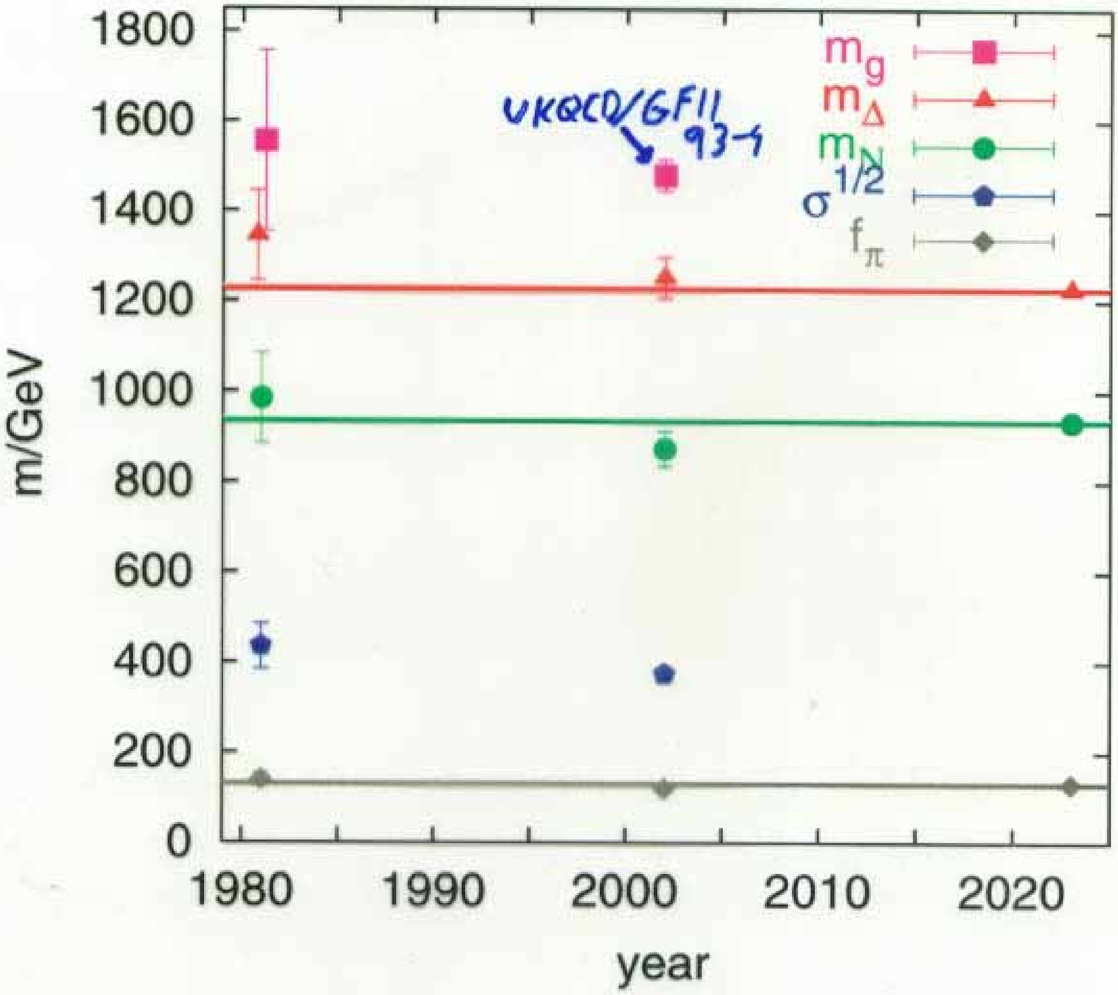
Baryons from the lattice: past, present and future

Gunnar Bali (Glasgow)

- **Lattice Summary**
- **Extrapolations and "Quenching"**
- **Spectrum**
- **Potentials**
- **Structure**
- **Conclusion**

N^* Pittsburgh, 10/10/02

Lattice Summary



1981: H. Hamber, G. Parisi, PRL 47, 1792 (81)

2002: CP-PACS: S. Aoki *et al.* hep-lat/0206009

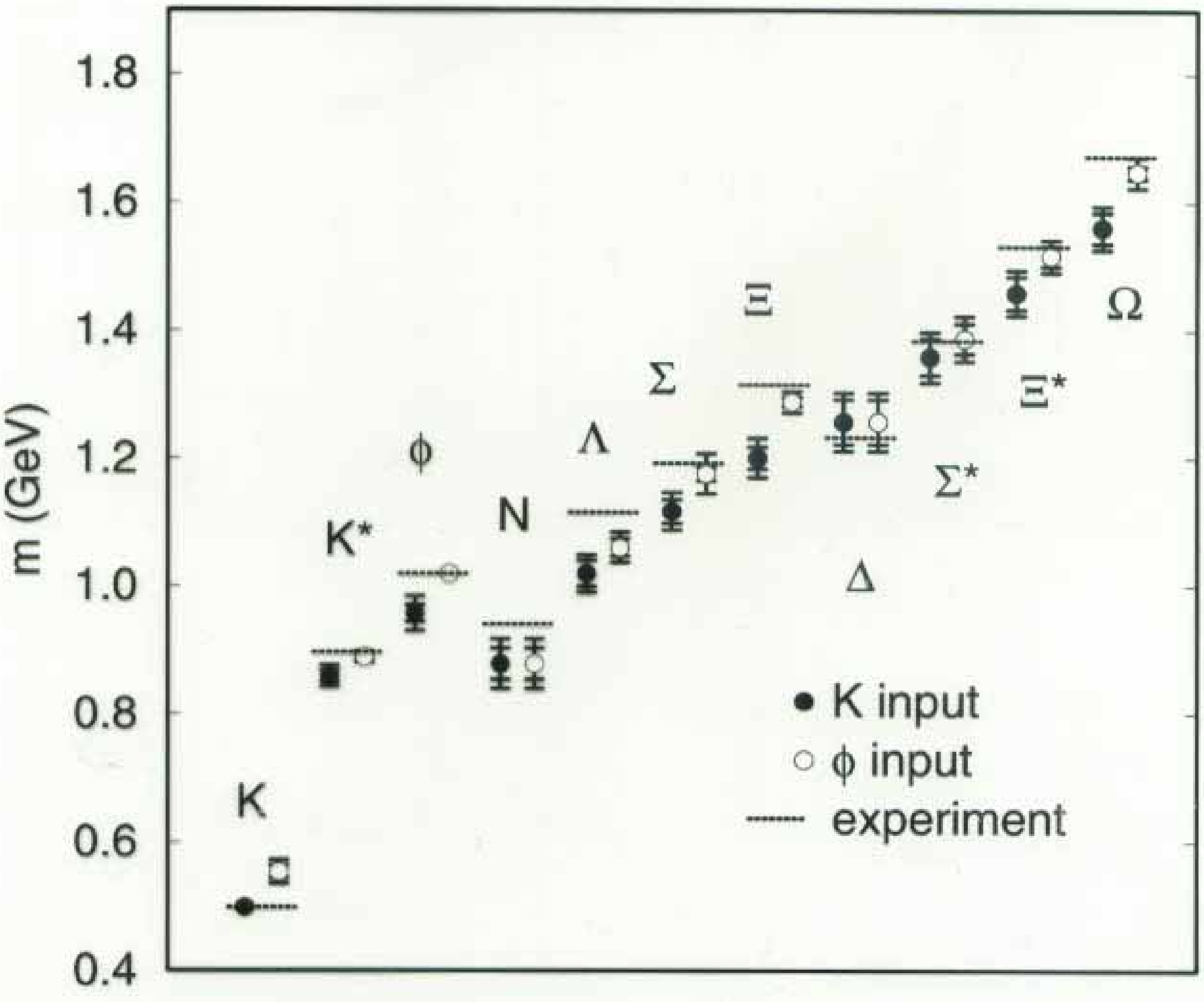
1981: wrong method, wrong theory

2002: right method, wrong theory

2023: right method, right theory ???

Light hadron spectrum

No sea quarks! CP-PACS: S. Aoki et al. '02

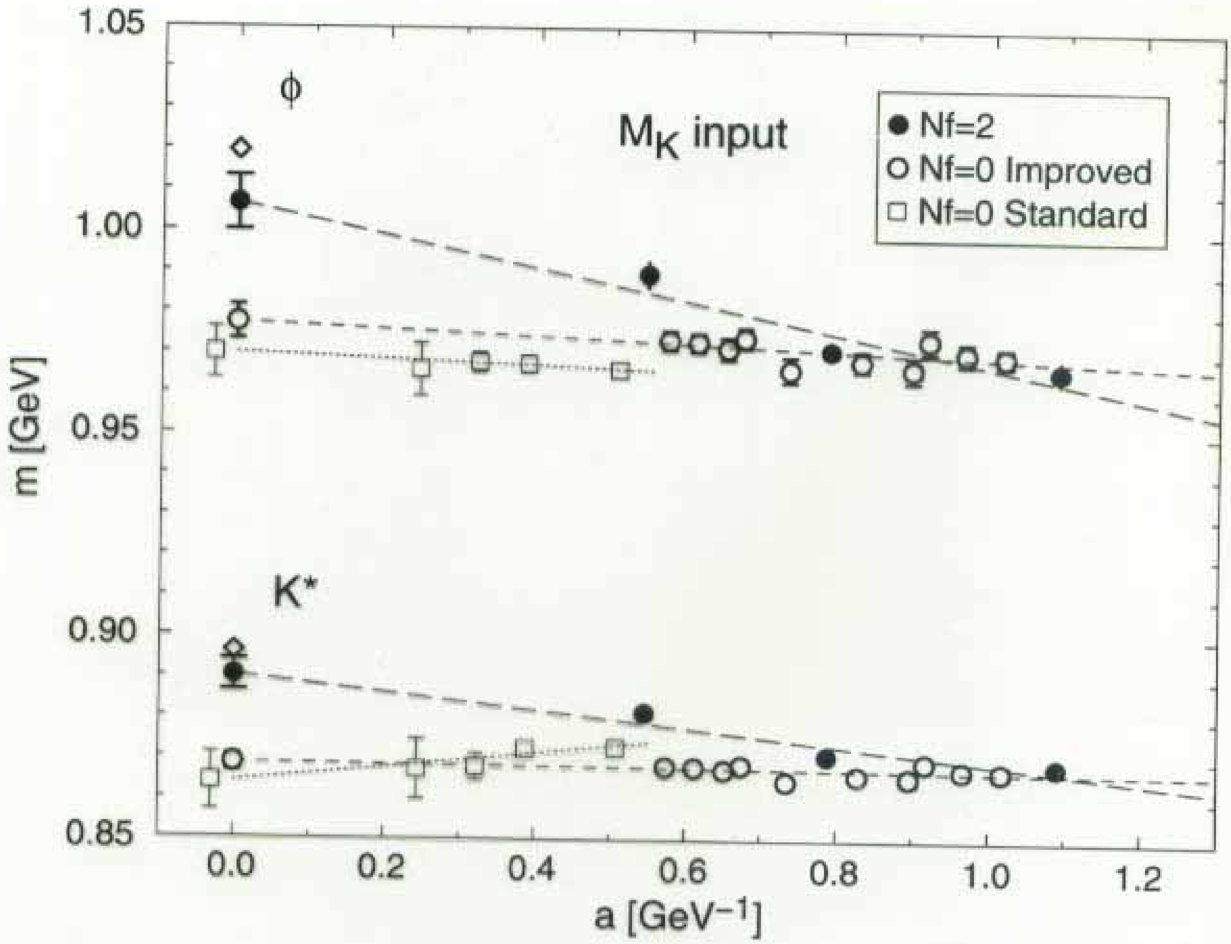


$La \approx 3$ fm, $a \approx 0.1, 0.075, 0.067, 0.05$ fm

QM: 10% error on mass

↝ 20% " " wave fctn ?

