An Instanton Induced Interaction

in a relativistic constituent quark model of baryons

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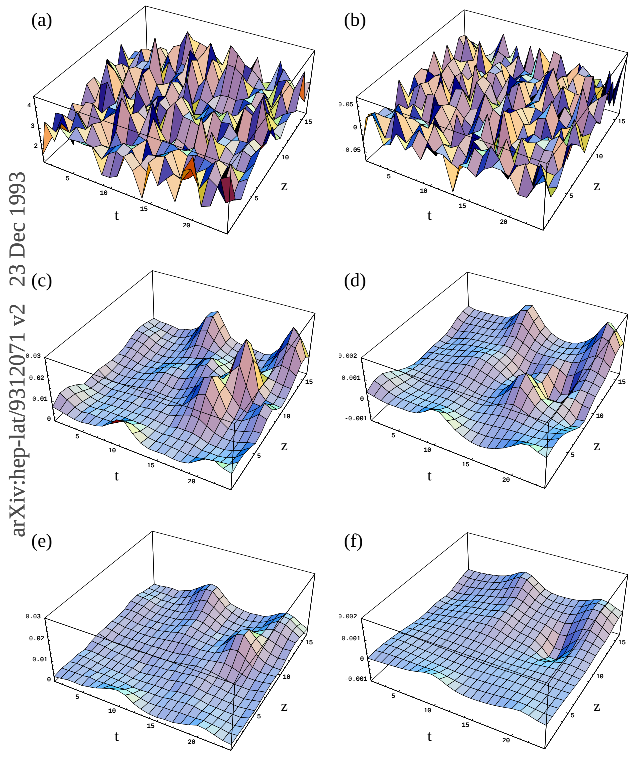
Outline

- Instantons in QCD:
  - χSB
  - $U_A(1)$-problem
- instanton induced quark interaction:
  - $q\bar{q}$-interaction: (pseudo)scalars
  - qq-interaction: octet-decuplet-splitting
- a relativistic constituent quark model:
  - basic assumptions and approximations: BSE $\to$ SE
  - parameterisation of confinement
- results:
  - mass spectra: Regge trajectories, parity doublets
  - e.m properties; form factors and helicity amplitudes
  - strong decays

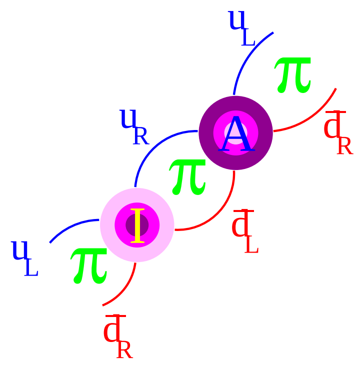
Instantons

Yang-Mills lattice simulations



Action density

Topological charge density



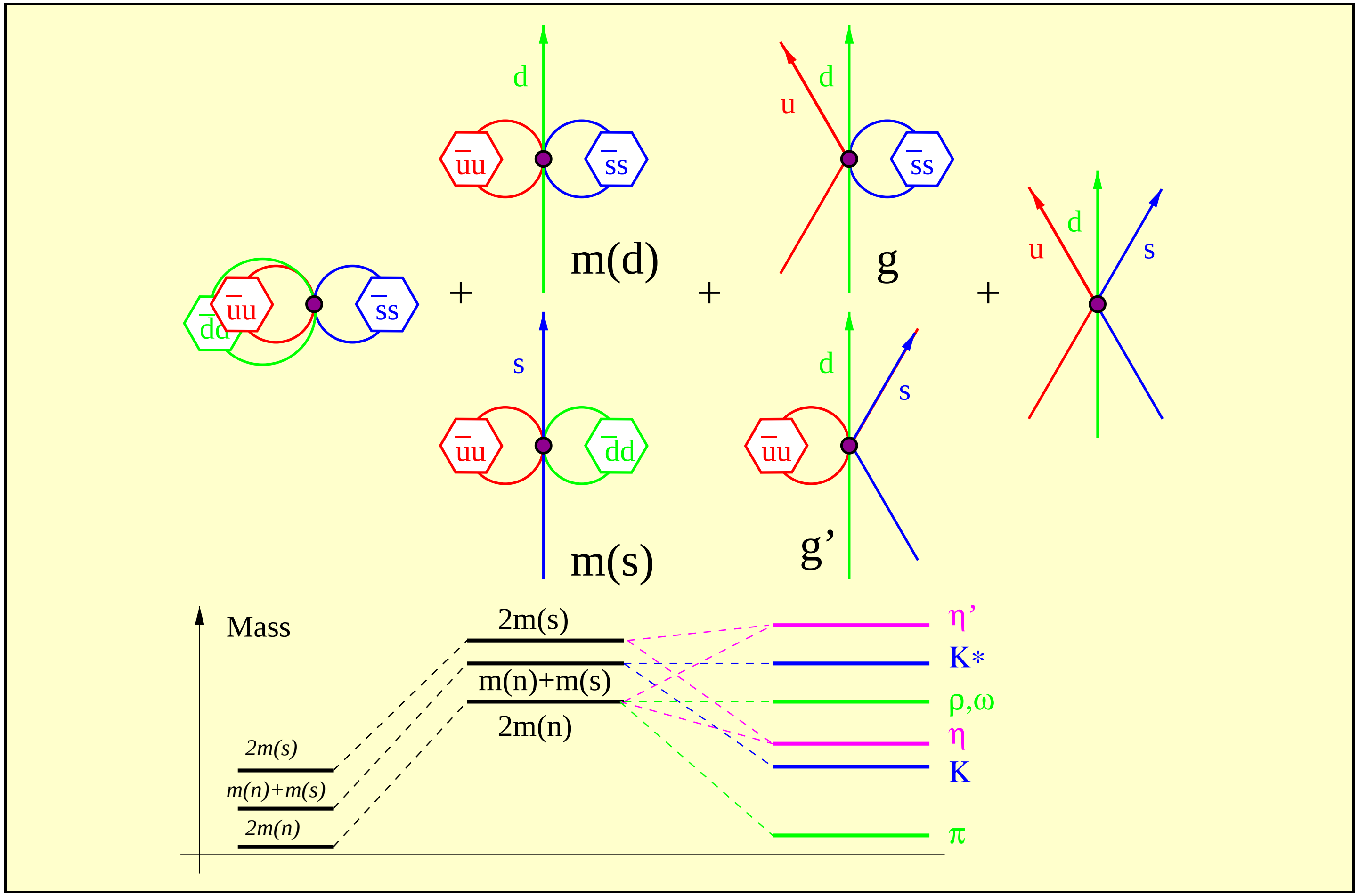
- Gas of instantons:
  $\langle$size$\rangle \approx$ 0.3-0.4 fm;
  $\langle$separation$\rangle \approx$ 0.9-1.0 fm;
- χ-symmetry breaking
  - effective interaction
  - $\langle \bar{q}q \rangle \neq 0$
  - constituent quark masses

effective masses and interaction

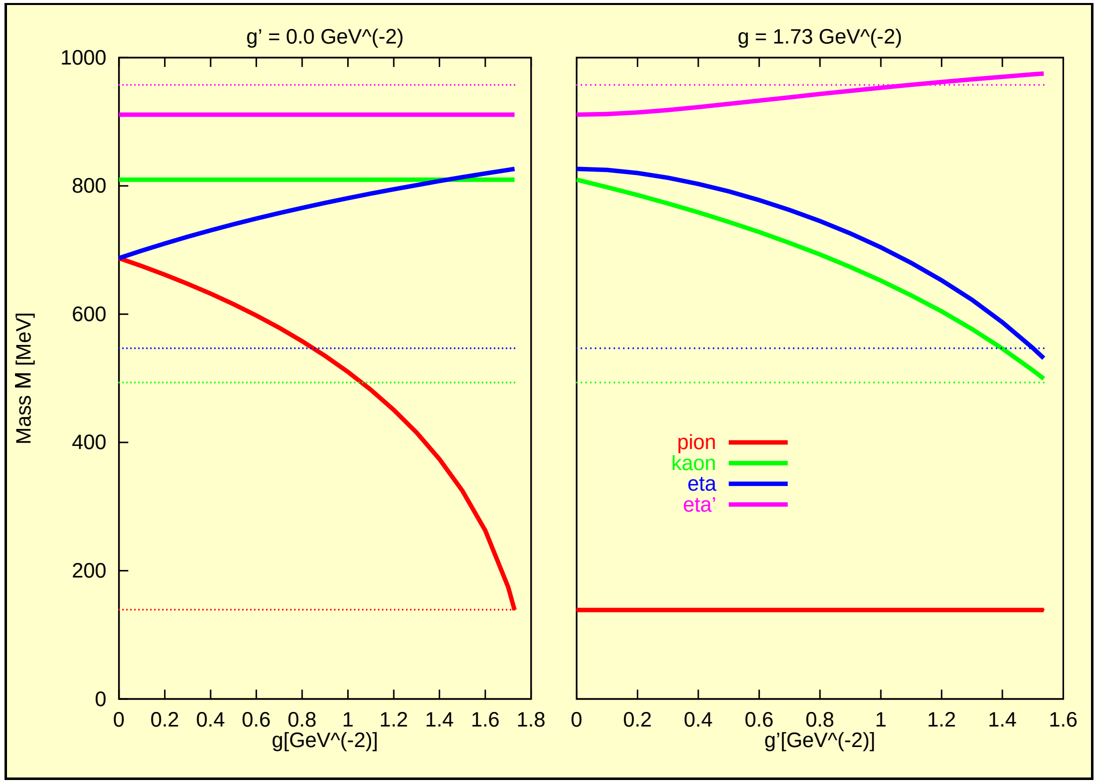
- after normal ordering:

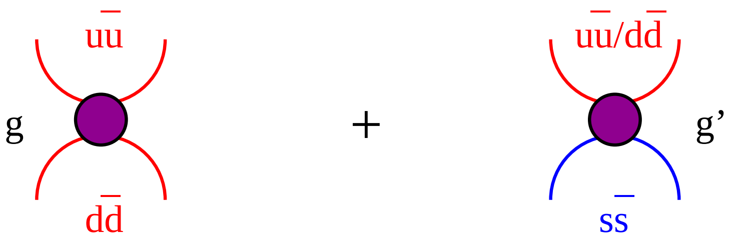
$$\Delta\mathcal{L}^{(0)}_{eff} \quad + \quad \Delta\mathcal{L}^{(1)}_{eff} \quad + \quad \Delta\mathcal{L}^{(2)}_{eff} \quad + \quad \Delta\mathcal{L}^{(3)}_{eff}$$

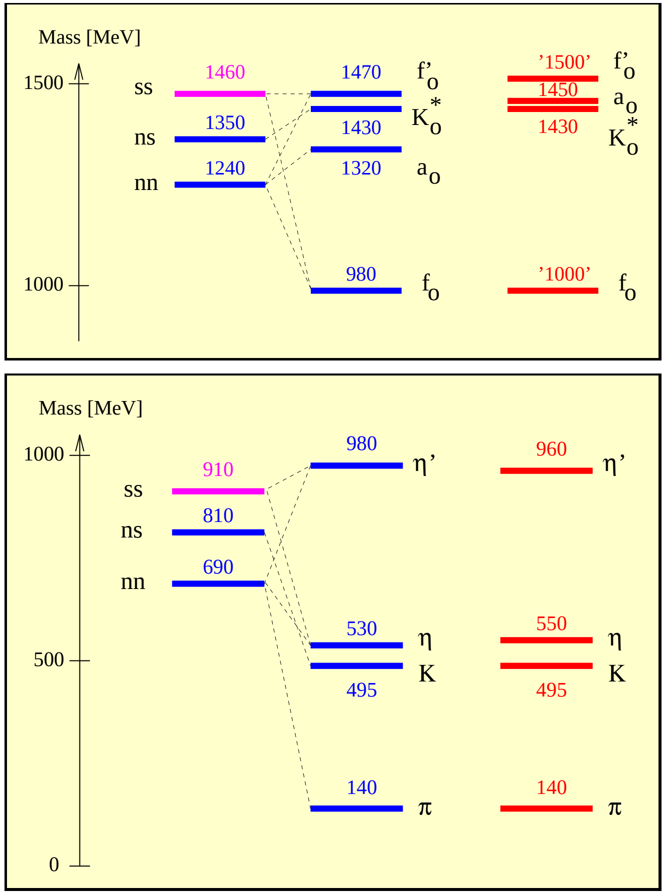
effective masses effective interaction



(pseudo)scalar mesons







$V_C + V_{III}$ Exp.

