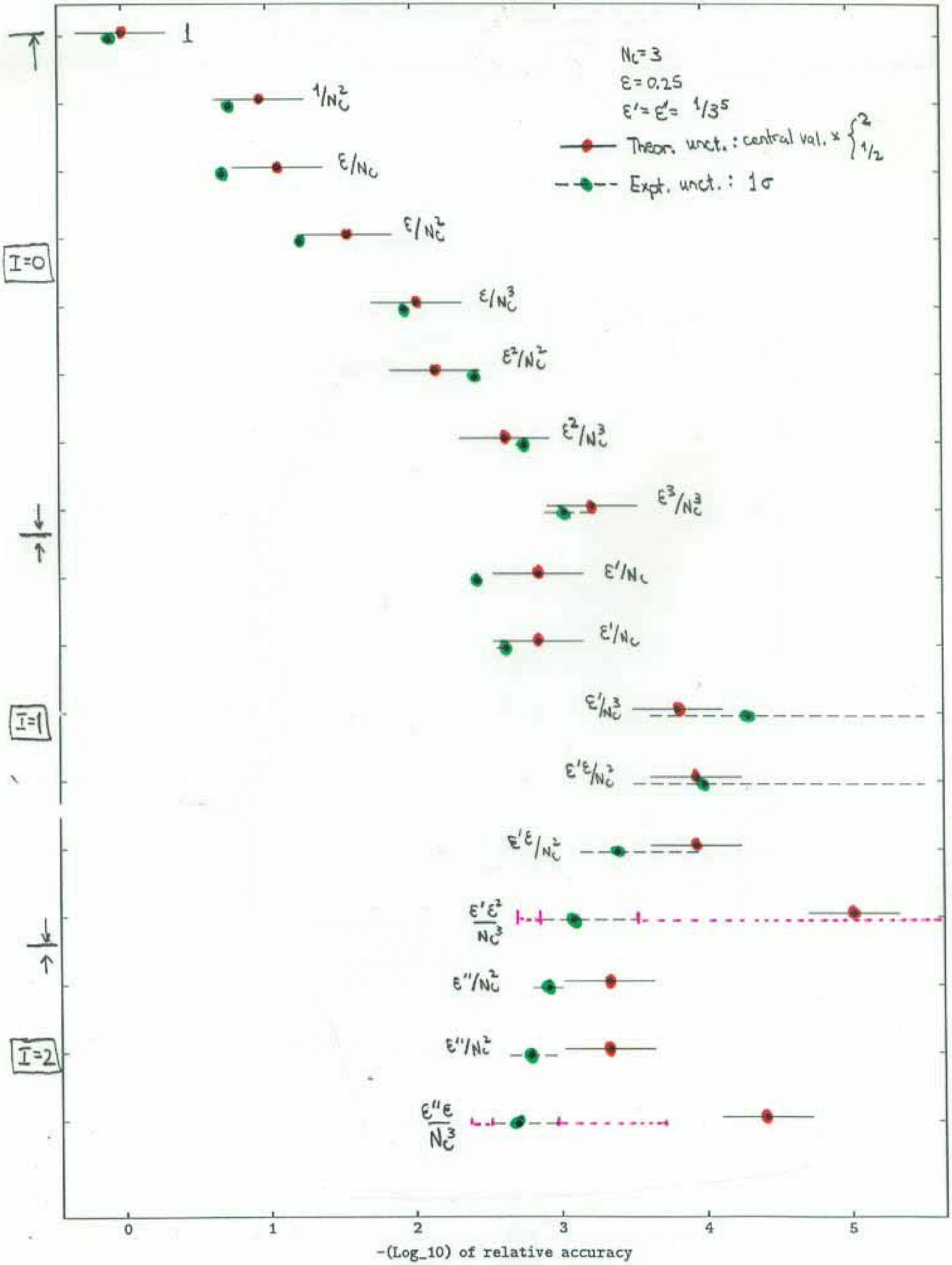
Finally, allow theoretical prediction to have uncertainty from unknown coefficient between $\frac{1}{2}$ and 2