$n_f = 2$ CP-PACS: S. Aoki et al. '00



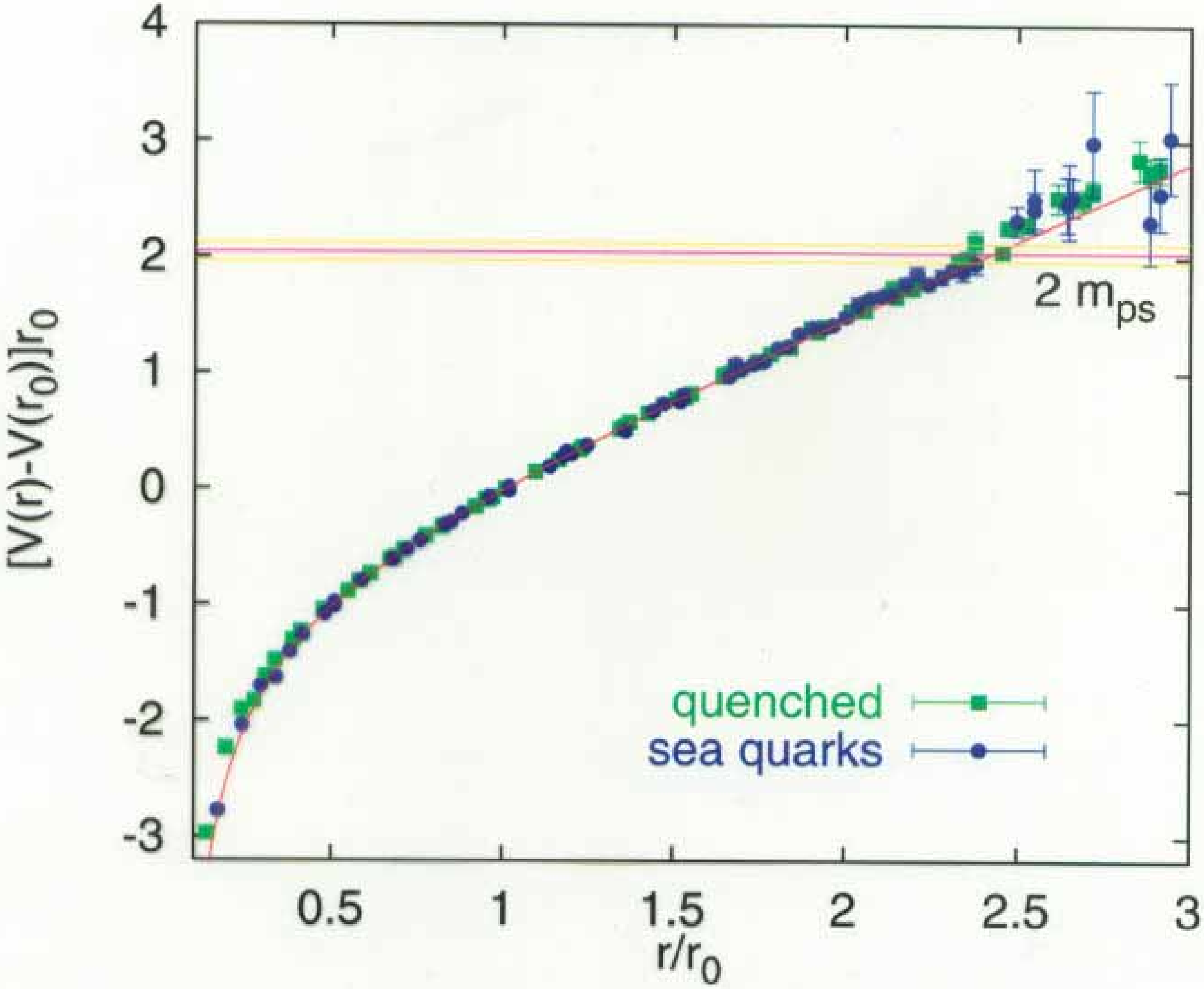
$La \approx 2.5$ fm, $a \approx 0.21, 0.16, 0.11$ fm,

$m_\pi/m_\rho \approx 0.8, 0.75, 0.69, 0.57 \gg 0.18$

$\overline{MS}$ masses at $\mu = 2$ GeV	m_{ud}/MeV from m_ρ	m_s/MeV from m_K	m_s/MeV from m_ϕ
$n_f = 0$	4.57 (18)	116(3)	144(6)
	4.36 (15)	110(6)	132(6)
$n_f = 2$	3.44 (18)	88 (6)	90 (8)

Static $q\bar{q}$ potential

$a \approx 0.08$ fm, $m_q \approx m_s/3$, $n_f = 2$, $r_0 \approx 0.5$ fm



SESAM: GB *et al.*,    '00

The Small print

Decay of correlation functions at infinite time:

$$m_O = \lim_{t \to \infty} m_O^{\text{eff}}(t) = a^{-1} \lim_{t \to \infty} \frac{d \ln C_O(t/a)}{d(t/a)}$$

$$C_O(t) = \langle \Psi_O(t) | \Psi_O(0) \rangle$$

Optimize Ψ_O such that $m_O \approx m_O^{\text{eff}}(t)$ at finite t/a!

For matrix elements, couplings, quark masses:

renormalization and operator mixing: perturbative, non-perturbative

Extrapolations:

Continuum: $a \to 0$

Infinite volume: $V \to \infty$

Chiral: $m_q \to m_{u,d} \approx 0$

Quenching: source of uncontrolled errors

$$\mathcal{L}_{QCD} = -\frac{1}{16\pi\alpha_s} FF + \bar{\psi}_f(\not{D} + m_f)\psi_f$$

"asymptotic freedom": $\alpha_s(q) \to 0$ as $q \to \infty$

"confinement": no free quarks or gluons

"chiral symmetry breaking":

$m_g = 0$

$m_u,\ m_d < 5$ MeV

But: $m_N \sim 1$ GeV

Global $U_V(1)$ symmetry: baryon number conservation.

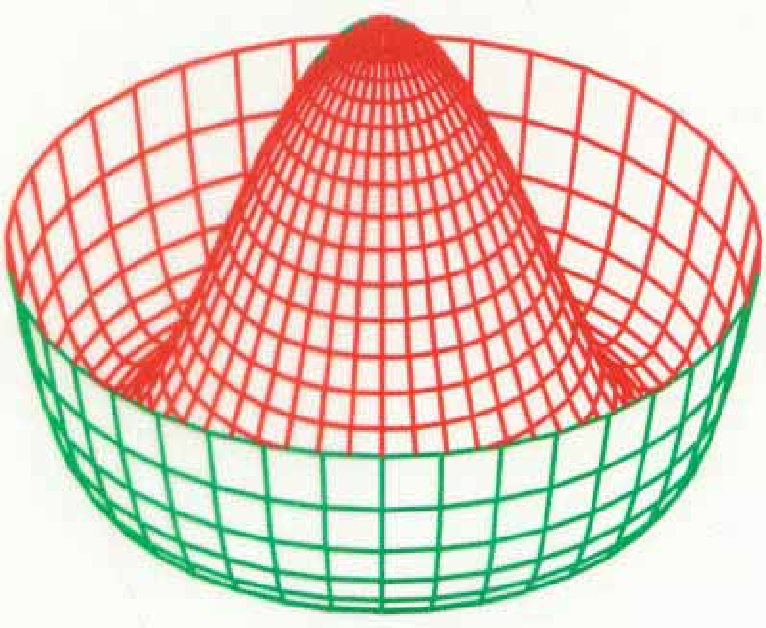
Broken $U_A(1)$ symmetry: $\partial_\mu j^5_\mu = -\frac{1}{16\pi^2} F * F$

$m_f = 0$: χ symmetry *spontaneously* broken at $T < T_c$:

$$SU_L(n_f) \otimes SU_R(n_f) \longrightarrow SU_V(n_f)$$

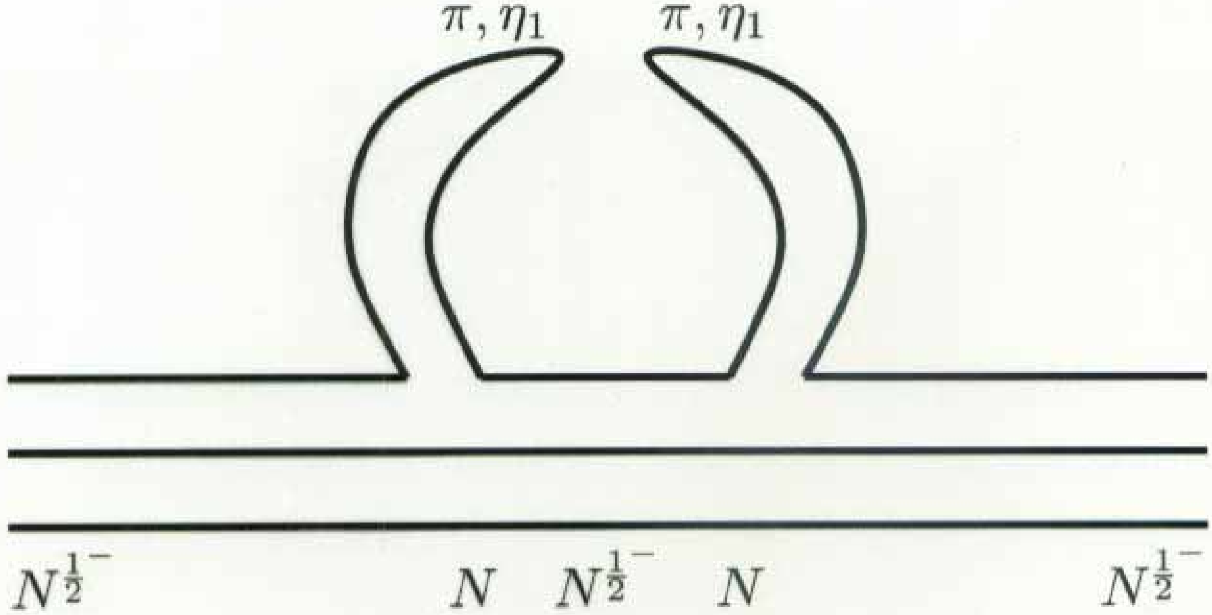
Quenched approximation:

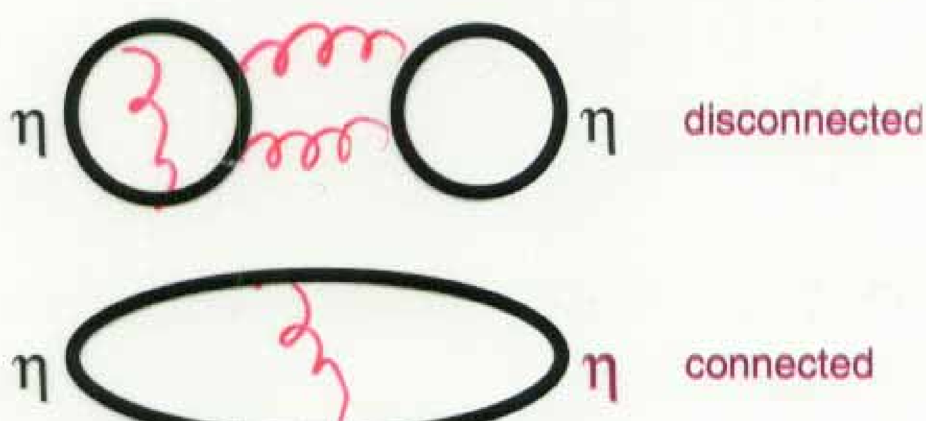
$$SU_L(n_f) \otimes SU_R(n_f) \otimes U_A(1) \longrightarrow SU_V(n_f)$$



n_f^2 instead of $n_f^2 - 1$ Nambu-Goldstone bosons!

$$N^{\frac{1}{2}^-} \longleftrightarrow \begin{matrix} N+\pi \\ N+\eta_1 \end{matrix} \; !$$