V_{III} mixes $u\bar{u}, d\bar{d}, s\bar{s}$ or $8_F, 1_F$ for (pseudo)scalars

The instanton-induced qq-interaction

In $qq, C = \bar{3}$-channel $\rightarrow$ instantaneous potential:
't Hooft's interaction (induced by instantons):

$$V^{(2)}_{\text{'t Hooft}}(\mathbf{x_1} - \mathbf{x_2}) = \underbrace{\delta^{(3)}(\mathbf{x_1} - \mathbf{x_2})}_{\text{point-interaction}} \cdot$$

$$-4 \underbrace{\left(g_{nn}\, \mathcal{P}^{\mathcal{F}}_{\mathcal{A}}(nn) + g_{ns}\, \mathcal{P}^{\mathcal{F}}_{\mathcal{A}}(ns)\right)}_{\text{flavour-dependent coupling}} \left[\mathbb{1} \otimes \mathbb{1} + \gamma^5 \otimes \gamma^5\right] \mathcal{P}^{\mathcal{D}}_{S_{12}=0}$$

G. 't Hooft, Phys. Rev. D 14, 3432 (1976)

M. A. Shifman, A. I. Vainstein, V. I. Zakharov, Nucl. Phys. B 163, 46 (1980)

- flavour-dependent: $\mathcal{P}^{\mathcal{F}}_{\mathcal{A}}$: flavour-antisymmetric quark pairs.
- spin-dependent: $\mathcal{P}^{\mathcal{D}}_{S_{12}=0}$: antisymmetric in spin $S_{12} = 0$.
- point-interaction: $\delta^{(3)}(\mathbf{x_1} - \mathbf{x_2}) \longrightarrow \frac{1}{\lambda^3 \pi^{\frac{3}{2}}} \exp\left(-\frac{|\mathbf{x_1} - \mathbf{x_2}|^2}{\lambda^2}\right)$

$\Rightarrow$ does <u>not</u> act on: flavour-decuplet, spin-symmetric states;

$\Rightarrow$ no $\vec{L} \cdot \vec{S}$, no tensor forces.

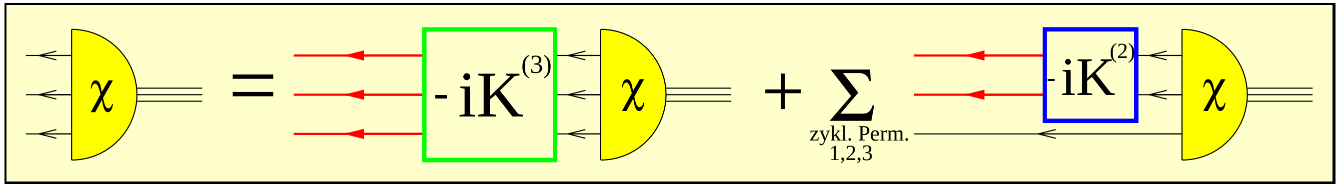
a relativistic quark model: assumptions

Pretend to describe baryons in the framework of quantum field theory: Basic quantities: **Bethe-Salpeter amplitudes**

$$\chi_{\bar{P}}(x_1, x_2, x_3) := \langle 0 | T\Psi(x_1)\Psi(x_2)\Psi(x_3) | \bar{P} \rangle$$

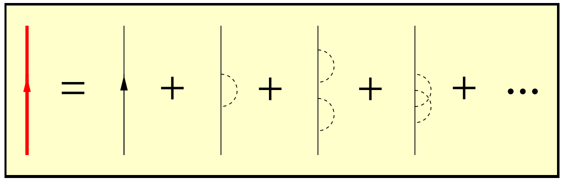
(with $\bar{P} = p_1 + p_2 + p_3$; $\bar{P}^2 = M^2$ baryon mass; $\Psi(x_i)$ quark field operator)

which fulfill the **Bethe-Salpeter Equation**:

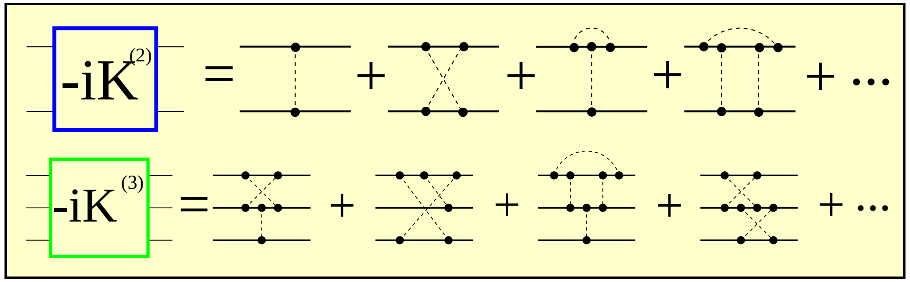


Full quark propagators:

$$S^F(x - x') = \langle 0 | T\Psi(x)\bar{\Psi}(x') | 0 \rangle$$



Irreducible 2- and 3 -body interaction kernels:



momentum space

From Poincaré invariance:

$$\chi_{\bar{P}}(x_1, x_2, x_3) = e^{-iX\bar{P}} \int \frac{d^4 p_\xi}{(2\pi)^4} \frac{d^4 p_\eta}{(2\pi)^4} e^{-ip_\xi \xi} e^{-ip_\eta \eta} \chi_{\bar{P}}(p_\xi, p_\eta)$$

$\Rightarrow$ 8-dimensional integral equation

$$\begin{aligned} \chi_{\bar{P}}(p_\xi, p_\eta) &= S_1^F(\tfrac{1}{3}\bar{P} + p_\xi + \tfrac{1}{2}p_\eta) \otimes S_2^F(\tfrac{1}{3}\bar{P} - p_\xi + \tfrac{1}{2}p_\eta) \otimes S_3^F(\tfrac{1}{3}\bar{P} - p_\eta) \\ &\times (-i) \int \frac{d^4 p'_\xi}{(2\pi)^4} \frac{d^4 p'_\eta}{(2\pi)^4} K(\bar{P}, p_\xi, p_\eta, p'_\xi, p'_\eta)\, \chi_{\bar{P}}(p'_\xi, p'_\eta) \end{aligned}$$

with integral kernel: $K = K^{(3)} + \bar{K}^{(2)}$,

$$\bar{K}^{(2)}(\bar{P}, p_\xi, p_\eta, p'_\xi, p'_\eta) := \sum_{(123)} K^{(2)}(P_{12}, p_\xi, p'_\xi) \otimes [S_3^F(\tfrac{1}{3}\bar{P} - p_\eta)]^{-1} (2\pi)^4 \delta^{(4)}(p_\eta - p'_\eta)$$

Bethe-Salpeter Equation, approximations...

Model assumptions (inspired by NonRelativistic Constituent Quark Model):

- Free quark propagators replace full propagators:

$$S_i^F(p_i) \approx \frac{i}{\gamma(p_i) - m_i \ + \ i\epsilon}$$

$\rightarrow m_i$ effective constituent quark masses

- Instantaneous approximation: No dependence of interaction kernels on p_ξ^0 und p_η^0 (in the restframe of the baryon):

$$\begin{aligned} K^{(3)}(\bar{P}, p_\xi, p_\eta, p'_\xi, p'_\eta) \,|_{\bar{P}=(M,\vec{0})} &= V^{(3)}(\mathbf{p}_\xi, \mathbf{p}_\eta, \mathbf{p}'_\xi, \mathbf{p}'_\eta) \\ K^{(2)}(P_{12}, p_\xi, p'_\xi) \,|_{\bar{P}=(M,\vec{0})} &= V^{(2)}(\mathbf{p}_\xi, \mathbf{p}'_\xi) \end{aligned}$$

$\Leftrightarrow$ 'Retardation effects are neglected'

$\Rightarrow$ Salpeter amplitude $\Phi_M(\mathbf{p}_\xi, \mathbf{p}_\eta) := \int \frac{dp_\xi^0}{2\pi} \frac{dp_\eta^0}{2\pi} \chi_{\bar{P}} \left((p_\xi^0, \mathbf{p}_\xi), (p_\eta^0, \mathbf{p}_\eta) \right) \Big|_{\bar{P}=(M,\mathbf{0})}$

satisfies the Salpeter equation $\mathcal{H}\Phi_M = M\Phi_M; \quad \langle \Phi_M | \Phi_M \rangle = 2M$

Salpeter Equation, Salpeter Hamiltonian

... approximate treatment of $V^{(2)}$ ...:

$$(\mathcal{H}\Phi_M)(\mathbf{p}_\xi,\mathbf{p}_\eta) = \sum_{i=1}^{3} H_i\ \Phi_M(\mathbf{p}_\xi,\mathbf{p}_\eta)$$

$$+ \quad \left(\Lambda_1^+ \otimes \Lambda_2^+ \otimes \Lambda_3^+ + \Lambda_1^- \otimes \Lambda_2^- \otimes \Lambda_3^-\right)$$

$$\gamma^0 \otimes \gamma^0 \otimes \gamma^0 \int \frac{d^3p'_\xi}{(2\pi)^3}\ \frac{d^3p'_\eta}{(2\pi)^3}\ V^{(3)}(\mathbf{p}_\xi,\mathbf{p}_\eta,\mathbf{p}'_\xi,\mathbf{p}'_\eta)\ \Phi_M(\mathbf{p}'_\xi,\mathbf{p}'_\eta)$$

$$+ \quad \left(\Lambda_1^+ \otimes \Lambda_2^+ \otimes \Lambda_3^+ - \Lambda_1^- \otimes \Lambda_2^- \otimes \Lambda_3^-\right)$$

$$\gamma^0 \otimes \gamma^0 \otimes \mathbb{1} \int \frac{d^3p'_\xi}{(2\pi)^3}\ \left[V^{(2)}(\mathbf{p}_\xi,\mathbf{p}'_\xi) \otimes \mathbb{1}\right]\ \Phi_M(\mathbf{p}'_\xi,\mathbf{p}_\eta)$$