$\Rightarrow$ Large N_c theoretical prediction

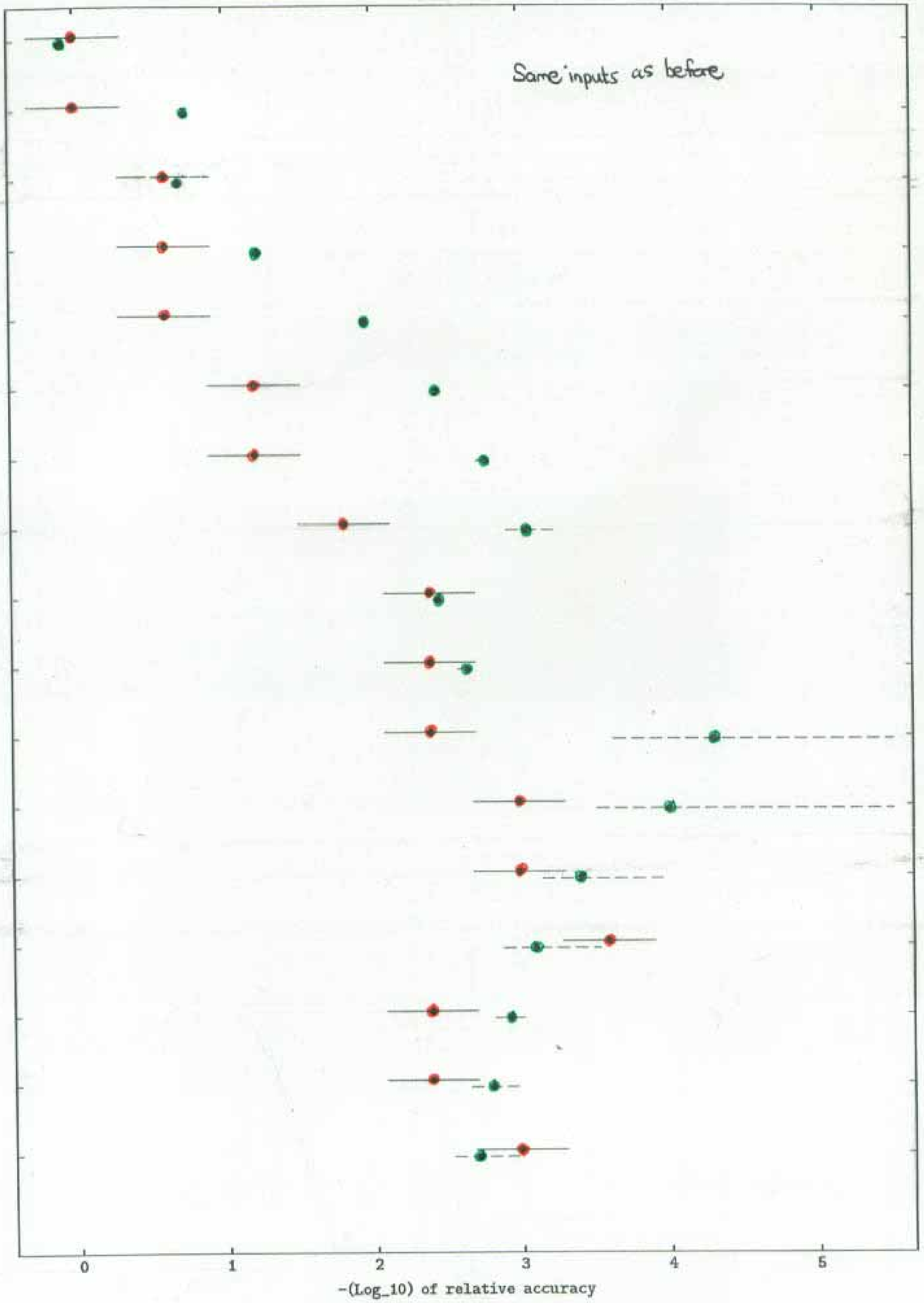
$$\frac{1}{2}\frac{\varepsilon^2}{N_c^2} \text{ to } 2\frac{\varepsilon^2}{N_c^2}$$

$$\approx \underline{\underline{0.34\% \text{ to } 1.38\%}}$$

Combined $1/N_c$ and flavor expansions vs. experiment



Flavor expansion with no $1/N_c$ suppressions



The $1/N_c$ expansion, including SU(3) and isospin breaking, is <u>necessary</u> and <u>sufficient</u> to understand the ground-state baryon mass hierarchy

Additional work on ground-state baryons includes work on:

- Axial couplings
- Magnetic moments
- Charge radii
- Quadrupole moments
- Heavy-quark baryons

So how about work on N^*'s?