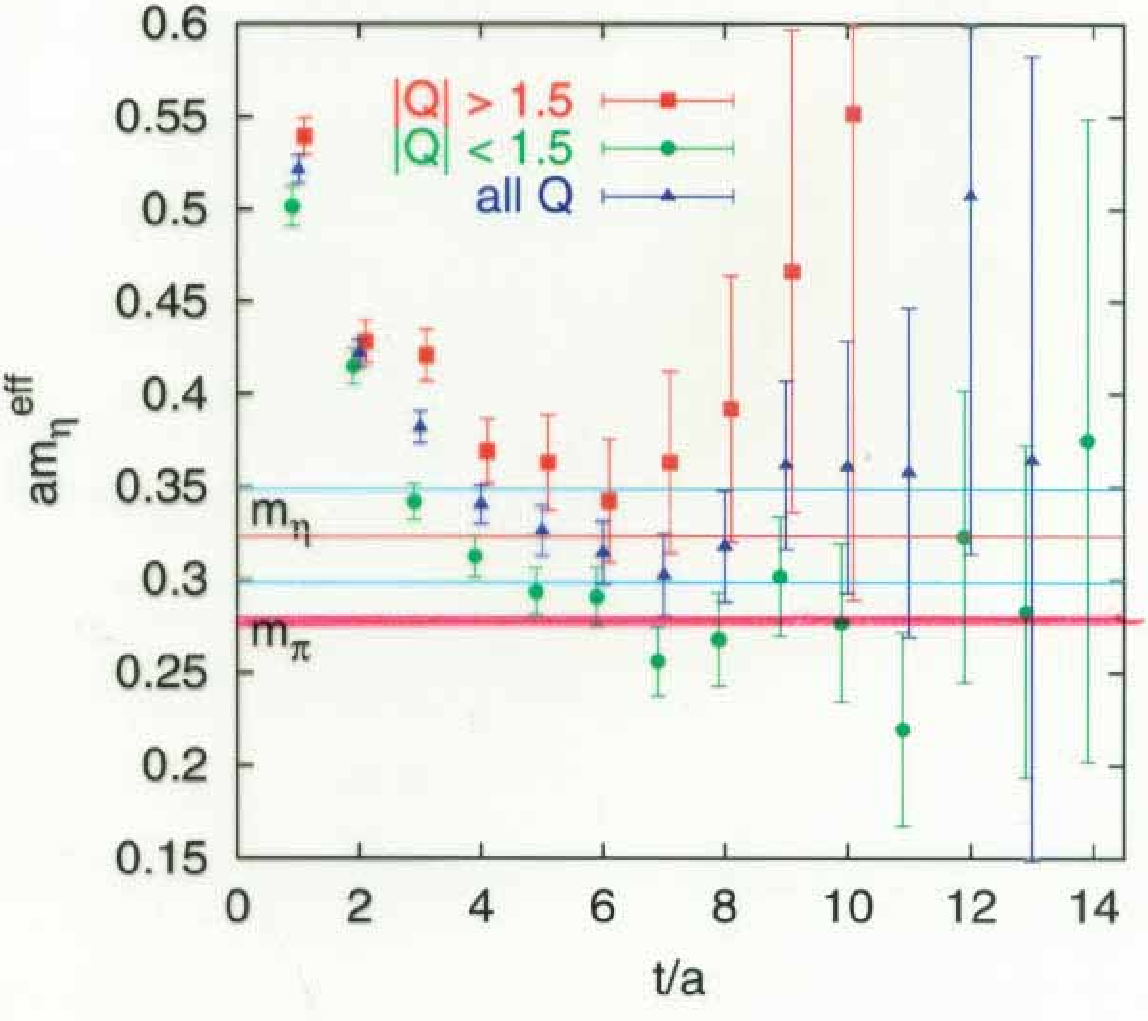


$$C_{\eta_1}(t) = C_{\mathrm{conn}}(t) - C_{\mathrm{disc}}(t) \xrightarrow{t\to\infty} c\, e^{-m_{\eta_1} t}$$

η mass for $n_f = 2$ "QCD" SESAM: GB et al. '01

$$m^{\text{eff}}(t) = a^{-1} \ln[C(t)/C(t+a)]$$

$$Q = (F, *F)/(16\pi)^2$$



$a \approx 2.35$ GeV, $L_\sigma a \approx 1.35$ fm, $m_\pi/m_\rho \approx 0.69$.

χ Lagrangian: $\mathcal{L}_\chi = \frac{1}{2}(\partial_\mu \phi^a)^2 + \cdots + \mathcal{O}(m_\pi/\Lambda_{\chi SB})$

$\Lambda_{\chi SB} \approx 4\pi f_\pi > 1$ GeV.

Baryon sizes: Λ_B^{-1}, $\Lambda_B < 400$ MeV.

χ extrapolation:

"full" QCD:

$$m_B = m_B(0) + c_2 m_\pi^2 + c_3 m_\pi^3 + c_{4L} m_\pi^4 \ln m_\pi + c_4 m_\pi^4 + \cdots$$

qQCD:

$$m_B^q = m_B^q(0) + c_1^q m_\pi + c_{2L}^q m_\pi^2 \ln m_\pi + c_2^q m_\pi^2 + c_3^q m_\pi^3 + \cdots$$

Analytic terms: powers of $m_\pi^2 \propto m_q$

Non-analytic terms: everything else

c_i are related to low energy parameters in $\mathcal{L}_\chi$, e.g. $c_3 = -3g_A^2/(32\pi f_\pi^2)$.

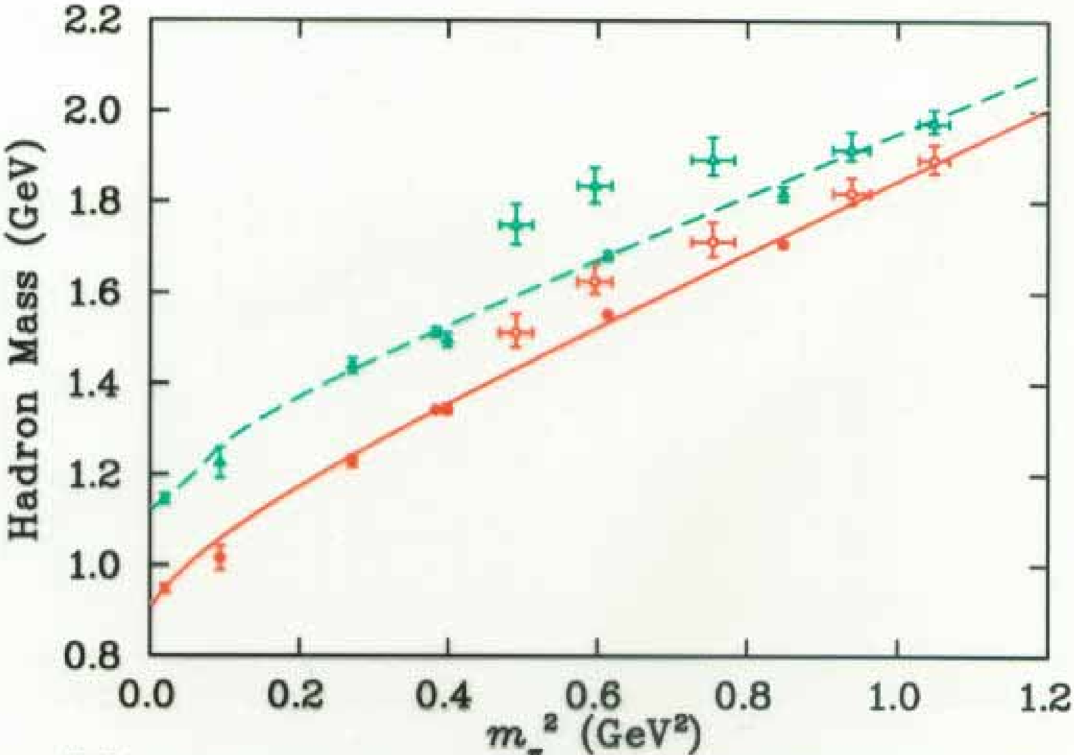
Lattice QCD: little evidence for non-analytic behaviour

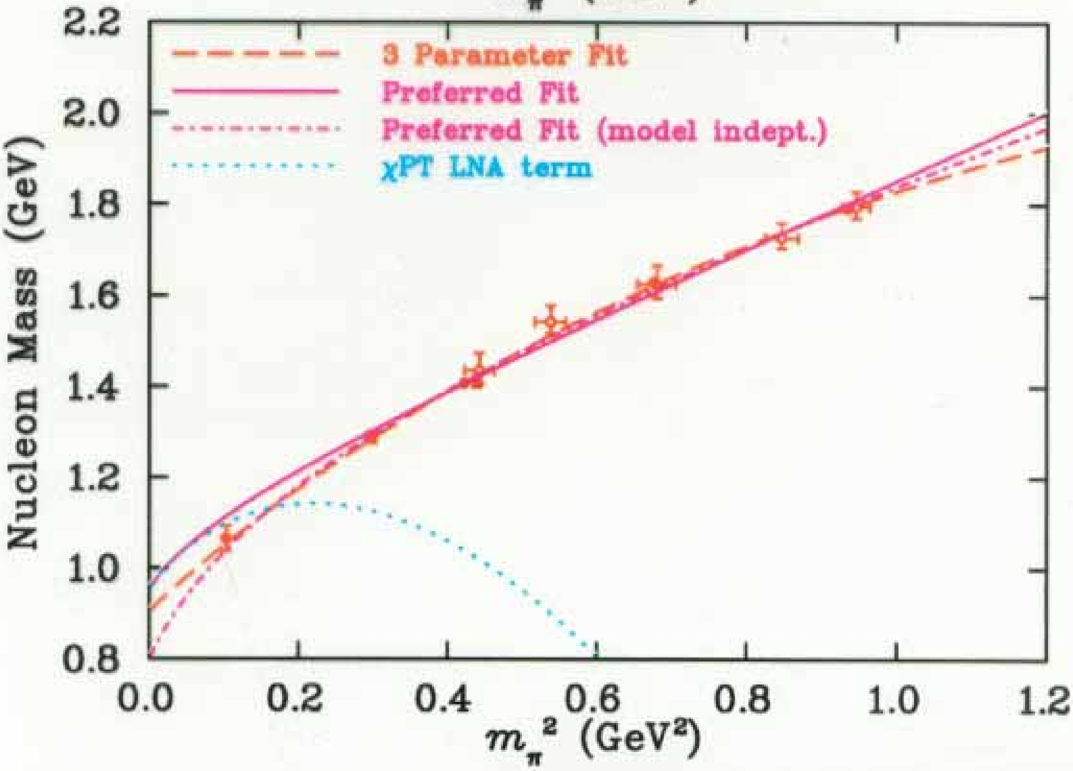
Adelaide: Leinweber, Thomas, Young, Wright, ...:

interpolation between heavy quark limit and χPT.

One new parameter per hadron: "size of pion cloud".

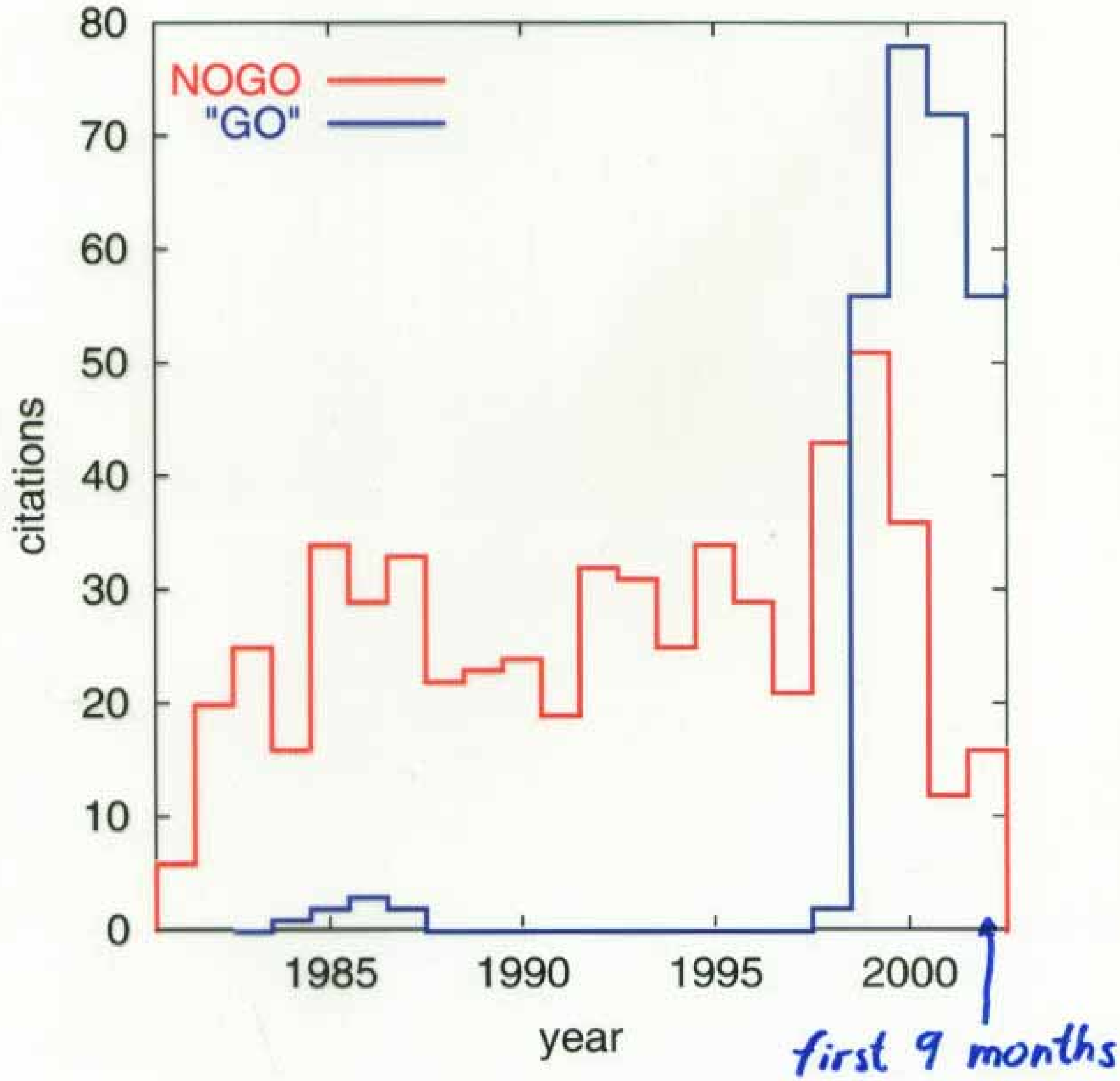
Adelaide: χ fit of CP-PACS/UKQCD m_N and m_Δ.