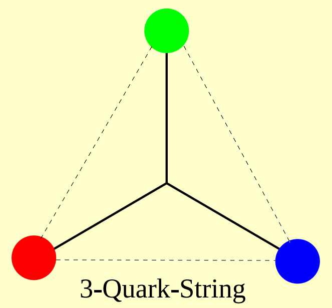
$+$ cycl. perm. (123)

- $\Lambda_i^\pm(\mathbf{p_i}) := \frac{\omega_i \pm H_i}{2\omega_i}$ Energy projectors
- $H_i(\mathbf{p_i}) := \gamma^0\ (\boldsymbol{\gamma}\cdot\mathbf{p_i} + m_i)$ Dirac Hamiltonian

Confinement

Quark confinement is realized by a phenomenological string potential for 3 quarks (*Ansatz* similar to NRCQM):

$$V^{(3)}_{\text{Conf}}(\mathbf{x_1},\mathbf{x_2},\mathbf{x_3}) = \mathbf{A_3} + \mathbf{B_3}\sum_{\mathbf{i<j}} |\mathbf{x_i}-\mathbf{x_j}|$$



In contrast to the nonrelativistic quark model the relativistic quasi-potential $V^{(3)}_{\text{Conf}}$ depends on the Dirac structure for three quarks. We choose:

$$\mathbf{A}_3 = a\,\frac{3}{4}\left[\mathbb{1}\otimes\mathbb{1}\otimes\mathbb{1} + \gamma^0\otimes\gamma^0\otimes\mathbb{1} + \gamma^0\otimes\mathbb{1}\otimes\gamma^0 + \mathbb{1}\otimes\gamma^0\otimes\gamma^0\right]$$

$$\mathbf{B}_3 = b\,\frac{1}{2}\left[-\mathbb{1}\otimes\mathbb{1}\otimes\mathbb{1} + \gamma^0\otimes\gamma^0\otimes\mathbb{1} + \gamma^0\otimes\mathbb{1}\otimes\gamma^0 + \mathbb{1}\otimes\gamma^0\otimes\gamma^0\right]$$

$\rightarrow$ Spin-orbit effects are small (compatible with exp.)

$\rightarrow$ Regge trajectories are quantitatively correct.

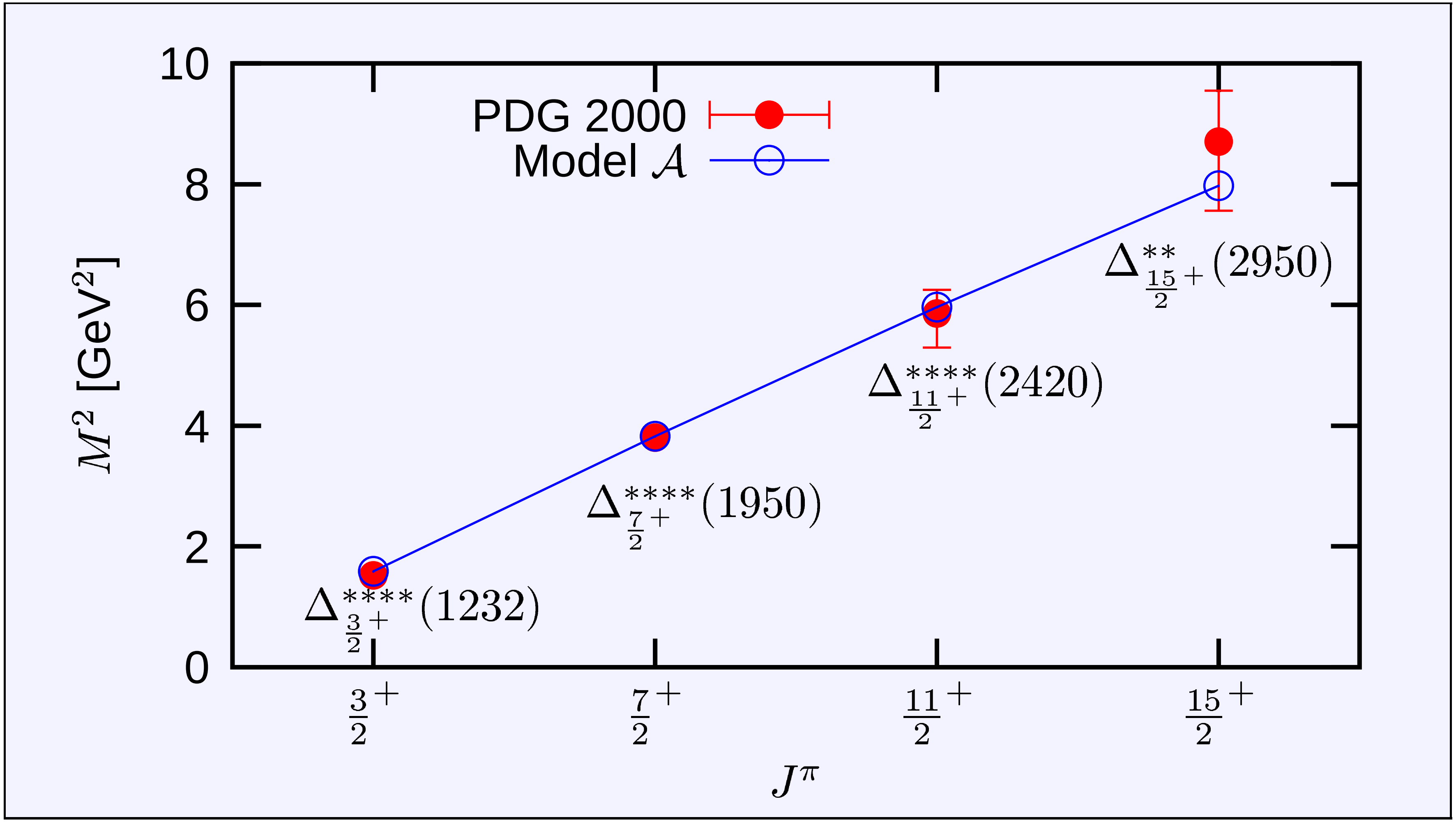
Model parameters

		parameter	value
quark-	'nonstrange'	m_n	330 Mev
masses	'strange'	m_s	670 Mev
confinement	offset	a	-744 MeV
	slope	b	470 MeV fm^{-1}
't Hooft's	nn-coupling	g_{nn}	136.0 MeV fm^3
force	ns-coupling	g_{ns}	94.0 MeV fm^3
	effective range	λ	0.4 fm

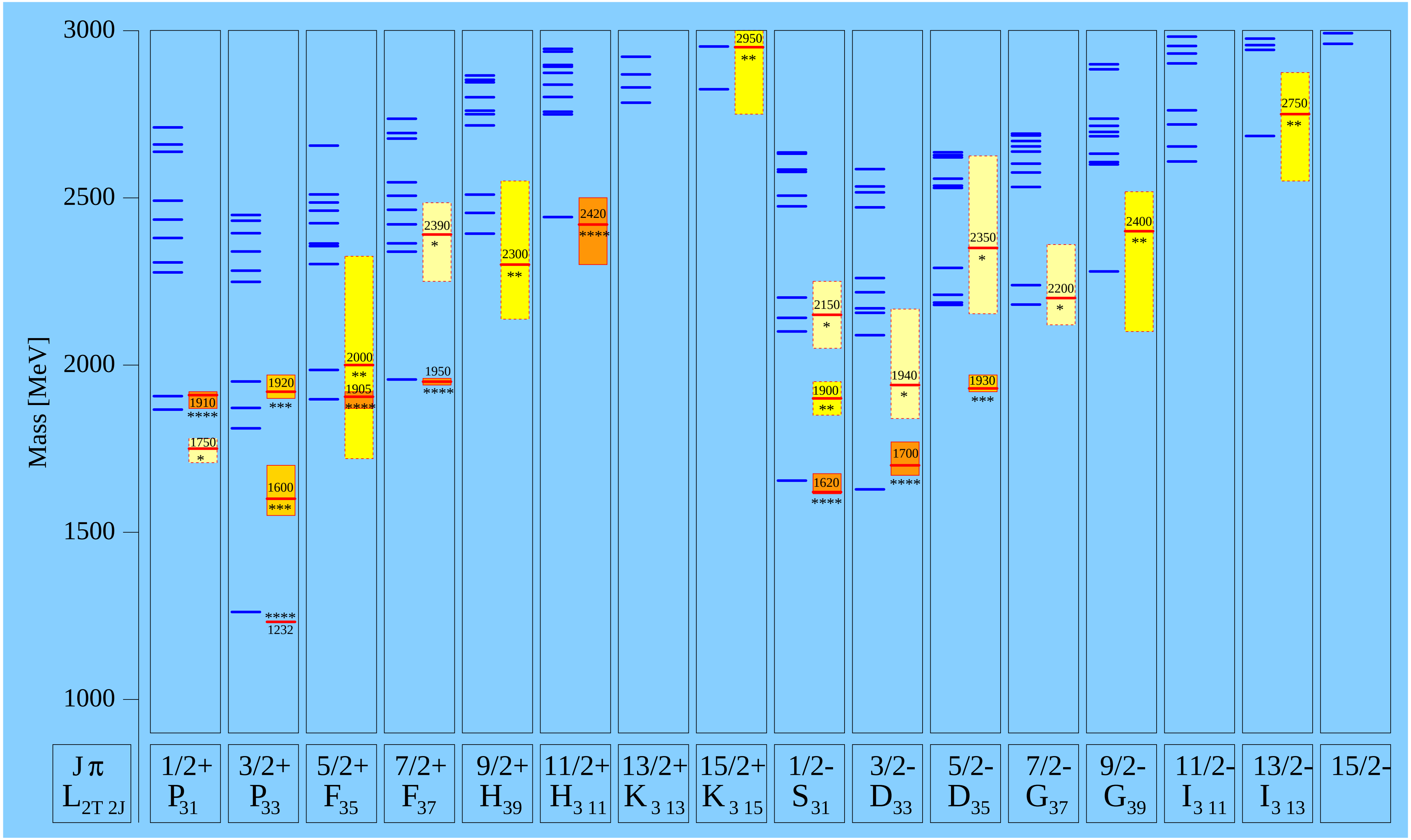
Parameters are fixed by

- the Δ-Regge trajectory
  $\longrightarrow$ Confinement parameters a, b and m_n
- baryon ground-states (octet und decuplet)
  $\longrightarrow$ g_{nn}, g_{ns}, λ and m_s