Masses of N^*'s in $1/N_c$

- Building on the ground-state results, one expects every coefficient in the N^* Hamiltonian to be of "natural" size

BUT IN FACT, most turn out to be smaller than expected, a few are of expected size, and none are > expected

⇒ $1/N_c$ is working, but there is <u>additional</u> dynamics at work

Carlson, Carone, Goity, Lebed:
PLB 438, 327 (1998)
PRD 59, 114008 (1999)
Goity, Schat, Scoccola:
PRL 88, 102002 (2002)
hep-ph/0209174

<u>Sample of results</u>:

1) It is perfectly natural that $\Lambda(1405)$ is the lightest N^*, despite having a strange quark — The hyperfine operator does not contribute to its mass, but pushes up the mass of nonsinglet baryons by 2-300 MeV

2) The $N^*(1535)-N^*(1650)$ and $N^*(1520)-N^*(1700)$ mixing angles can be obtained from pion production decays, photoproduction, or masses, and all three agree numerically

3) Operators with flavor dependence (isospin exchange) are vital to obtain good results, e.g., for mixing angles

4) The spin-orbit coupling is relatively small, but nevertheless explains $\Lambda(1520)-\Lambda(1405)$ splitting

TABLE I: List of operators and the coefficients resulting from the best fit to the known **70**-plet masses and mixings [11].

Operator	Fitted coef. [MeV]
$O_1 = N_c \, 1$	$c_1 = \ 449 \pm 2$
$O_2 = l_i \, s_i$	$c_2 = \ 52 \pm 15$
$O_3 = \frac{3}{N_c} \, l_{ij}^{(2)} \, g_{ia} \, G_{ja}^c$	$c_3 = \ 116 \pm 44$
$O_4 = \frac{4}{N_c+1} \, l_i \, t_a \, G_{ia}^c$	$c_4 = \ 110 \pm 16$
$O_5 = \frac{1}{N_c} \, l_i \, S_i^c$	$c_5 = \ 74 \pm 30$
$O_6 = \frac{1}{N_c} \, S_i^c \, S_i^c$	$c_6 = \ 480 \pm 15$
$O_7 = \frac{1}{N_c} \, s_i \, S_i^c$	$c_7 = \ -159 \pm 50$
$O_8 = \frac{2}{N_c} \, l_{ij}^{(2)} s_i \, S_j^c$	$c_8 = \ 3 \pm 55$
$O_9 = \frac{3}{N_c^2} \, l_i \, g_{ja} \{S_j^c, G_{ia}^c\}$	$c_9 = \ 71 \pm 51$
$O_{10} = \frac{2}{N_c^2} t_a \{S_i^c, G_{ia}^c\}$	$c_{10} = \ -84 \pm 28$
$O_{11} = \frac{3}{N_c^2} \, l_i \, g_{ia} \{S_j^c, G_{ja}^c\}$	$c_{11} = \ -44 \pm 43$
$\bar{B}_1 = t_8 - \frac{1}{2\sqrt{3}N_c} O_1$	$d_1 = \ -81 \pm 36$
$\bar{B}_2 = T_8^c - \frac{N_c-1}{2\sqrt{3}N_c} O_1$	$d_2 = \ -194 \pm 17$
$\bar{B}_3 = \frac{10}{N_c} \, d_{8ab} \, g_{ia} \, G_{ib}^c + \frac{5(N_c^2-9)}{8\sqrt{3}N_c^2(N_c-1)} O_1 + \frac{5}{2\sqrt{3}(N_c-1)} O_6 + \frac{5}{6\sqrt{3}} O_7$	$d_3 = \ -15 \pm 30$
$\bar{B}_4 = 3 \, l_i \, g_{i8} - \frac{\sqrt{3}}{2} O_2$	$d_4 = \ -27 \pm 19$