Chiral fermions

Standard lattice actions explicitely violate χ symmetry at finite lattice spacing a



Recent theoretical development:

Overlap and Domain Wall lattice fermions

N^* Spectrum

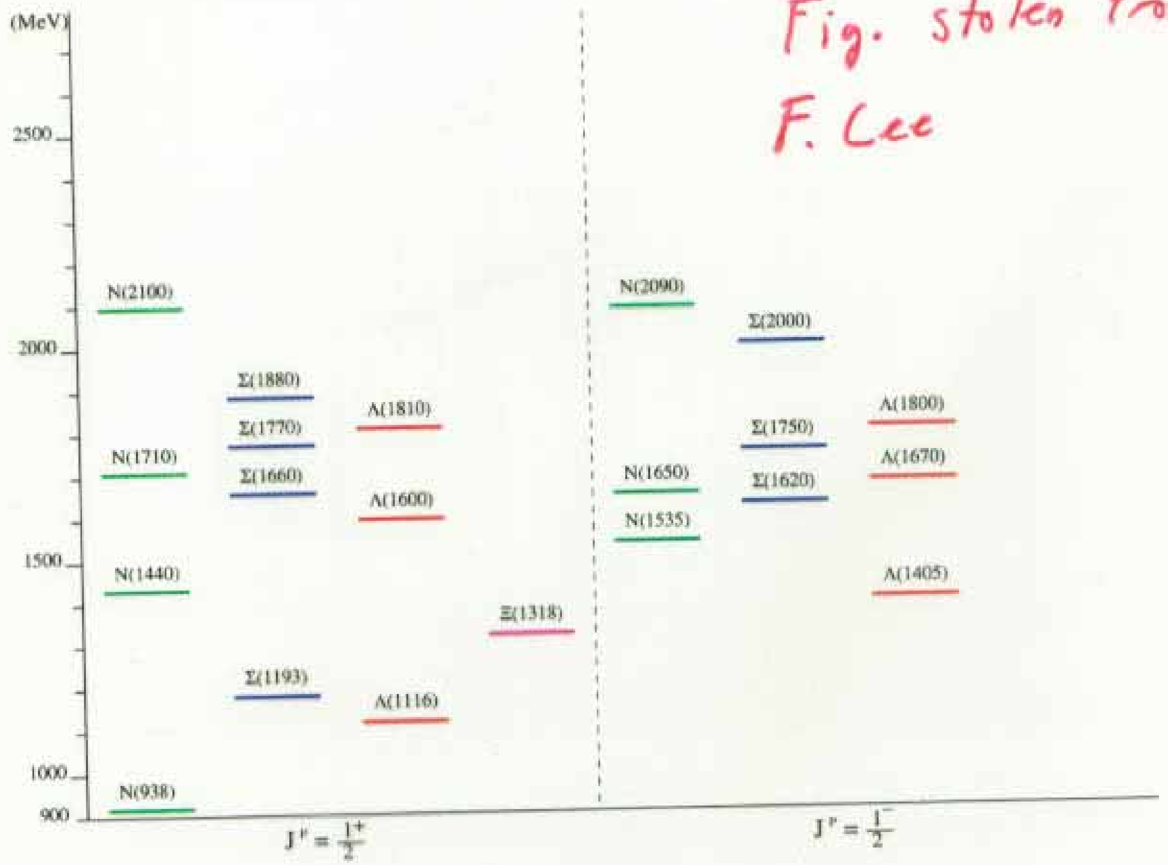


Fig. stolen from F. Lee

Questions:

nature of Roper $N(1405)$ resonance? Missing states?

rôle of excited glue?

qq interactions

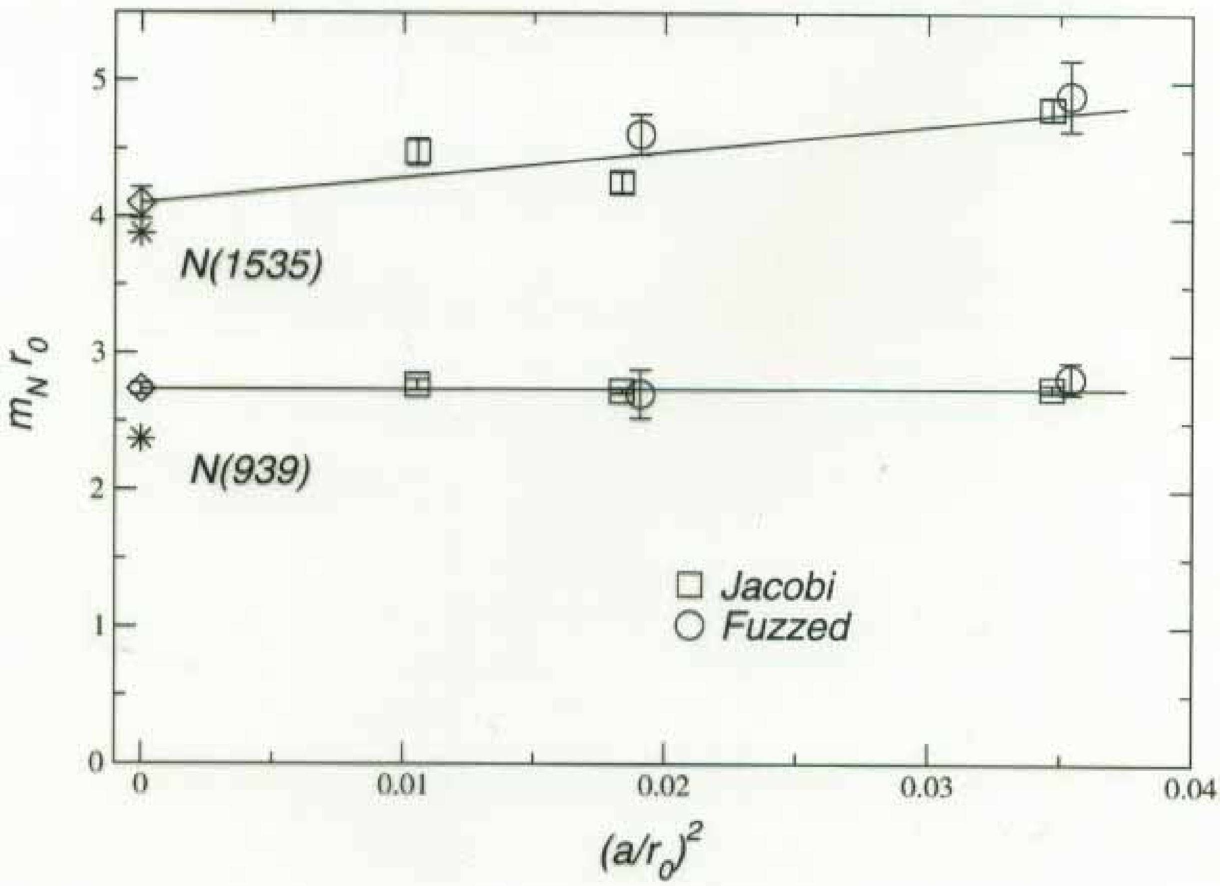
quark-diquark picture

Is exact chiral symmetry important?

So far all Lattice studies are in qQCD.

$N^{1/2^-}$ LHPC/UKQCD/QCDSF: M. Göckeler et al. '01

$$\left[\begin{array}{ccccccccc} 3\otimes 3\otimes 3 & = & 1 & \oplus & 8 & \oplus & 8 & \oplus & 10 \\ J^P & & \frac{1}{2}^- & & \frac{1}{2}^+ & & \frac{1}{2}^- & & \frac{3}{2}^+ \end{array}\right]$$



LHPC/UKQCD/QCDSF: Richards et al.: Wilson-Clover

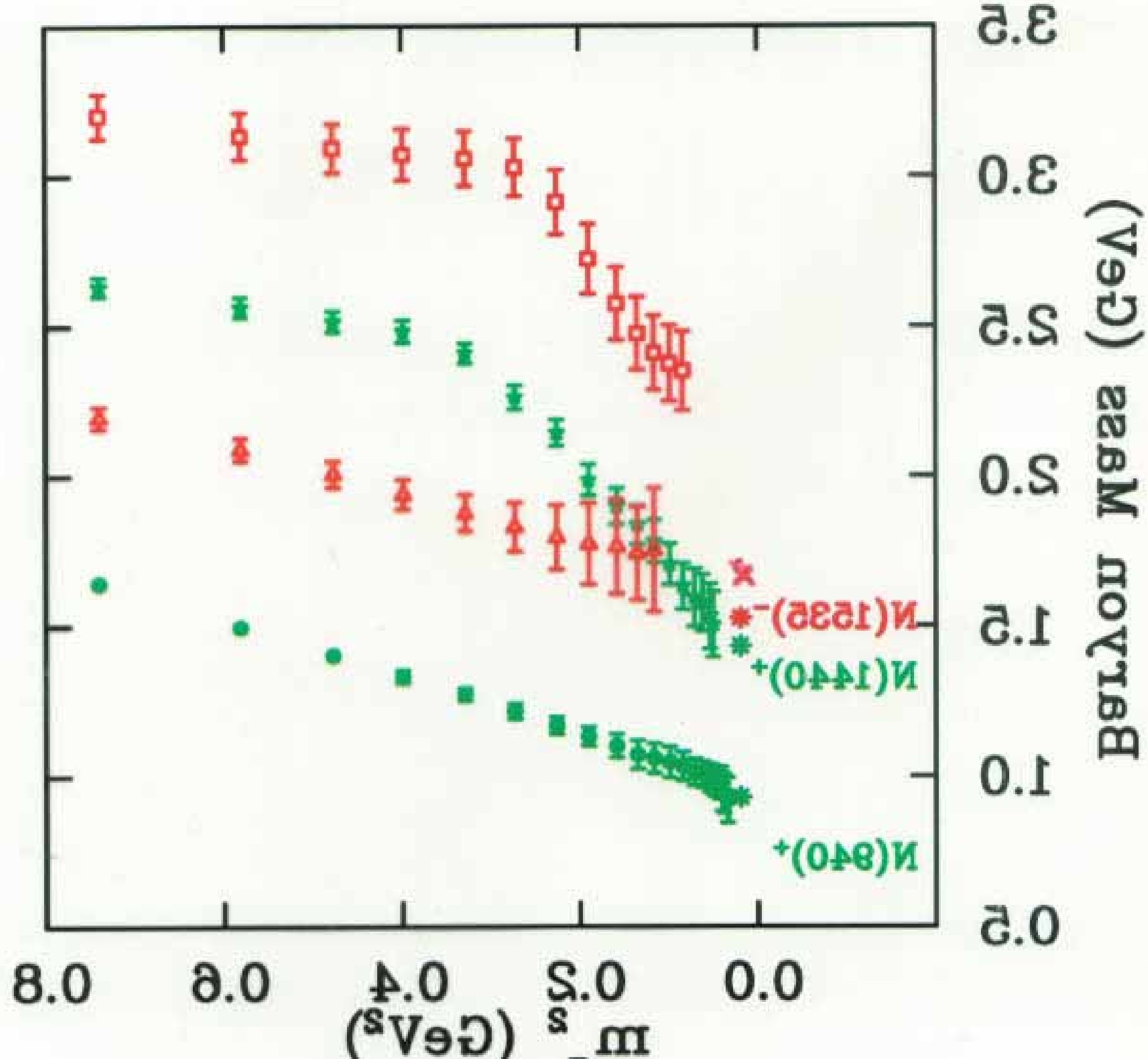
Riken-BNL: Sasaki et al.: Domain Wall

Lee (et al.): D_{234}, Overlap

Melnichouk et al.: "FLIC"