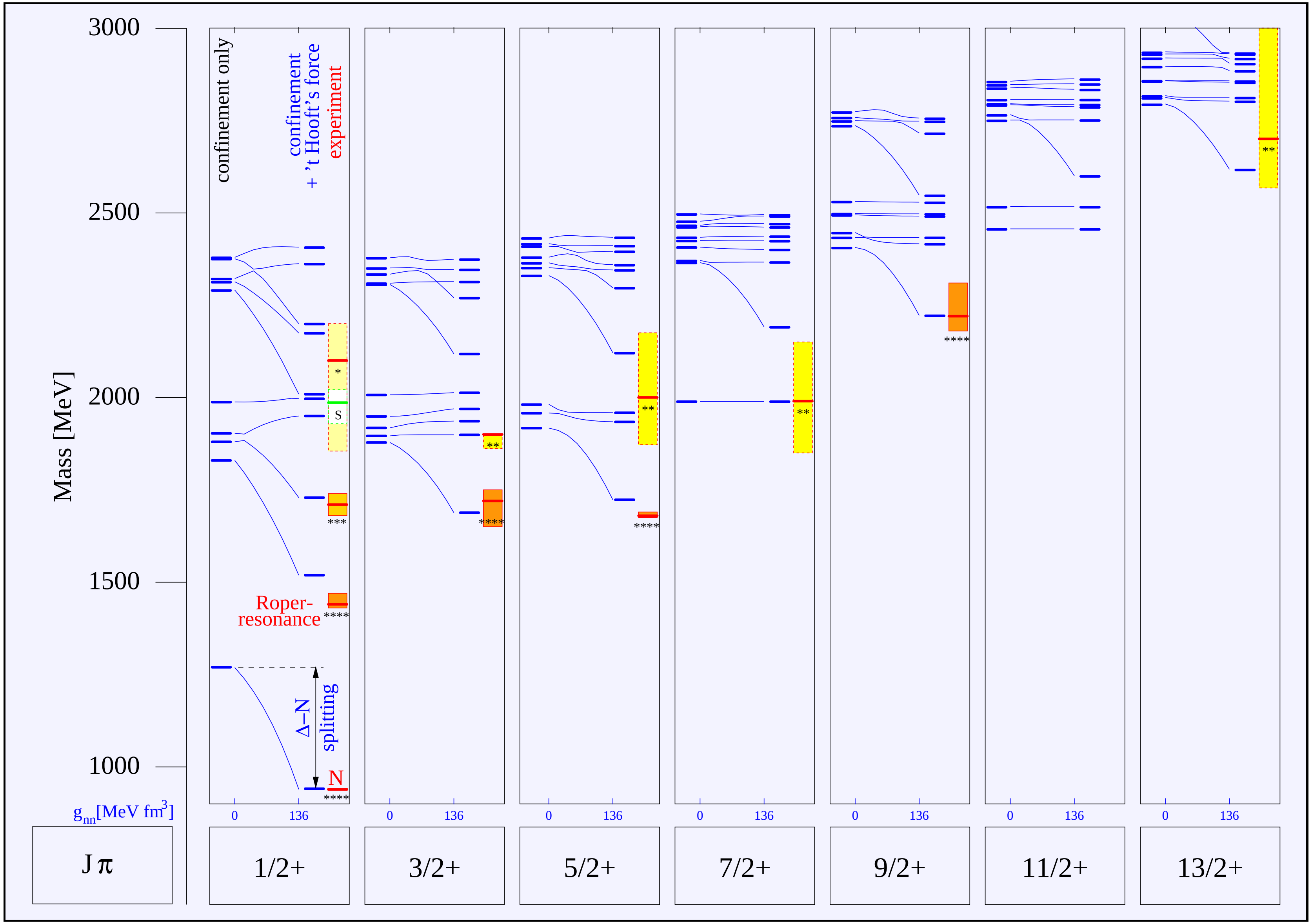
Δ-Regge trajectory



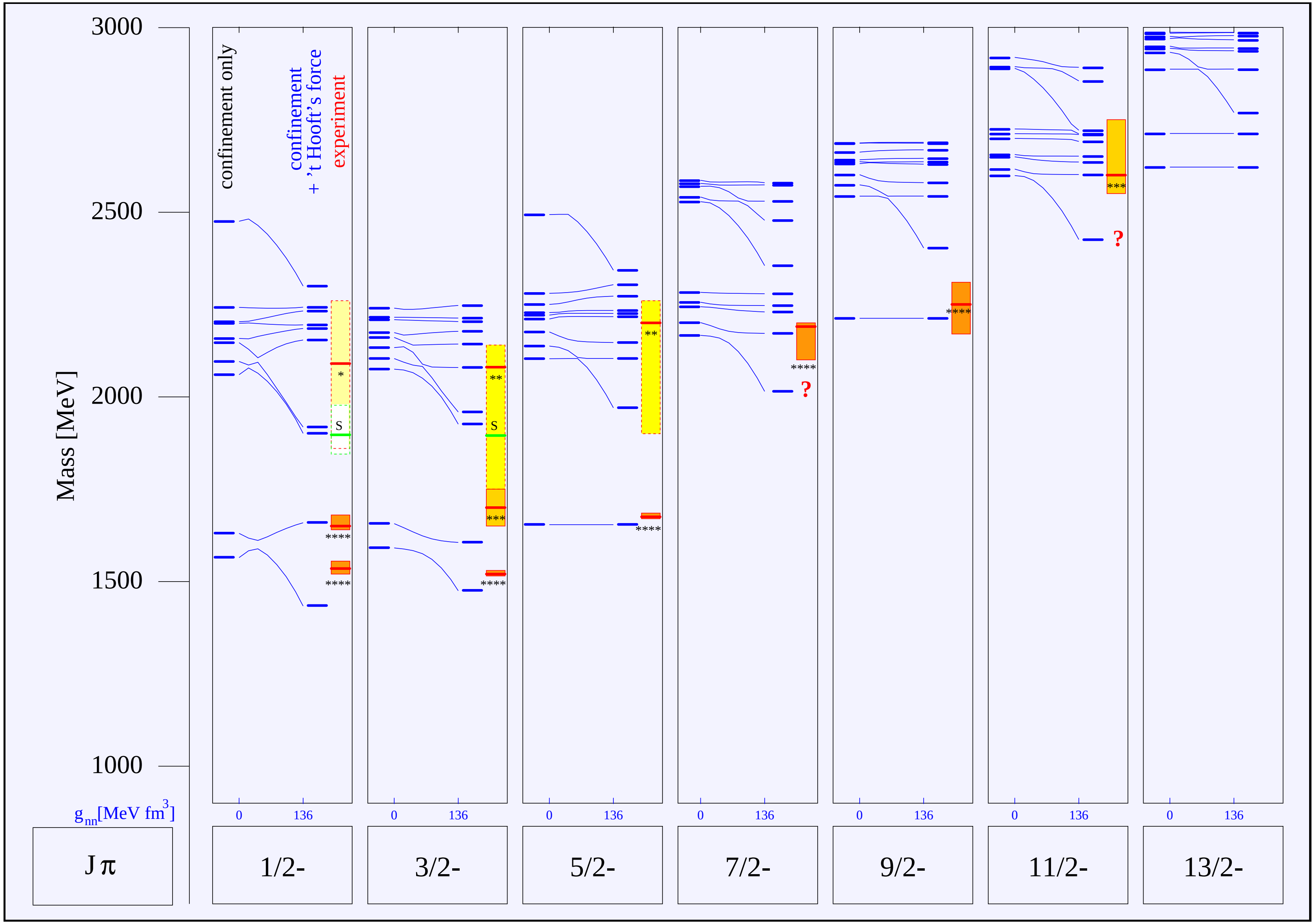
Δ-resonances



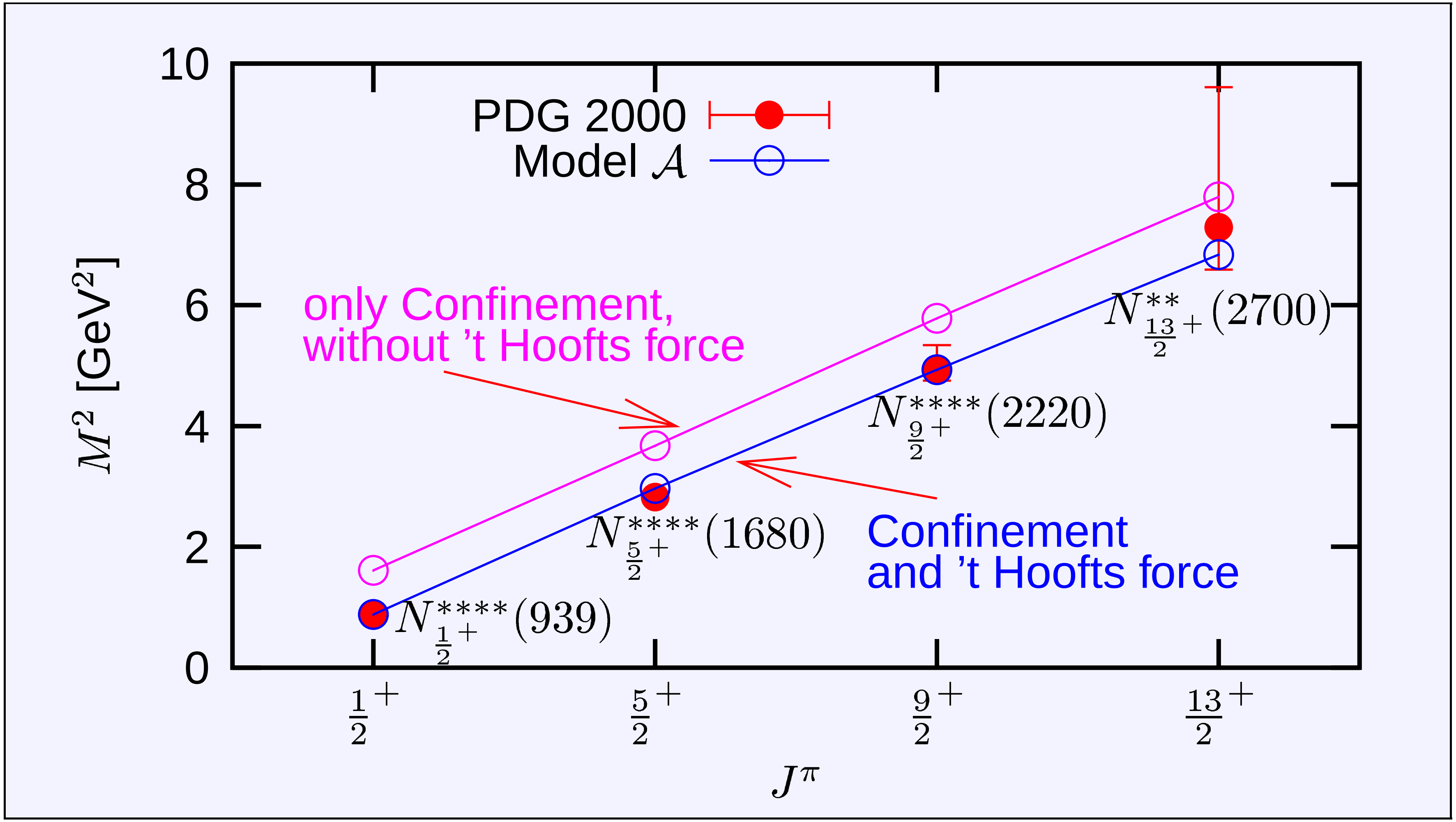
Instanton-induced effects in the N^{*+}-spectrum