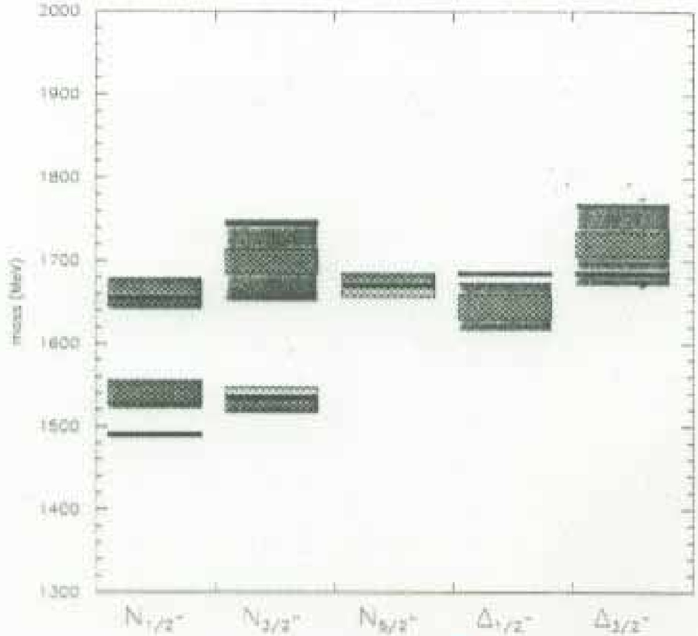
Expected size: ~ 500 MeV

Expected size: ~ 150-200 MeV

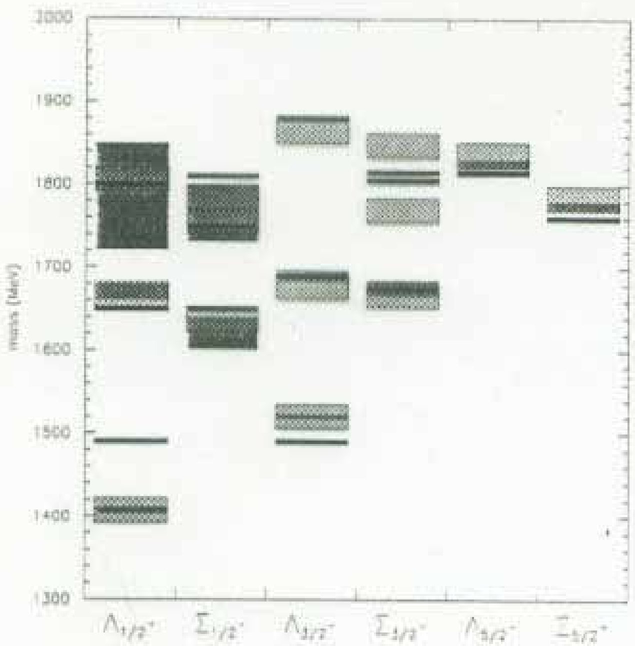
χ^2/d.o.f. = 1.29

1/Nc fit to N* baryon masses

Shaded boxes = DATA
Dark lines = Isgur/Karl Quark Model
Cross-hatching = 1/Nc RESULT



Strangeness 0



Strangeness -1

Excited baryon production and decays

- One may analyze $N^* \to N\gamma$: 19 measured modes, using the $1/N_c$ expansion [Carlson + Carone, PRD 58, 053005 (1998)]

  There are 1-quark and 2-quark operators of the same order in N_c

  e.g. $Q_{exc.} \vec{\varepsilon}_{exc} \cdot \vec{\varepsilon}_\gamma$ vs. $\left(\sum_{\alpha \neq exc.} Q_\alpha \frac{\vec{S}_\alpha}{N_c} \right) \cdot \vec{S}_{exc.} \left(\vec{\varepsilon}_{exc} \cdot \vec{\varepsilon}_\gamma \right)$

  (exc = excited quark)

  As before, all of the coefficients turn out to be <u>at most</u> of expected size

  <u>BUT</u> a detailed fit shows that the 2-quark operators are much less relevant

  $\Rightarrow$ $1/N_c$ works, but additional dynamics is evident

- Using this empirical observation, that 1-quark operators dominate, one may now predict vast numbers (24) of $N^* \to \Delta\gamma$ amplitudes [Carlson + Carone, PLB 441, 363 (1998)]

  – Careful analysis at modern machines (e.g. JLab) can extract such amplitudes and test the one-quark ansatz

- One may also study the $56' \to 56$ + meson decays using $1/N_c$ and the 1-quark ansatz [Carlson + Carone, PLB 484, 260 (2000)]

  Is the Roper a qqq radial excitation or something else?

  Measuring the decay modes and using the $1/N_c$ expansion may sort this out

What Next?

1) The $1/N_c$ expansion is manifest in baryons.
It works beautifully for ground-state baryons
HOWEVER,

2) There is additional dynamics at work, as clearly indicated in N^*s
So $1/N_c$ will be a starting point for future models

- I believe that the key missing ingredient is chiral symmetry
Models that respect both pieces of physics should be very successful

A complete analysis of the large amount of available and forthcoming data, building upon the success of the $1/N_c$ expansion, should illuminate for us the precise nature of the N^* resonances