$N_{1/2^\pm}$

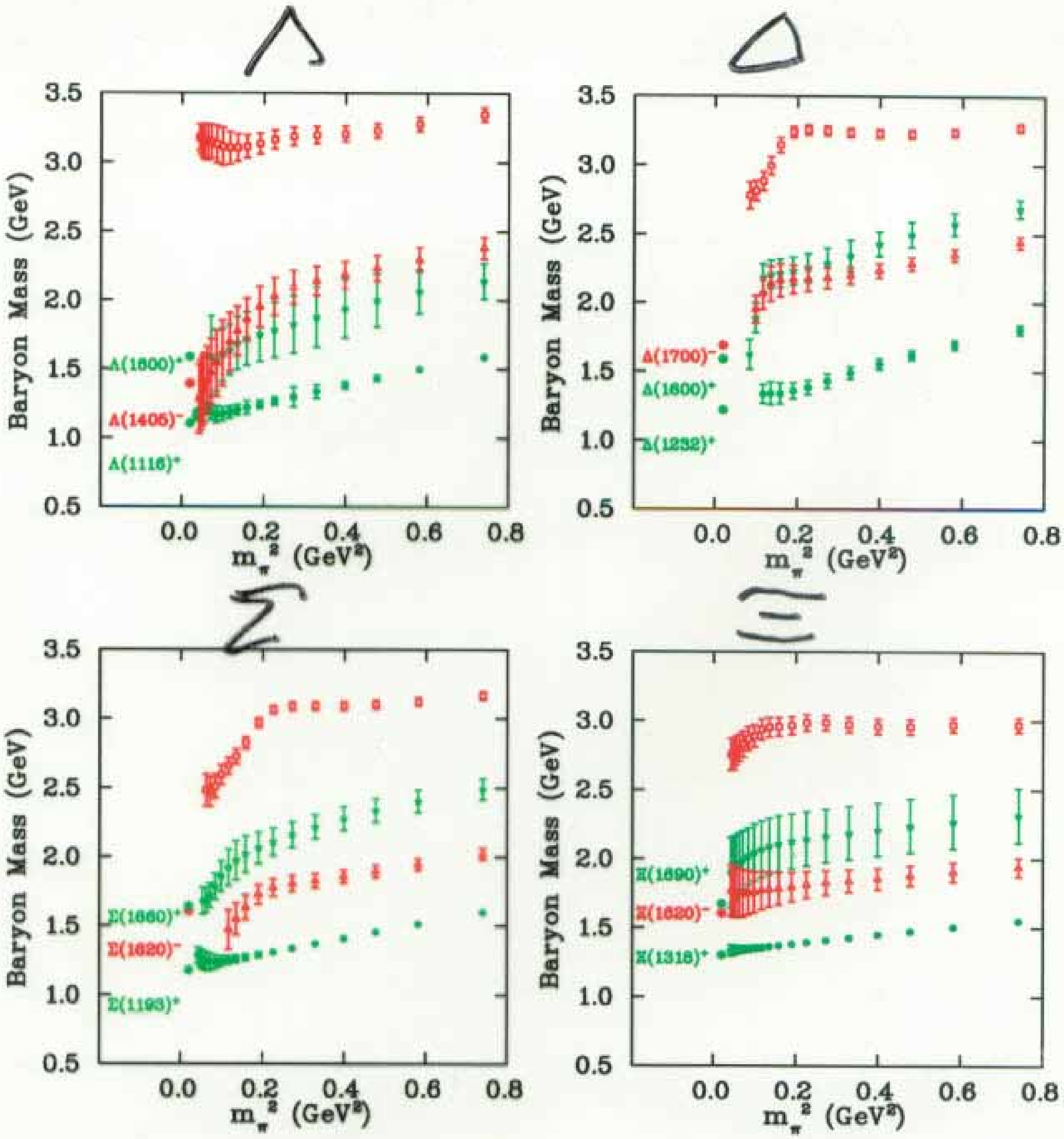
Lee, Dong, Draper, Horváth, Liu, Matur, Zhang '02



$a \approx 0.2$ fm, $L \approx 3$ fm, $m_\pi > 180$ MeV $\Rightarrow Lm_\pi > 2.7$

What about $N(1650)^-$?

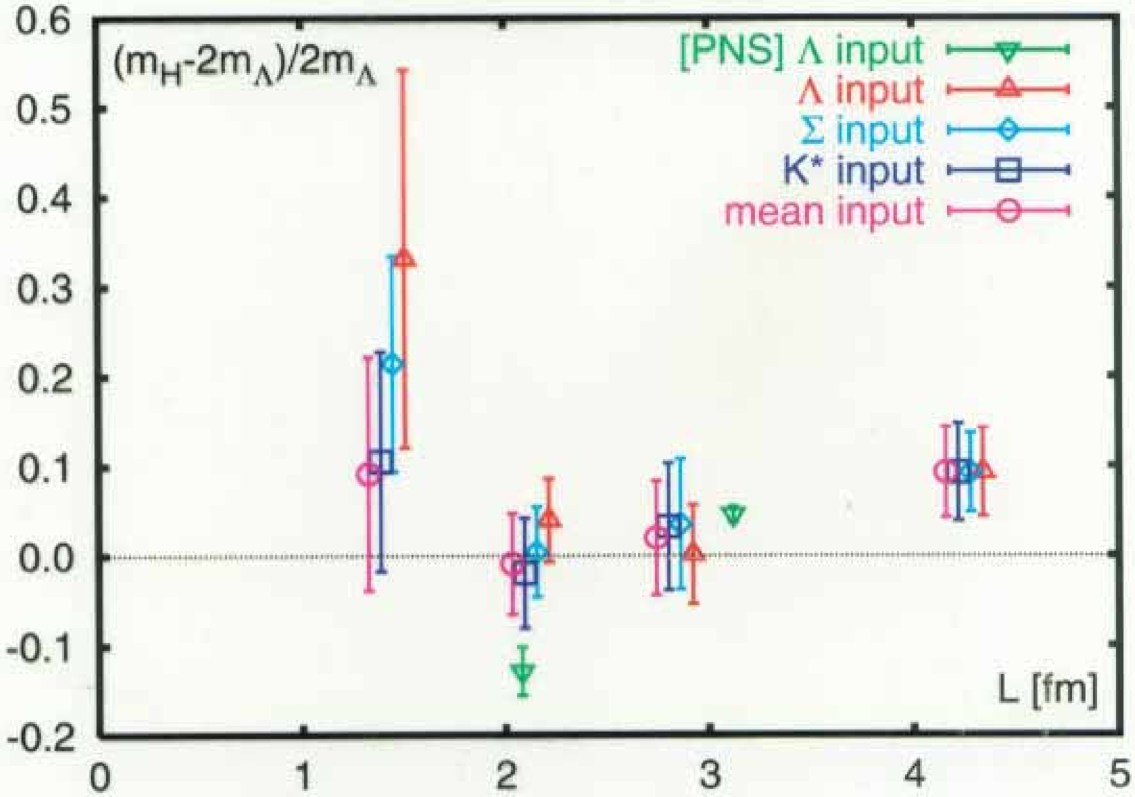
FSE ?



F. Lee parallel Session

H dibaryon Wetzorke, Karsch '02

Is there a $uuddss$ state below the 2231 MeV $\Lambda\Lambda$ threshold?



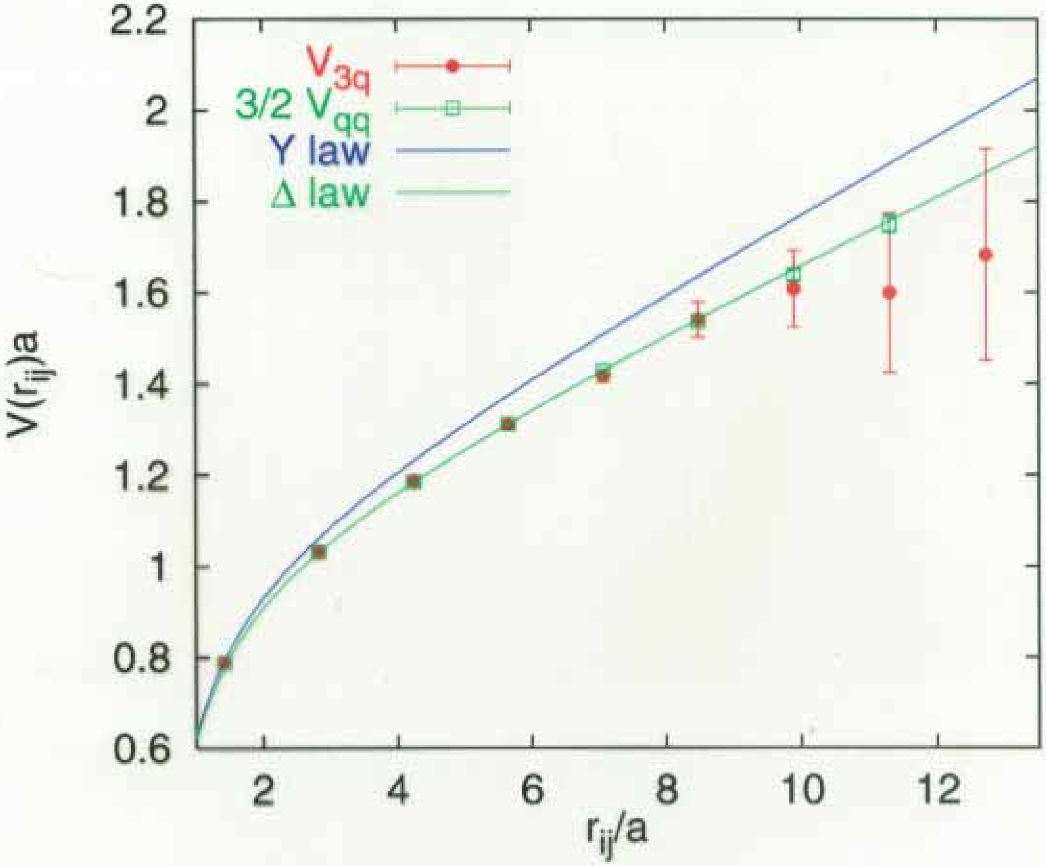
$a \approx 0.18$ fm, Symanzik improved glue and clover quarks.

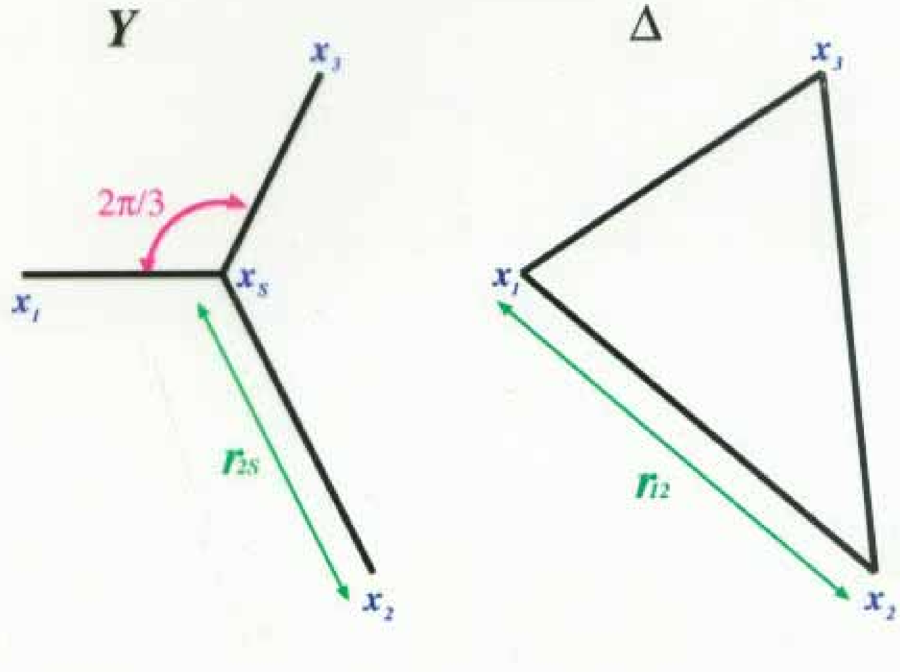
No!