Instanton-induced effects in the N^{*-}-spectrum



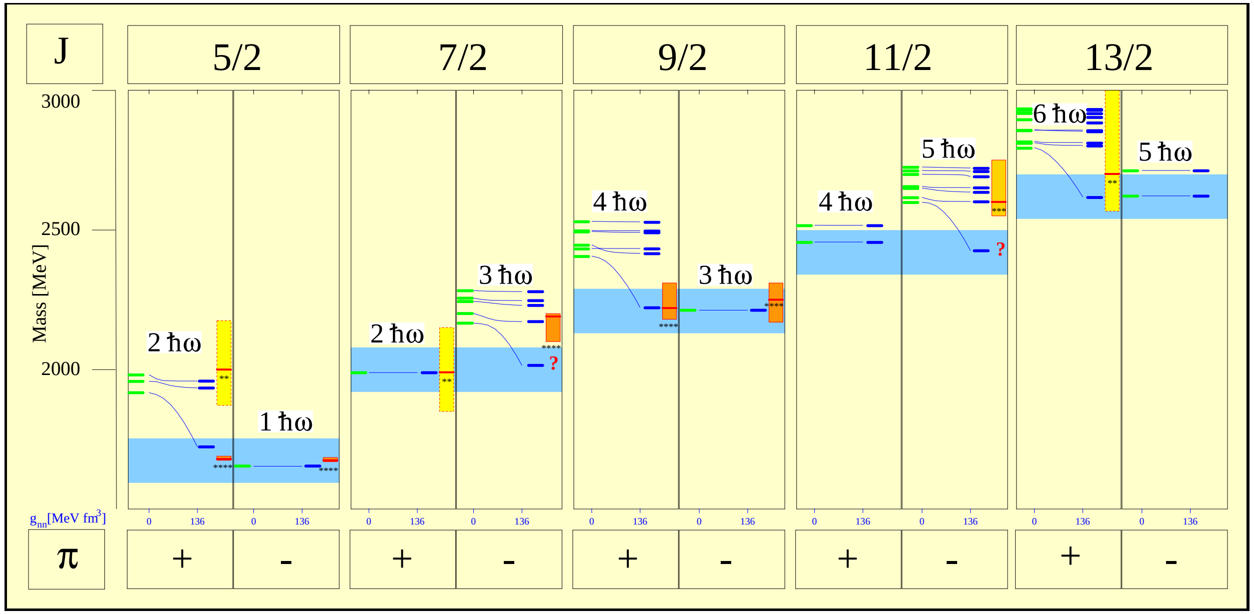
N-Regge trajectory

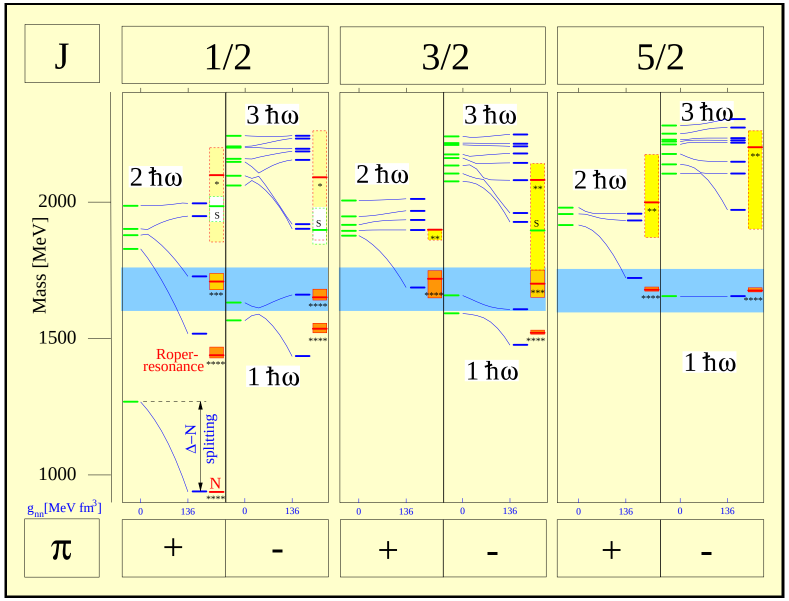


$\longrightarrow$ 't Hooft's force produces a constant shift in M^2

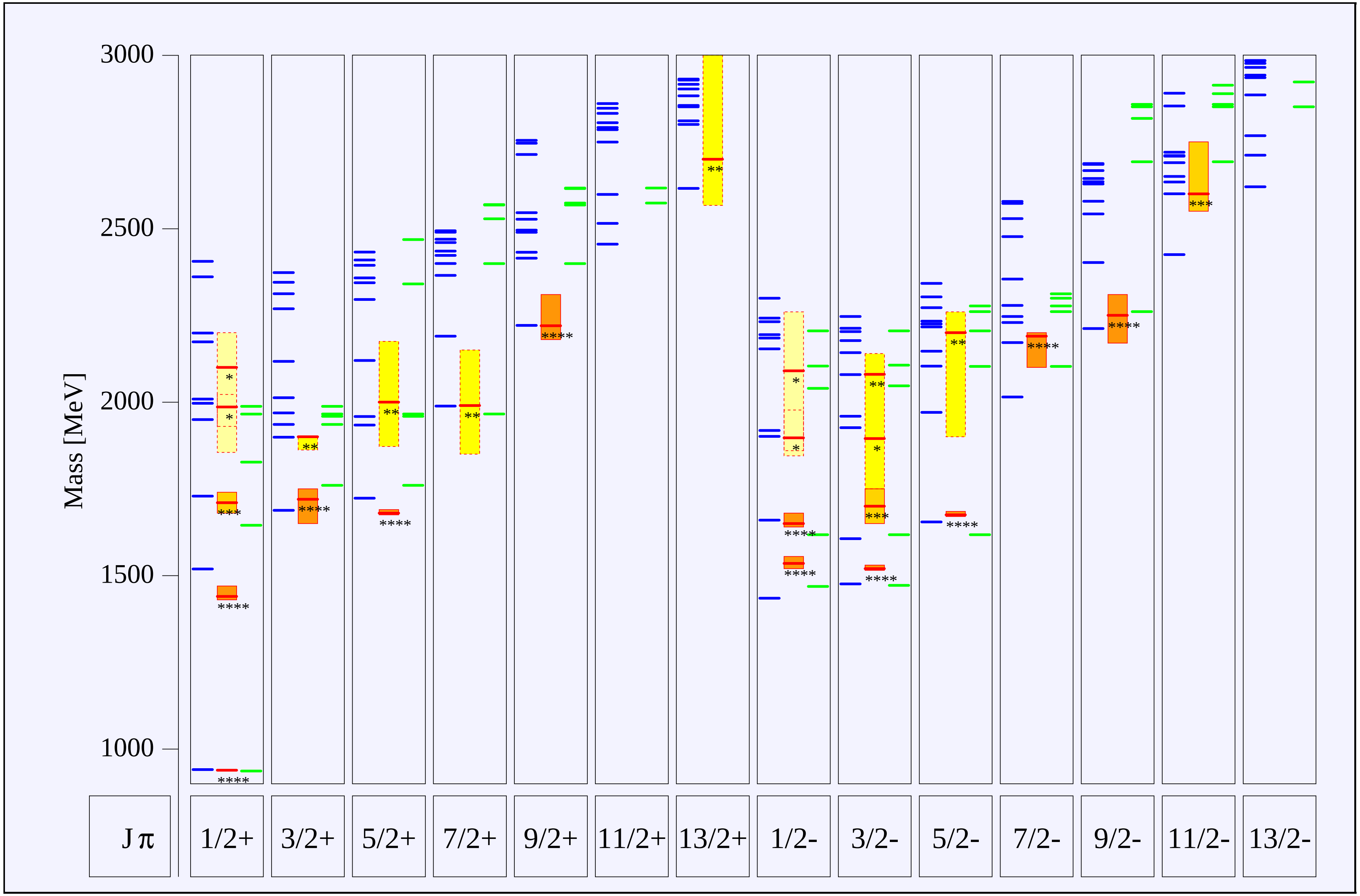
Furthermore, parity doublets in the nucleon spectrum

Parity doublets (high spin)





BSE ↔ NRCQM $(V_{\text{conf.}} + V_{III})$



Relativistic effects

Effects of ’t Hooft’s force in two different confinement models

$$V^{(3)}_{\rm conf}(\mathbf{x_1},\mathbf{x_2},\mathbf{x_3}) = 3\,a\,\frac{1}{4}\left[\mathbb{1}\otimes\mathbb{1}\otimes\mathbb{1}+\gamma^0\otimes\gamma^0\otimes\mathbb{1}+\text{cycl. perm.}\right]$$

Modell A (discussed so far)

$$+\; b\sum_{i<j}|\mathbf{x_i}-\mathbf{x_j}|\;\frac{1}{2}\left[-\mathbb{1}\otimes\mathbb{1}\otimes\mathbb{1}+\gamma^0\otimes\gamma^0\otimes\mathbb{1}+\text{cycl. perm.}\right]$$

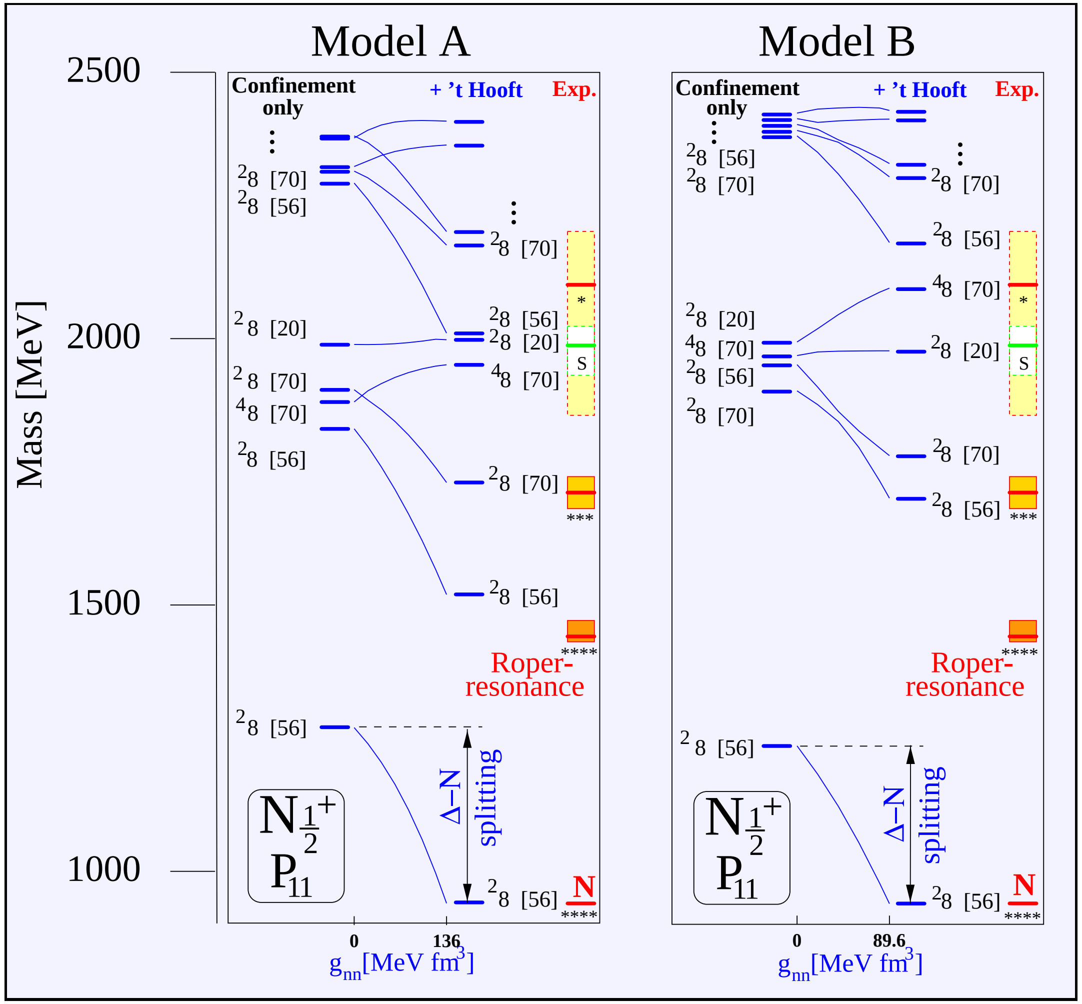
$$V^{(3)}_{\rm conf}(\mathbf{x_1},\mathbf{x_2},\mathbf{x_3}) = \left[3a+b\sum_{i<j}|\mathbf{x_i}-\mathbf{x_j}|\right]\frac{1}{4}\left[\mathbb{1}\otimes\mathbb{1}\otimes\mathbb{1}+\gamma^0\otimes\gamma^0\otimes\mathbb{1}+\text{cycl. perm.}\right]$$

Modell B (alternative)

Although both models give exactly the same results in the nonrelativistic limit, large differences appear in a fully relativistic treatment (Salpeter equation) !

Example: Roper resonance position . . .

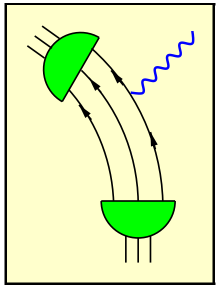
interplay of confinement and instanton effects...



- "Correct" mass splittings result from a specific interplay of relativistic effects, the Dirac structure of the confinement potential and 't Hooft's interaction
- A reliable test of a residual interaction in fact requires a fully relativistic treatment

Electromagnetic and strong coupling amplitudes

Mandelstam formalism: Matrix element in initial state rest frame (IA):



$$\langle B\bar{P}_B | J^\mu(0) | B^* M\rangle = -3 \int \frac{d^4 p_\xi}{(2\pi)^4} \frac{d^4 p_\eta}{(2\pi)^4}$$

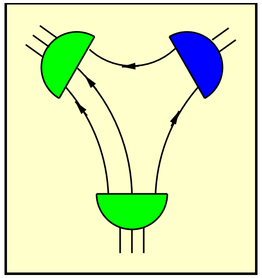
$$\overline{\Gamma}_{\bar{P}}(p'_\xi, p'_\eta)\, S_F^1(p_1) \otimes S_F^2(p_2) \otimes S_F^3(p'_3)\, \hat{Q}\gamma^\mu S_F^3(p_3)\, \Gamma_M(\vec{p}_\xi, \vec{p}_\eta)$$

- Reconstruction of the vertex function in the rest frame

$$\Gamma_M(\vec{p}_\xi, \vec{p}_\eta) = -i \int \frac{d^3 p'_\xi}{(2\pi)^3} \frac{d^3 p'_\eta}{(2\pi)^4}\, V(\vec{p}_\xi, \vec{p}_\eta, \vec{p}'_\xi, \vec{p}'_\eta)\, \Phi_M(\vec{p}'_\xi, \vec{p}'_\eta)$$

- Boost $(M, \vec{0}) \to (\sqrt{M^2 + \vec{P}^2}, \vec{P})$

$$\langle B\bar{P}_B, \pi\bar{P}_\pi | S | B^* M\rangle = -3 \int \frac{d^4 p_\xi}{(2\pi)^4} \frac{d^4 p_\eta}{(2\pi)^4}$$



$$\overline{\Gamma}_{\bar{P}}(p'_\xi, p'_\eta)\, S_F^1(p_1) \otimes S_F^2(p_2) \otimes S_F^3(p'_3)\, \bar{\Gamma}(p) S_F^3(p_3)\, \Gamma_M(\vec{p}_\xi, \vec{p}_\eta)$$

mesonic Bethe-Salpeter amplitudes from
(M.Koll et al., Eur. Phys. J. A 9, 73 (2000))

Static electromagnetic properties

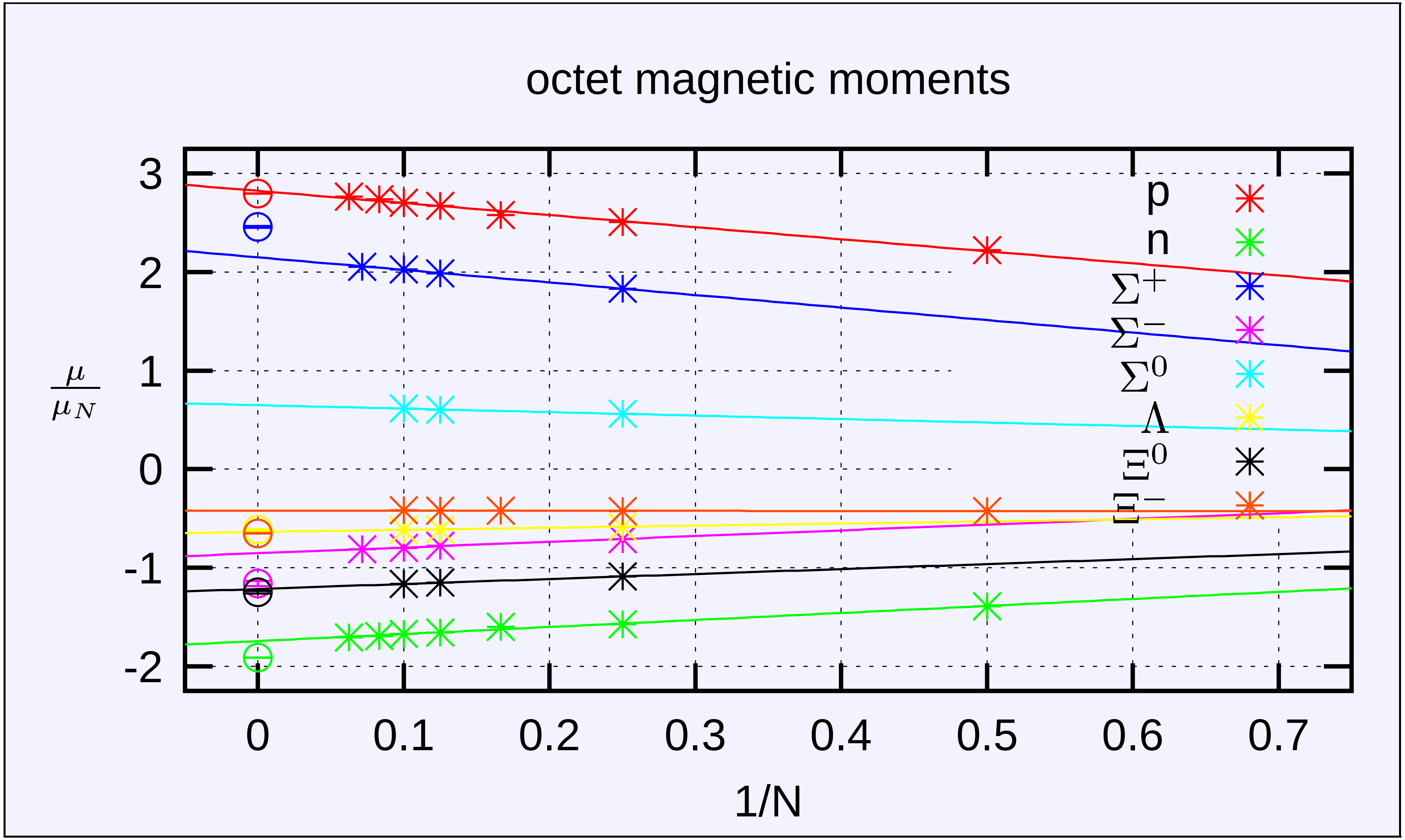
from $Q^2 \to 0$-limit of form factors:

	Calc.	Exp.	
μ_p	$2.74\mu_N$	$2.793\mu_N$	(PDG)
μ_n	$-1.70\mu_N$	$-1.913\mu_N$	(PDG)
$\sqrt{\langle r^2\rangle_E^p}$	0.82 fm	0.847 fm	(MMD)
$\langle r^2\rangle_E^n$	0.11 fm^2	0.113 ± 0.004 fm^2	(MMD)
$\sqrt{\langle r^2\rangle_M^p}$	0.91 fm	0.836 fm	(MMD)
$\sqrt{\langle r^2\rangle_M^n}$	0.86 fm	0.889 fm	(MMD)
g_A	1.21	1.2670 ± 0.0035	(PDG)
$\sqrt{\langle r^2\rangle_A}$	0.62 fm	0.61 ± 0.01 fm	(Bernard)