3 body forces

GB '00

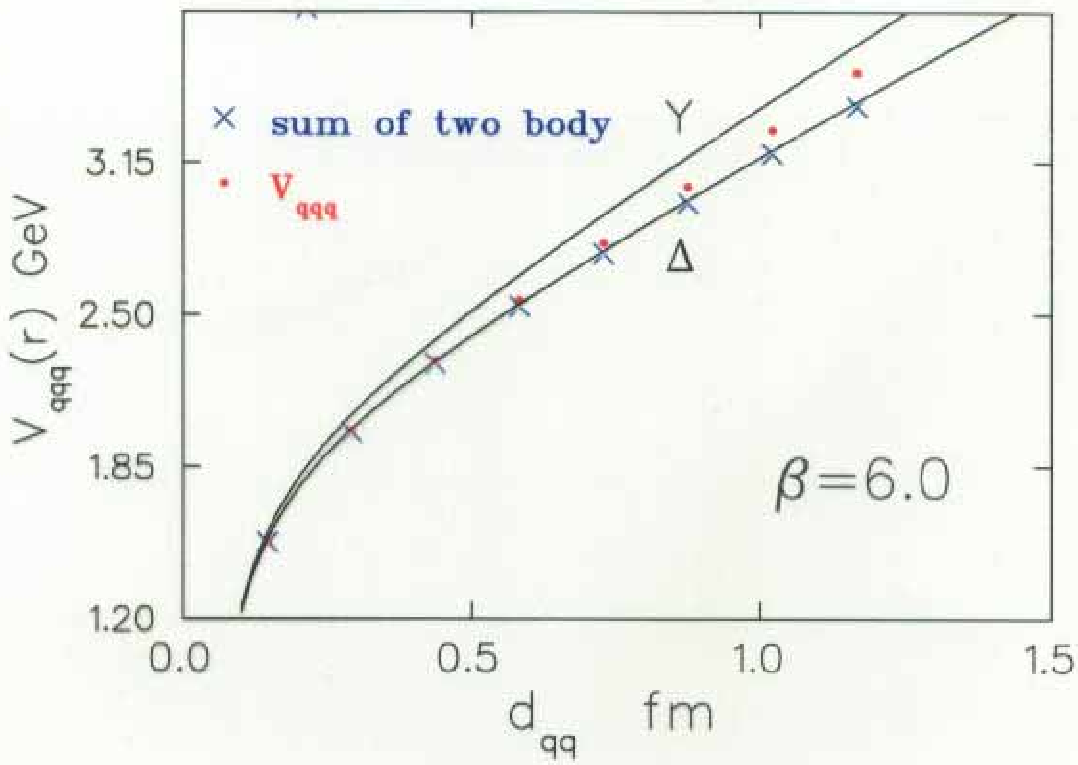
a ≈ 0.1 fm



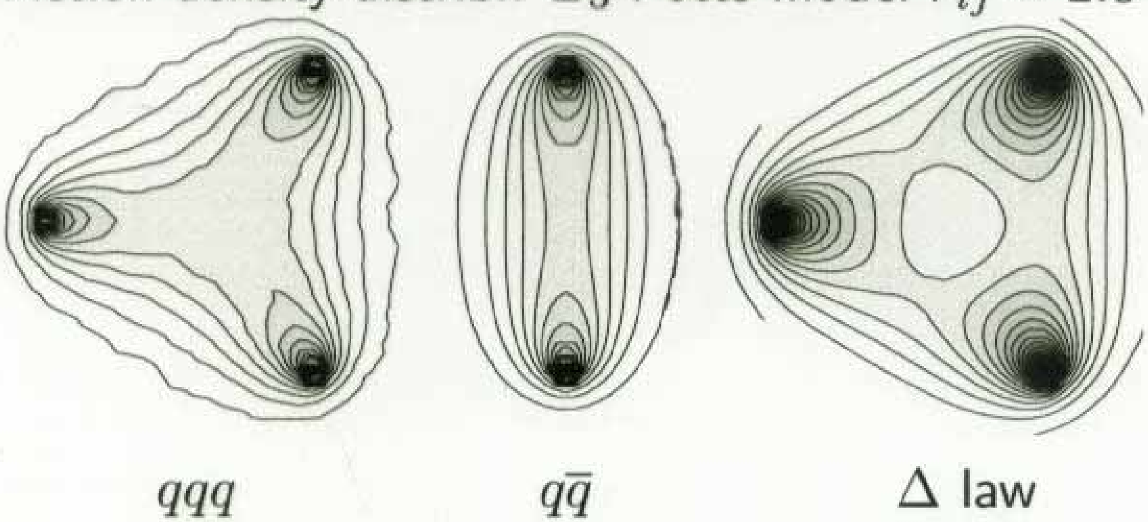


qqq potential

Alexandrou et al. '02



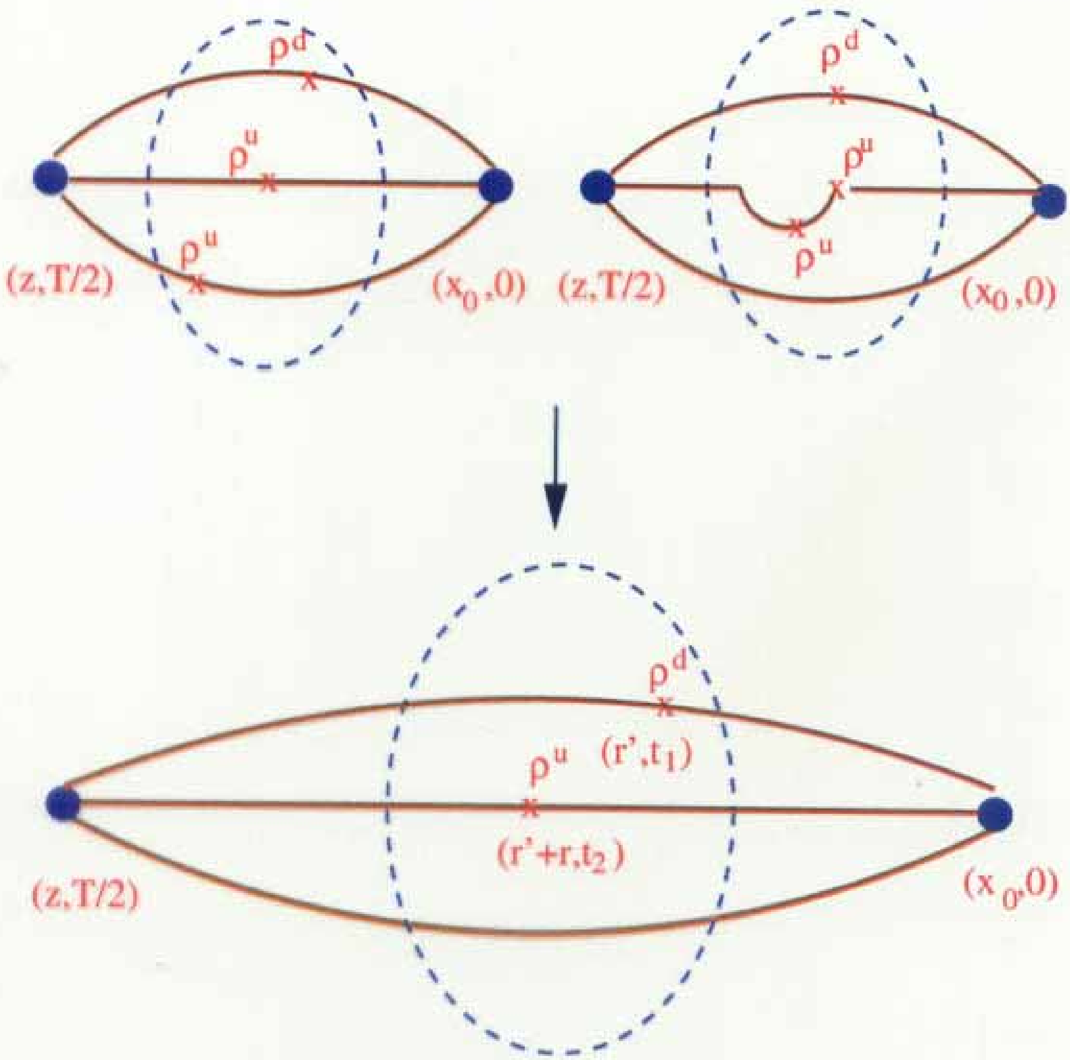
Action density distrib.: Z_3 Potts model $r_{ij} \approx 2.8$ fm:



Also work by Takahashi et al.

Density-density correlators

Alexandrou et al. '02



π form factor: $\int d^3r\, e^{i\mathbf{q}\mathbf{r}} C(\mathbf{r}) \xrightarrow{|\mathbf{q}|\to 0} \frac{(E+m_\pi)^2}{2E} \times F^2(\mathbf{q}^2)$

charge radius: $C(\mathbf{r}_1, \mathbf{r}_2) \Rightarrow r_{ch}^2 = \sum_{q=1}^{3} \langle e_q \mathbf{r}_q^2 \rangle_{\mathbf{r}_1,\mathbf{r}_2}$

Λ spin structure

QCDSF: Göckeler et al. '02

$$\langle \Lambda(p', s) | \bar{q}\gamma_\mu\gamma_5 q | \Lambda(p, s) \rangle = 2(p' - p)_\mu \Delta q$$

$SU(3)_F$: $\Delta s_\Lambda = \frac{1}{3}(2\Delta u_p - \Delta d_p + 2\Delta s_p)$

$\Delta u_\Lambda/d_\Lambda = \frac{1}{6}(\Delta u_p + 4\Delta d_p + \Delta s_p)$

quenched simulation, $a \approx 0.1$ fm, $L \approx 1.6$ fm

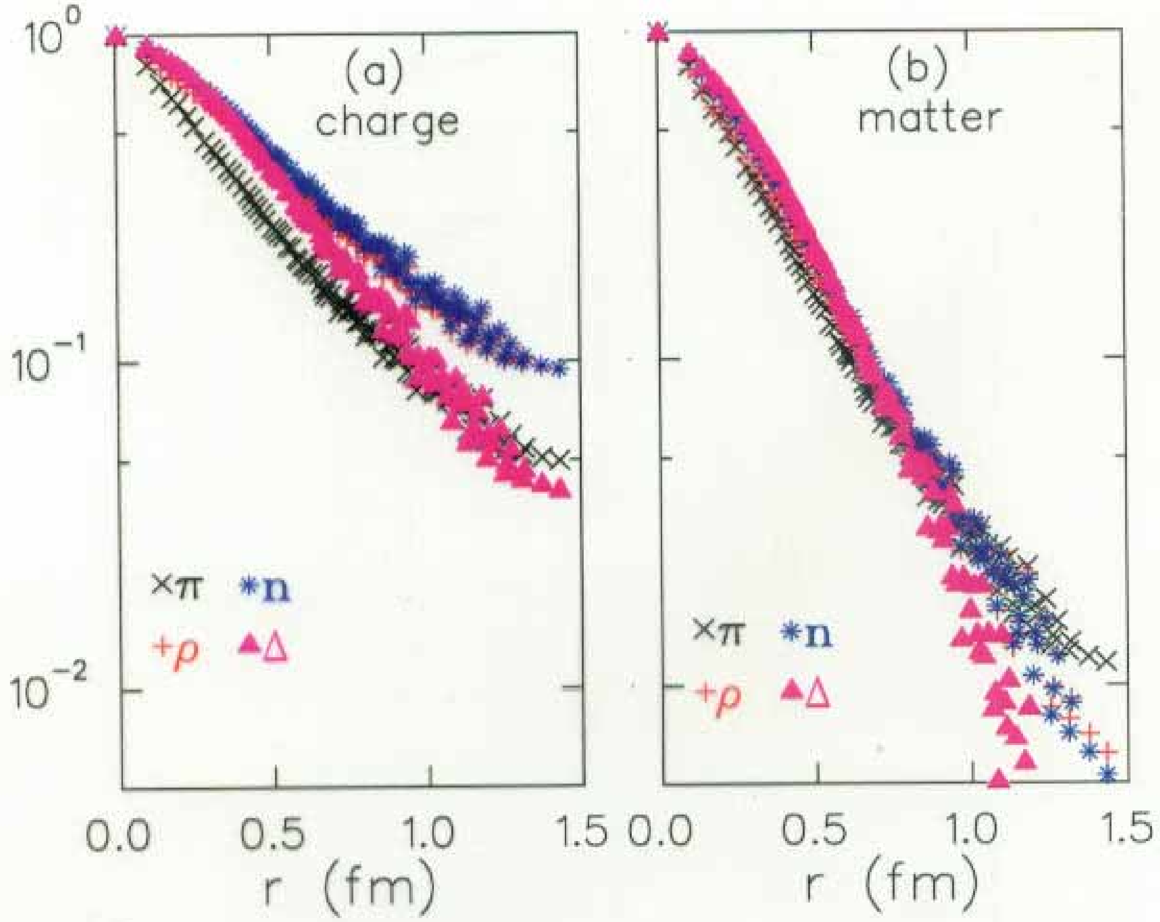
$\overline{MS}$, 2 GeV	$\Delta u_\Lambda = \Delta d_\Lambda$	Δs_Λ
quark model	0	1
exp. proton $+SU(3)_F$	-0.17(3)	0.63(3)
lat. proton $+SU(3)_F$	-0.016(9)	0.65(2)
lattice	-0.02(4)	0.68(4)

Quenched lattice gets Δq_p's and hence Δq_Λ's wrong.