MMD: Mergell, Meißner, Drechsel
Bernard: Bernard, Elouadrhiri, Meißner

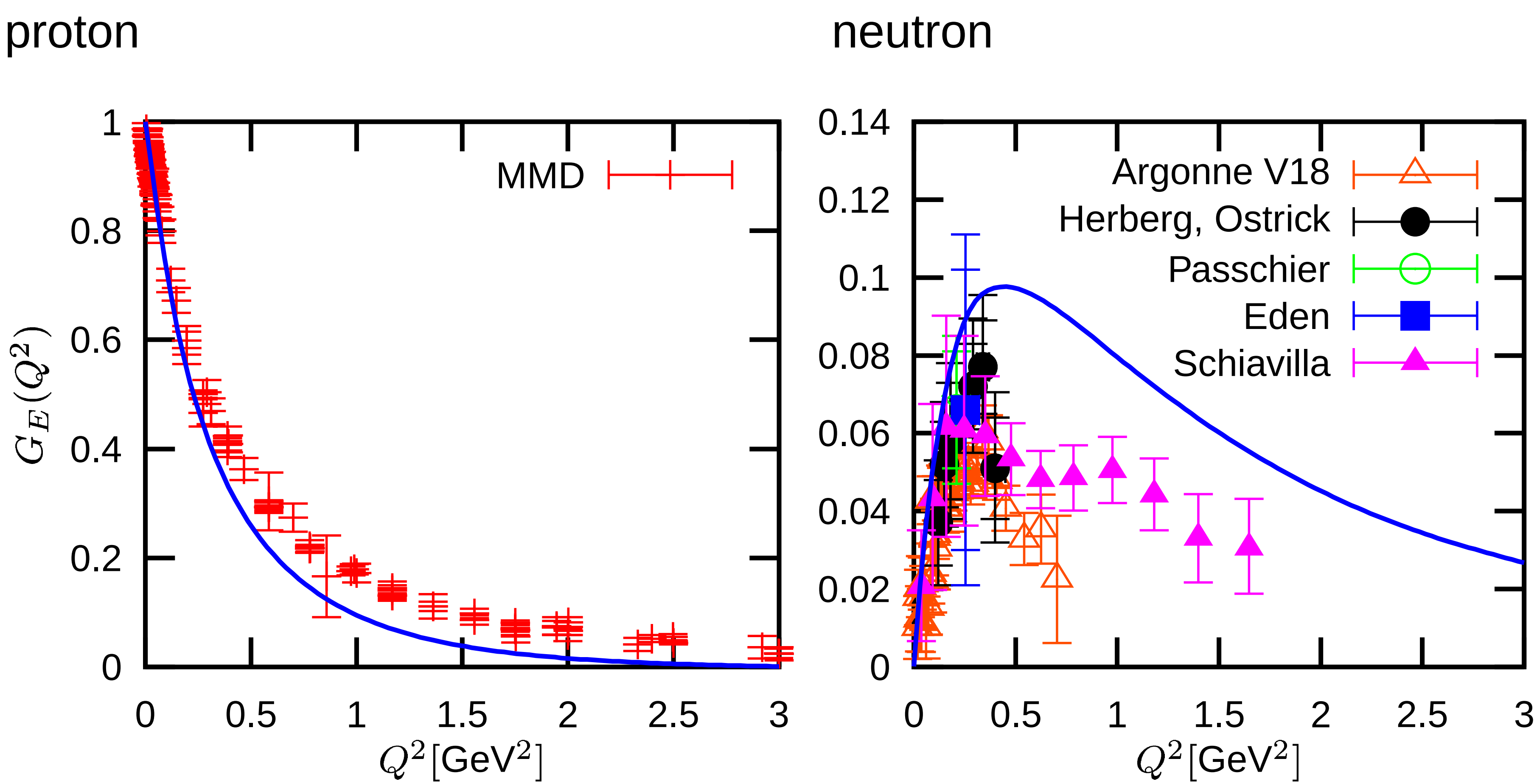
Octet magnetic moments

from “expectation values” of local operators with S.A.:



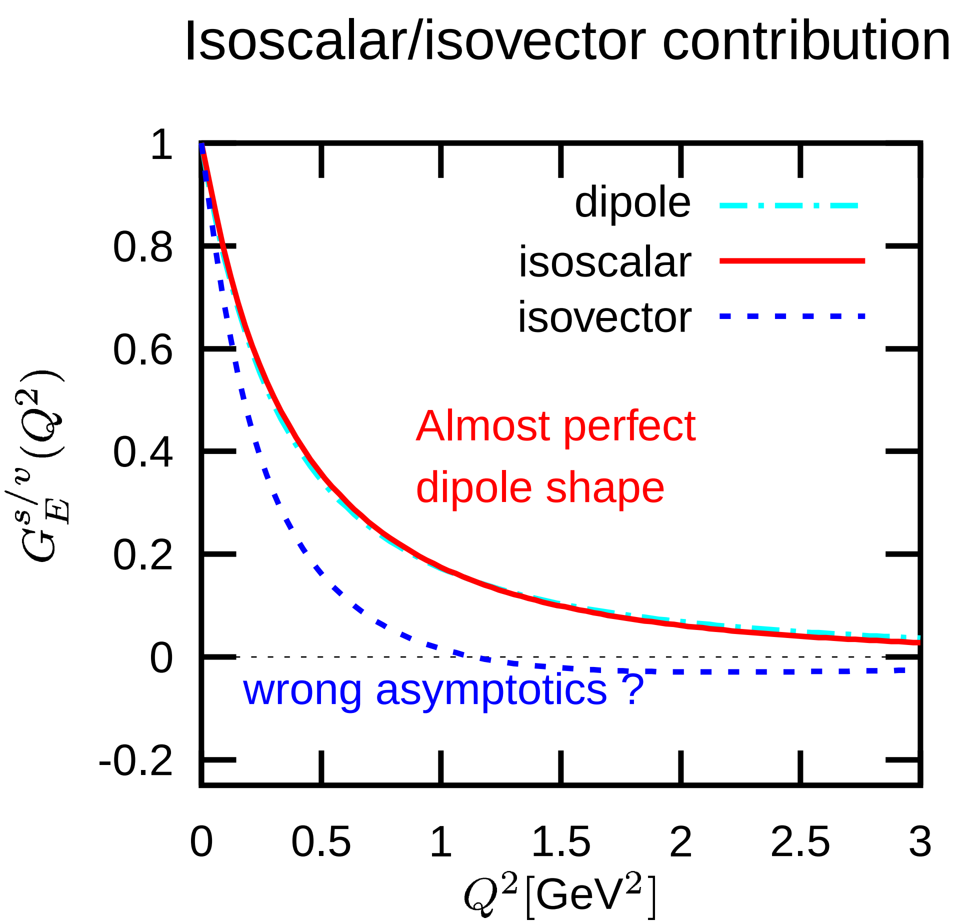
exp. $\leftarrow$ basis size (calc.)

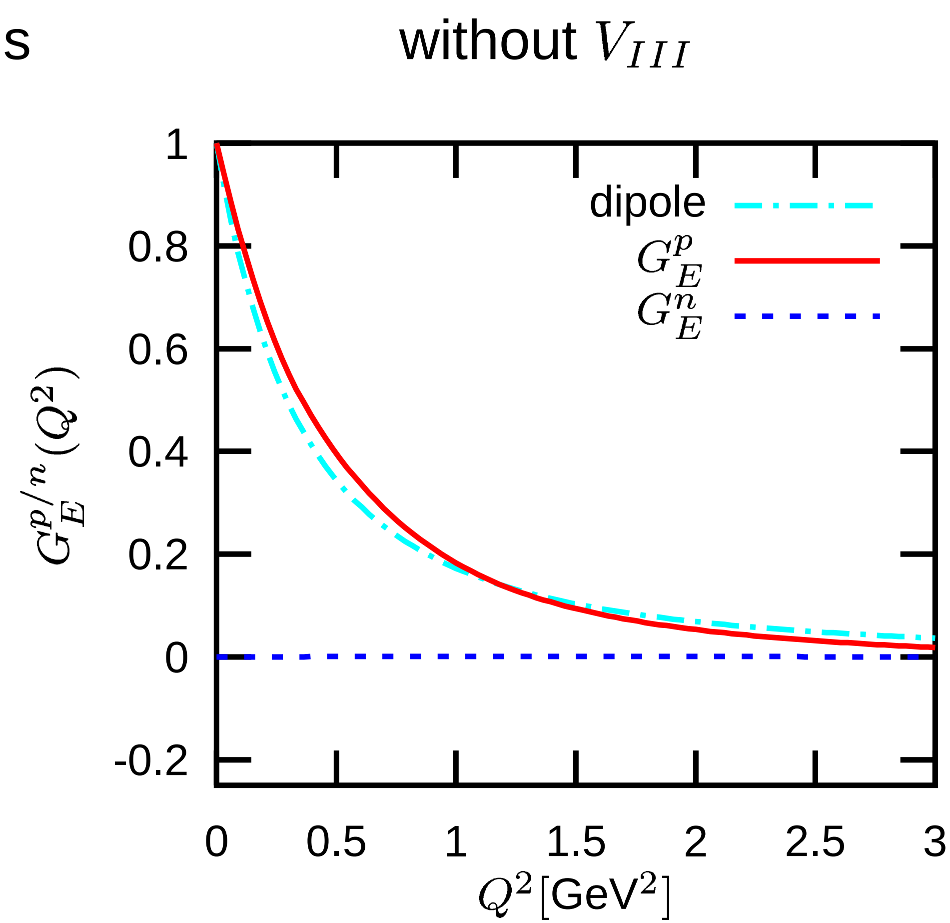
Electric form factor of proton and neutron