But results support $SU(3)_F$!

Also ∃ QCDSF '02 results on diquark distributions in nucleon !

$n_f = 0$, $m_\pi/m_\rho = 0.81$, $a \approx 0.1$ fm, $V = 16^3 \times 32$



	quenched, physical m_π/m_ρ	experiment
r_p	0.59(4) fm	0.81 fm
r_n^2/r_p^2	-0.29(12)	-0.146

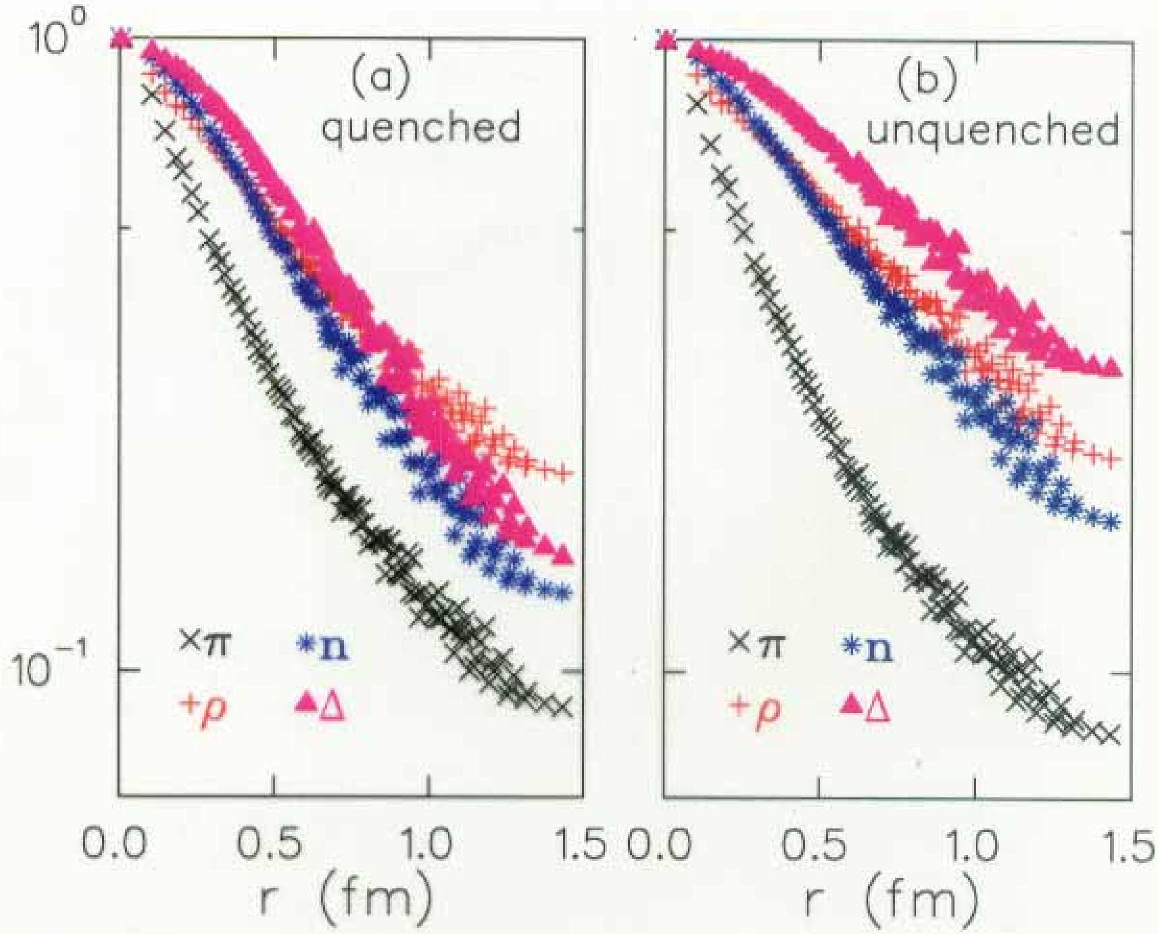
Finite size effects? Un-quenching?

QCDSF '01:

$n_f = 0$: $r_p \approx 0.60$ fm $\qquad$ $n_f = 2$: $r_p \approx 0.70$ fm.

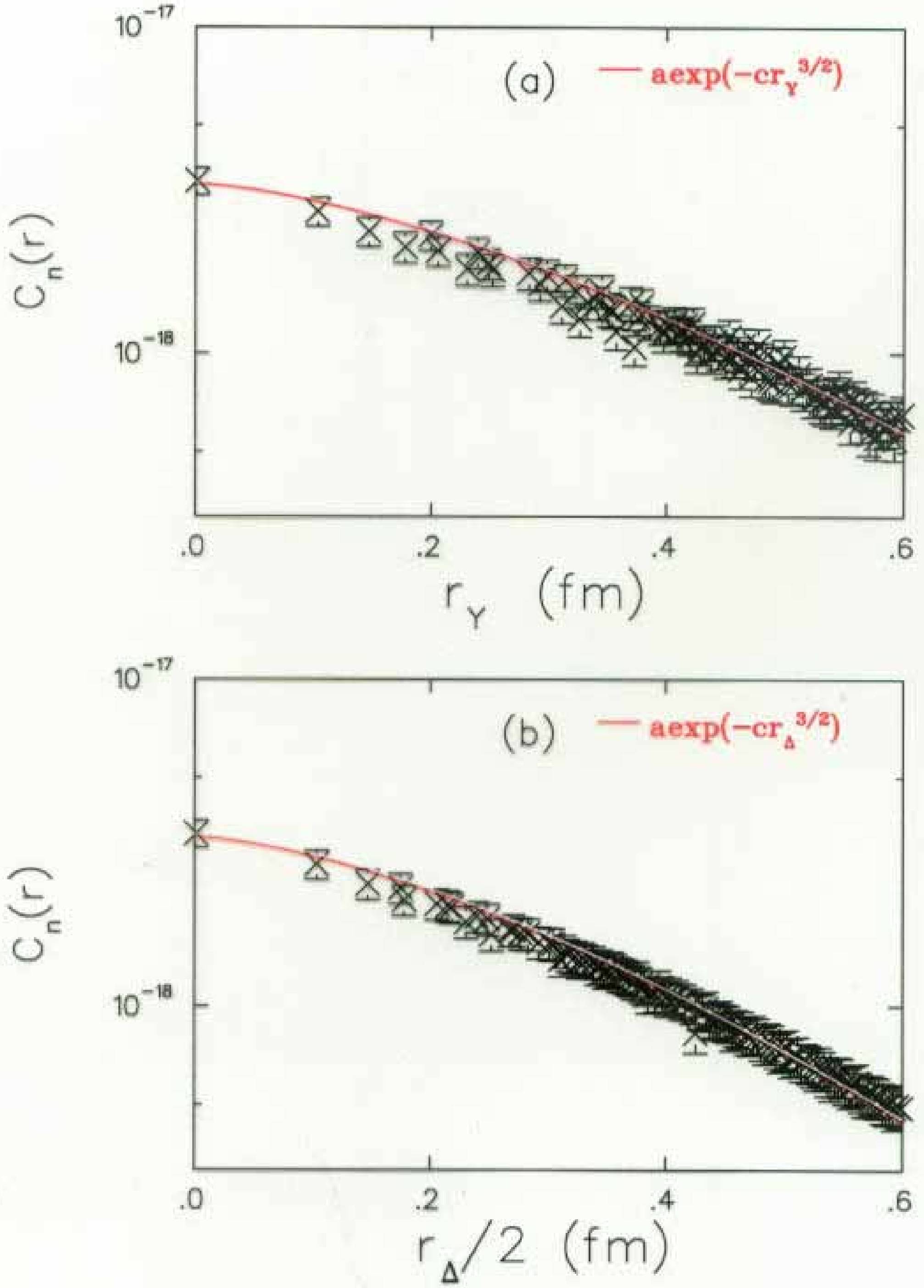
from dipole fit to G_e^p

Charge density distributions:



$n_f = 0, 2$, $m_\pi/m_\rho = 0.76$, $a \approx 0.1$ fm, $V = 16^3 \times 32$

Comparison: nonrelativistic potential model

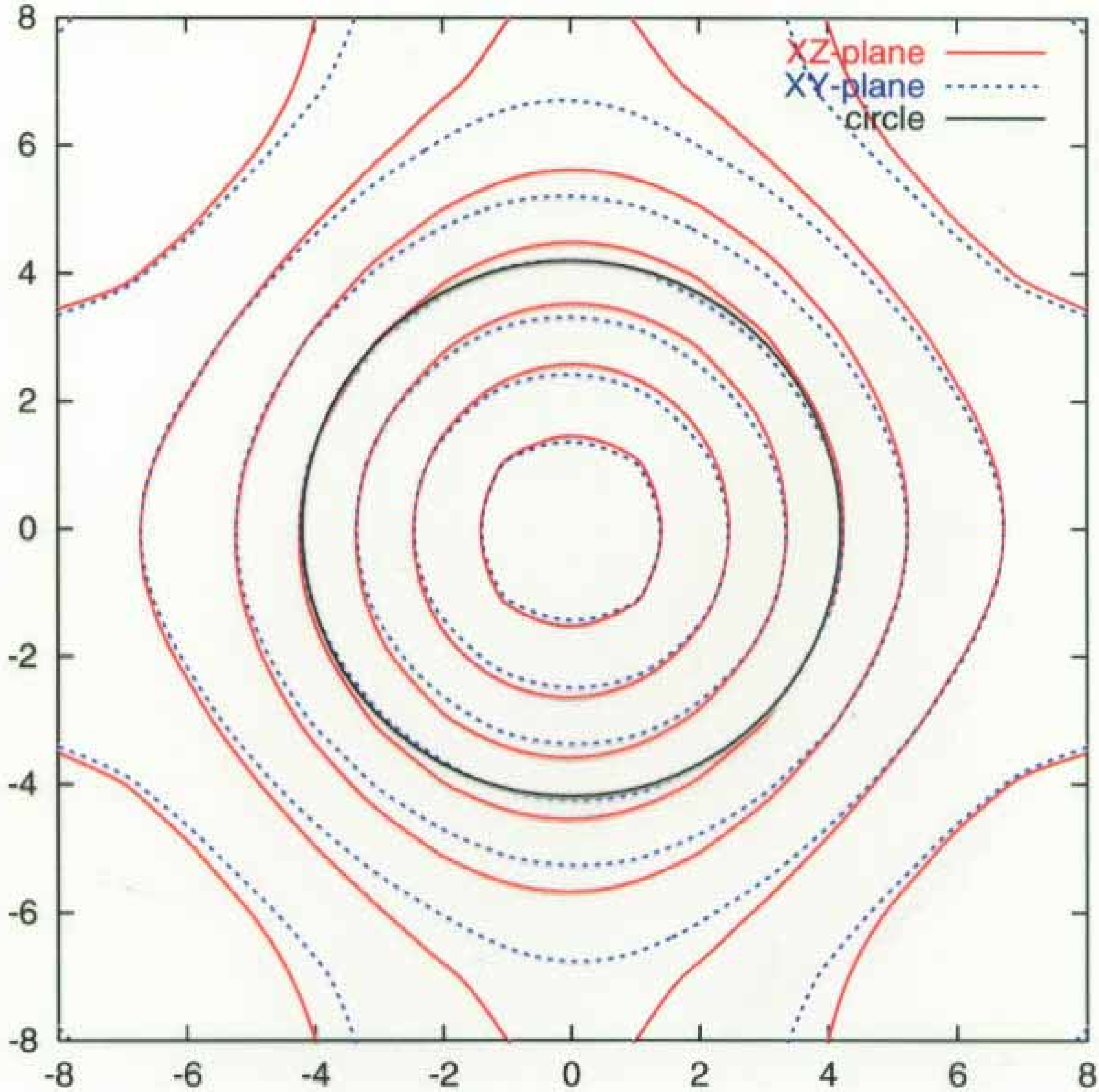


Δ law preferred !!!

Conclusions

- Quantitative results for $n_f > 0$ emerging
- Simple $n_f = 0$ problems (e.g. m_ρ) difficult in $n_f > 0$
- Many complicated observables have become accessible since $n_f > 0$ is so expensive anyway
- New methods, new computers, new experiments
- Chiral extrapolations are an important issue
- N^* spectroscopy maturing
- Status of Roper still controversial
- Proton dislikes QCD QQQ Mercedes
- Many new results on N, Δ, Λ structure and form factors

Δ^+, [redacted] *No J_3 projection*



C2

Slightly oblate? [redacted] quadrupole moment?

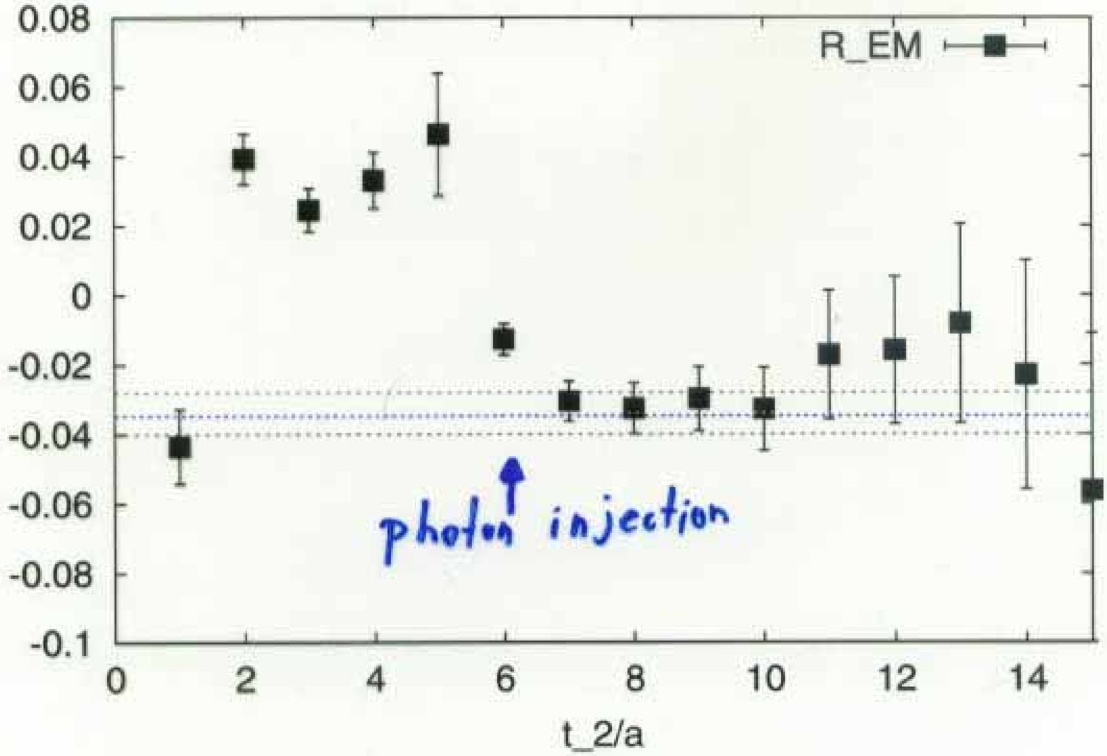
Effect stronger when sea quarks are included.

ρ prolate!

$R_{EM} = \mathcal{G}_{E2}/\mathcal{G}_{M1}$ for $\gamma N \to \Delta$ Tsapalis et al. '02

$$\langle \Delta(\mathbf{p}', \mathbf{s}') | J_\mu(\mathbf{q}) | N(\mathbf{p}, \mathbf{s}) \rangle = \bar{u}(\mathbf{p}', \mathbf{s}') \mathcal{O}_\mu u(\mathbf{p}, \mathbf{s})$$

$$\mathcal{O}^\mu = \mathcal{G}_{M1}(Q^2) K^\mu_{M1} + \mathcal{G}_{E2}(Q^2) K^\mu_{E2} + \mathcal{G}_{C2}(Q^2) K^\mu_{C2}$$



L = 16a

$n_f = 2$, $a^{-1} \approx 1.85$ GeV $\Rightarrow$ smallest $q^2 \approx 0.53$ GeV^2

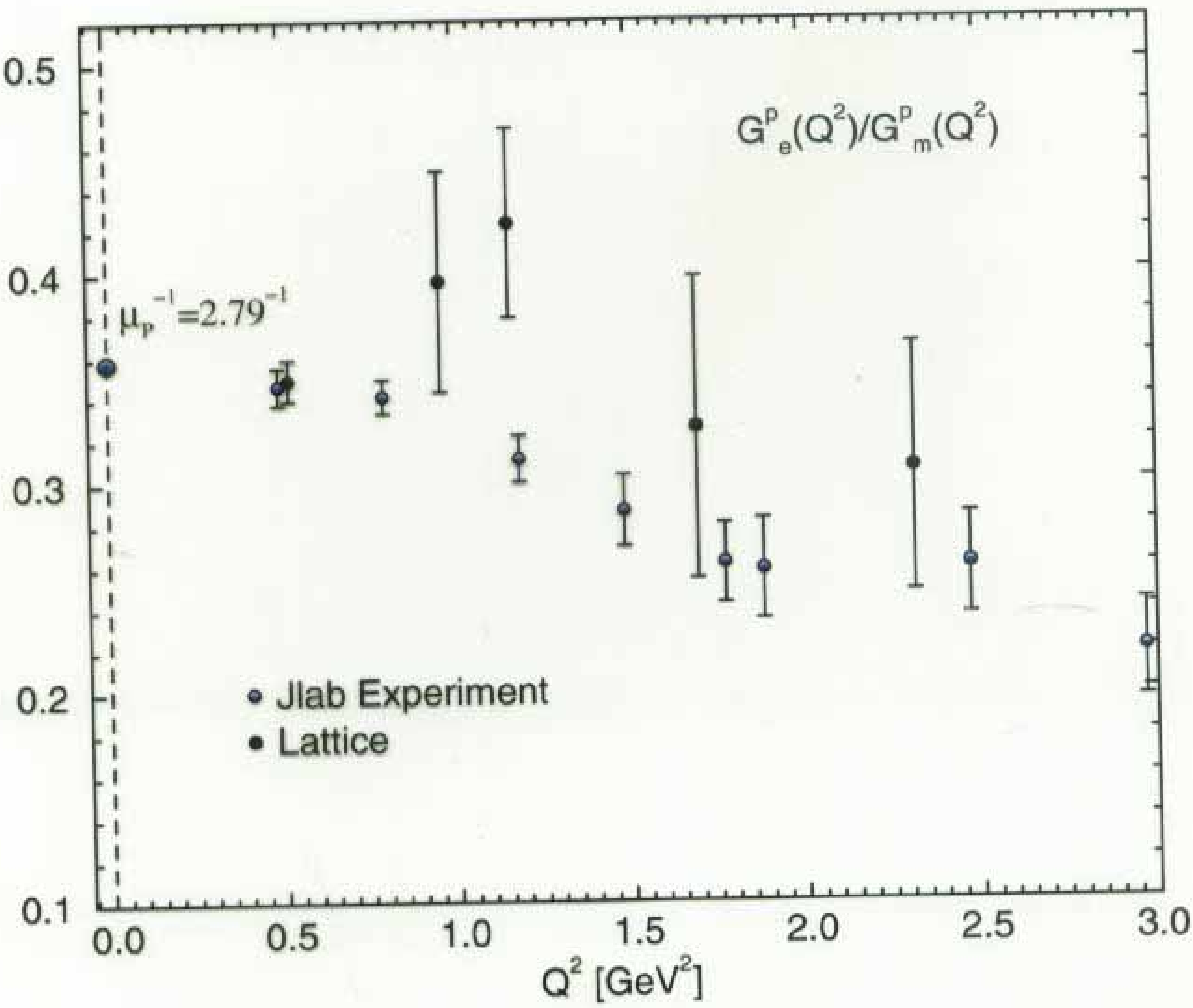
$R_{EM} \approx -0.03 \pm 0.01$: 3 times bigger than quenched

$R_{SM} = \mathcal{G}_{C2}/\mathcal{G}_{M1} \approx -0.03 \pm 0.02$.

$G_e^p,\ G_m^p$ QCDSF: S. Capitani et al. '99

$$\langle N(\mathbf{p}',\mathbf{s}')|J_\mu(\mathbf{q})|N(\mathbf{p},\mathbf{s})\rangle = \bar{u}(\mathbf{p}',\mathbf{s}')\mathcal{O}_\mu u(\mathbf{p},\mathbf{s})$$

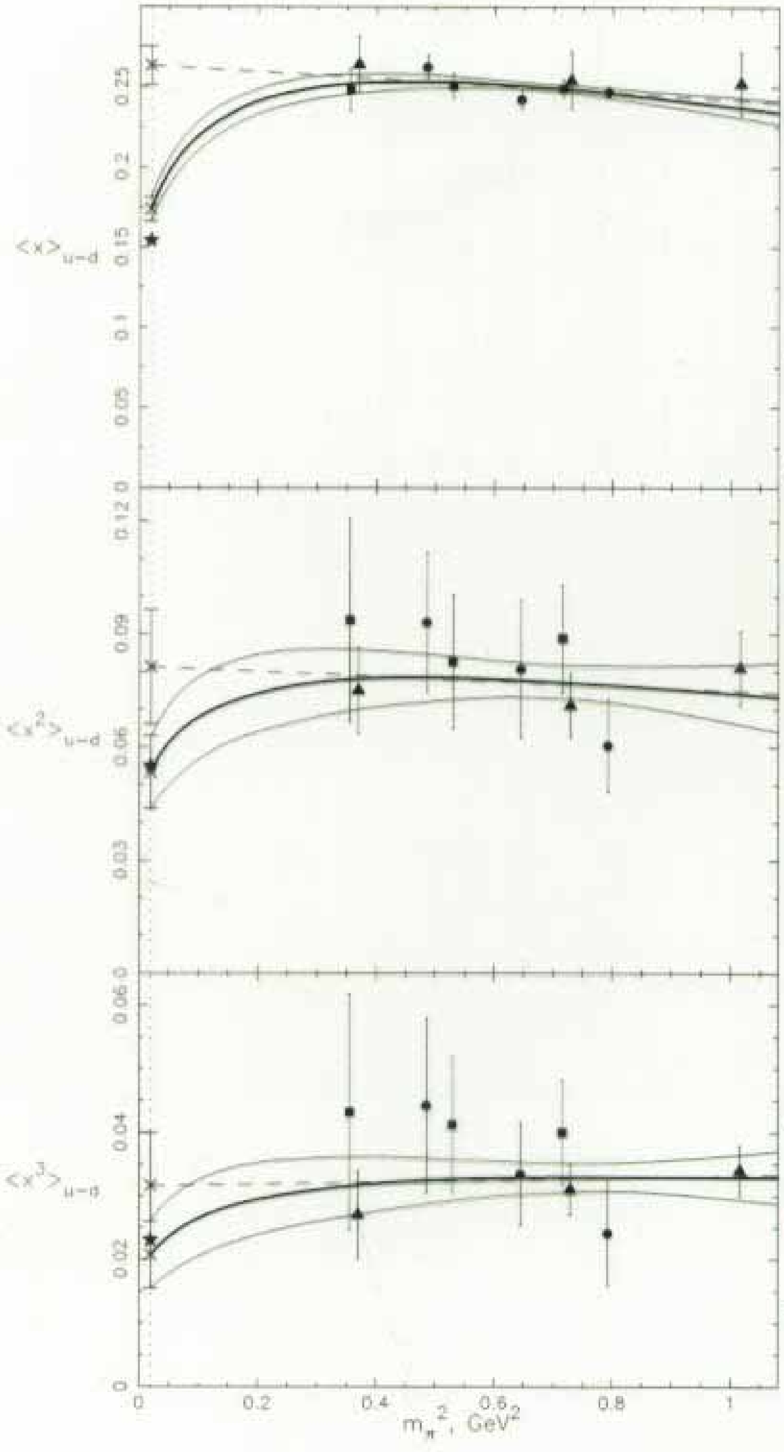
$$\mathcal{O}^\mu = \mathcal{G}_e^p(Q^2)K_e^\mu + \mathcal{G}_m^p(Q^2)K_m^\mu$$



$n_f = 0$, $a^{-1} \approx 2.9$ GeV, at physical m_π/m_ρ

Moments of light cone quark distributions

LHPC/SESAM: Dolgov et al. '02



quark distribution in proton

$n_f = 2$: circles

$n_f = 0$: squares,

QCDSF: triangles

χ extrapolation: Detmold et al. 01

Also $\exists$ results on helicity and transversity dist.

(Instantons)

$\overline{MS}$, $\mu = 2$GeV