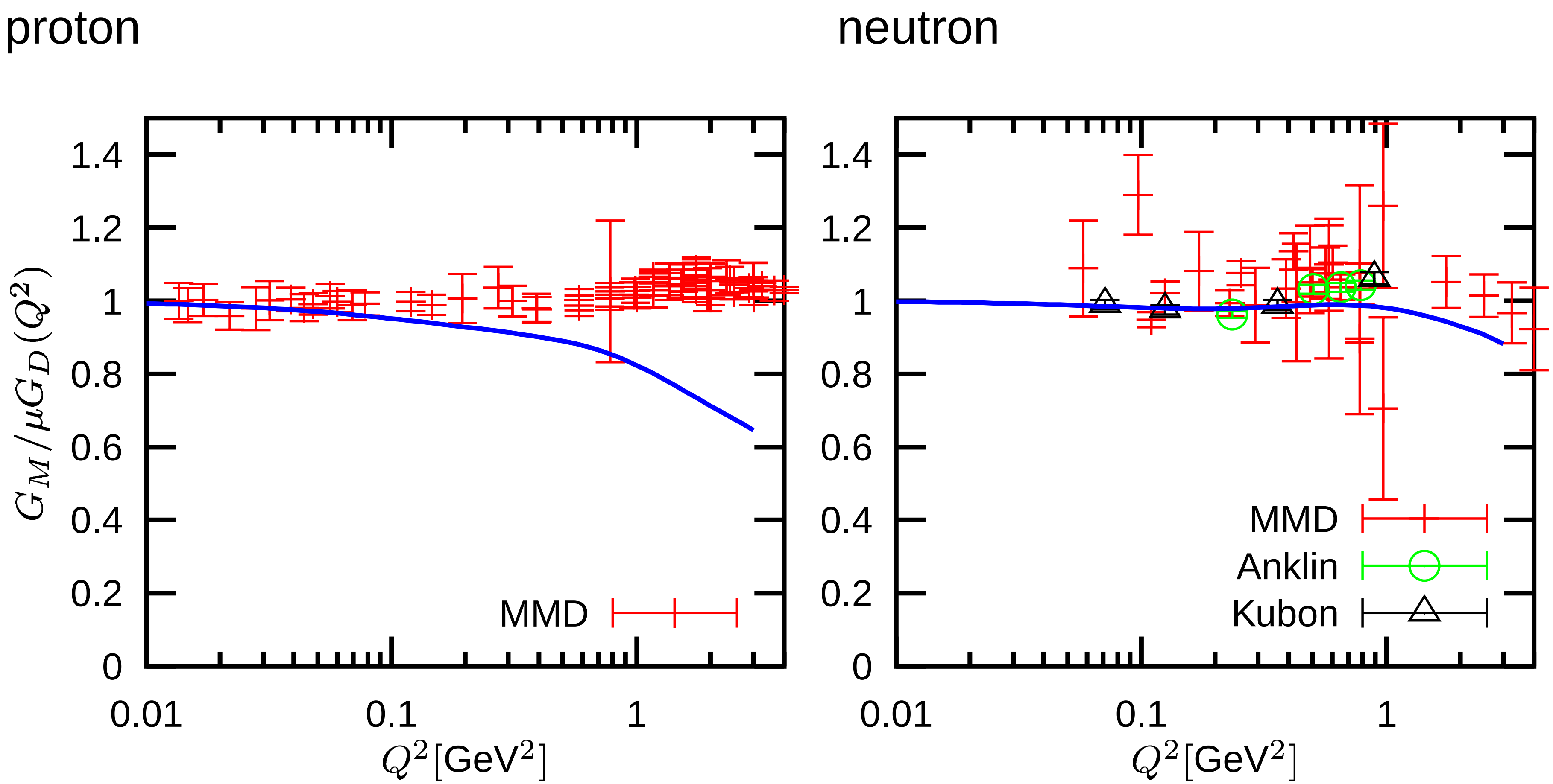
MMD: Mergell, Meißner, Drechsel

Electric form factor of proton and neutron



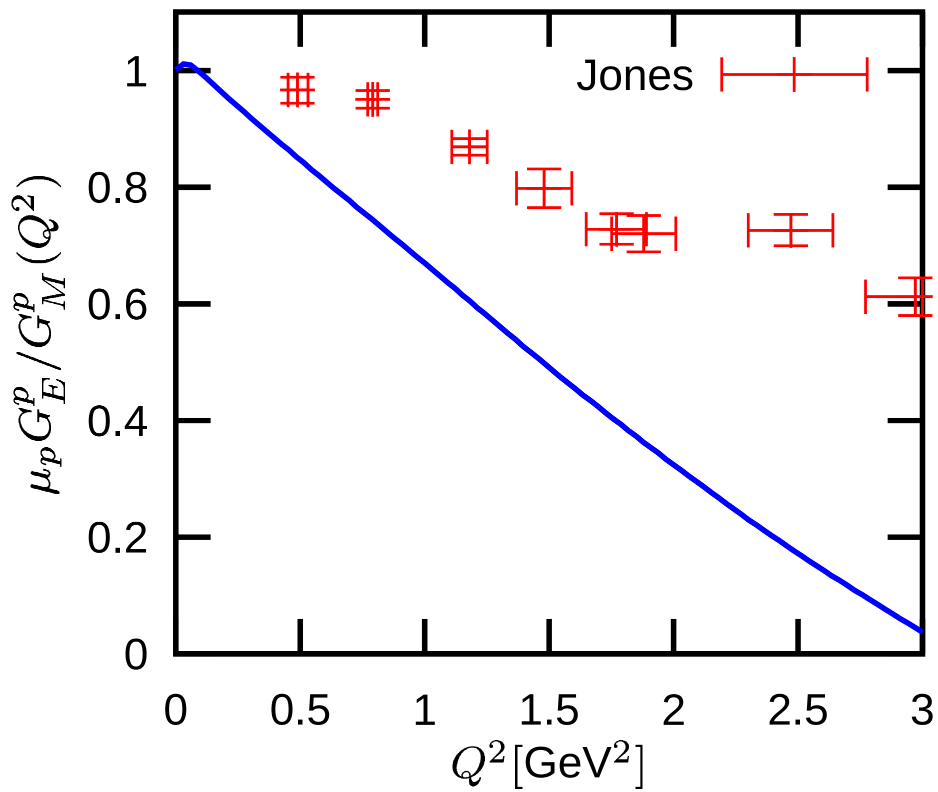


Magnetic form factor of proton and neutron

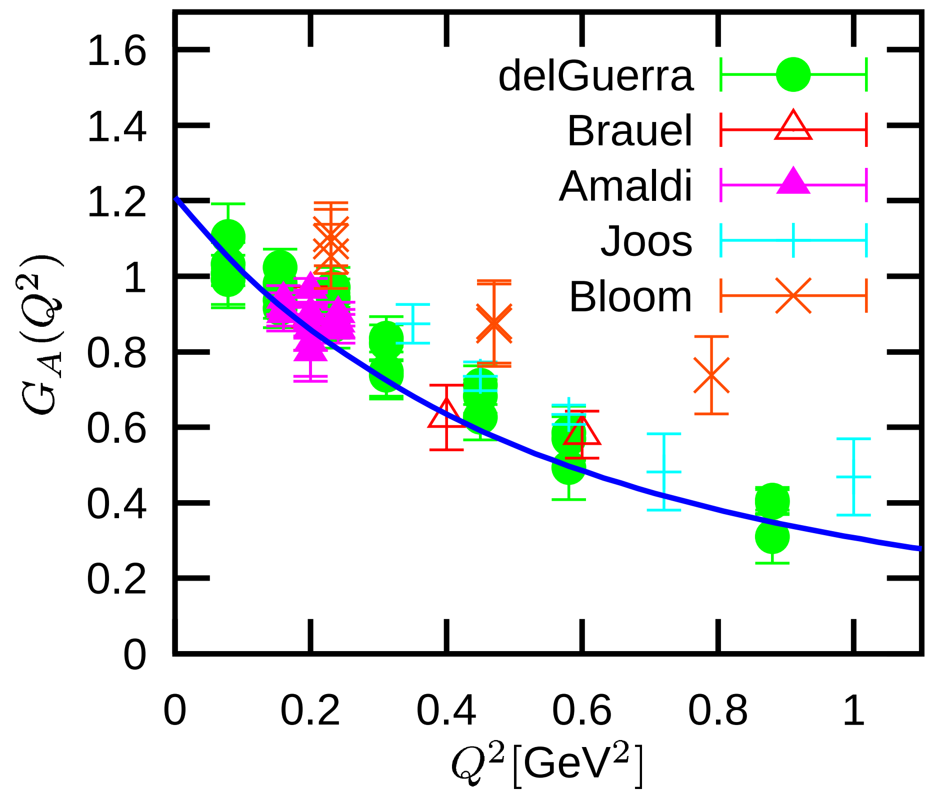


Electroweak nucleon form factors

G_E/G_M



axial form factor

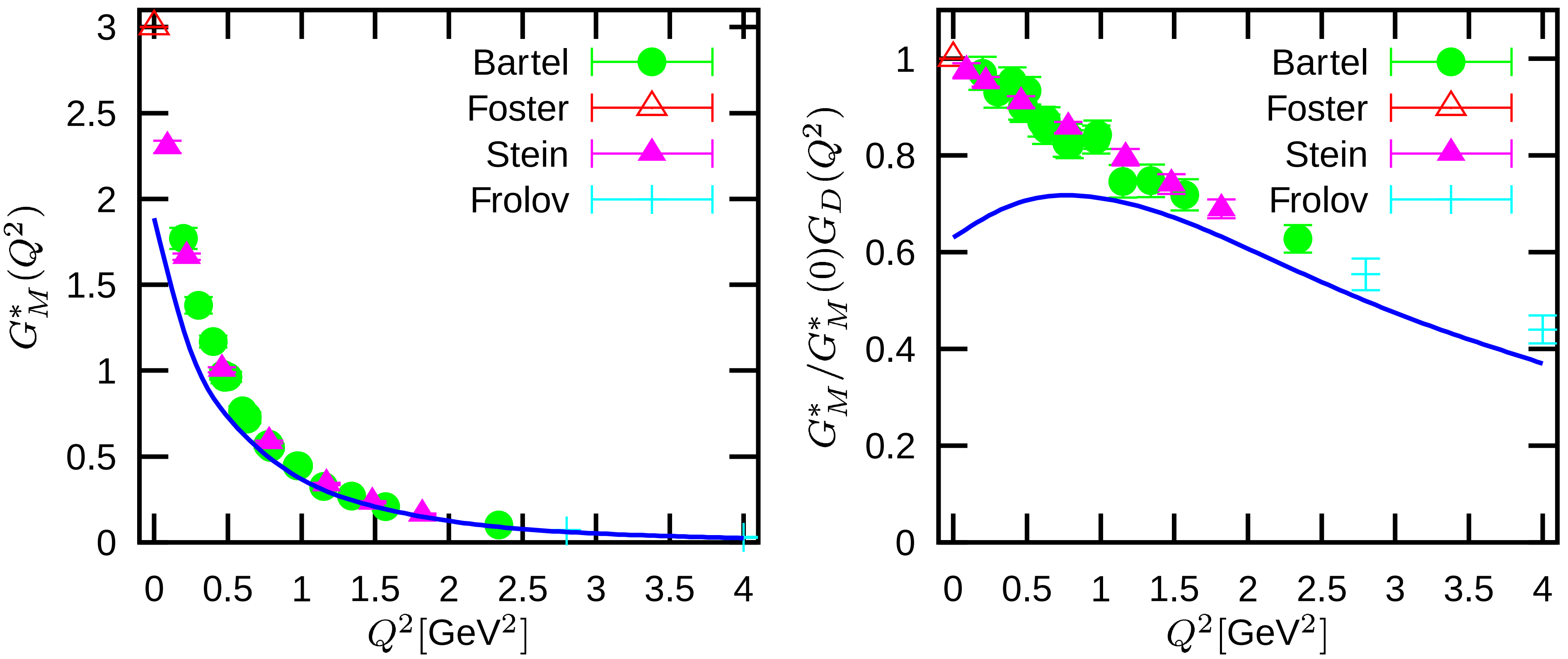


Photoncouplings $[10^{-3}\mathrm{GeV}^{-\frac{1}{2}}]$

state		Calc	PDG		Calc	PDG
$P_{33}(1232)$	$A^N_{1/2}$	-89	-135 ± 6			
	$A^N_{3/2}$	-152	-255 ± 8			
$S_{11}(1535)$	$A^p_{1/2}$	113	90 ± 30	$A^n_{1/2}$	-75	-46 ± 27
$S_{11}(1650)$	$A^p_{1/2}$	6	53 ± 16	$A^n_{1/2}$	-22	-15 ± 21
$D_{13}(1520)$	$A^p_{1/2}$	-53	-24 ± 9	$A^n_{1/2}$	1	-59 ± 9
	$A^p_{3/2}$	51	166 ± 5	$A^n_{3/2}$	-52	-139 ± 11
$D_{13}(1700)$	$A^p_{1/2}$	-18	-18 ± 13	$A^n_{1/2}$	23	0 ± 50
	$A^p_{3/2}$	-14	-2 ± 24	$A^n_{3/2}$	-60	-3 ± 44
$D_{15}(1675)$	$A^p_{1/2}$	5	19 ± 8	$A^n_{1/2}$	-35	-43 ± 12
	$A^p_{3/2}$	7	15 ± 9	$A^n_{3/2}$	-47	-58 ± 13
$P_{11}(1440)$	$A^p_{1/2}$	-48	-65 ± 4	$A^n_{1/2}$	27	40 ± 10
$S_{31}(1620)$	$A^N_{1/2}$	18	27 ± 11			
$D_{33}(1700)$	$A^N_{1/2}$	63	104 ± 15			
	$A^N_{3/2}$	68	85 ± 22			

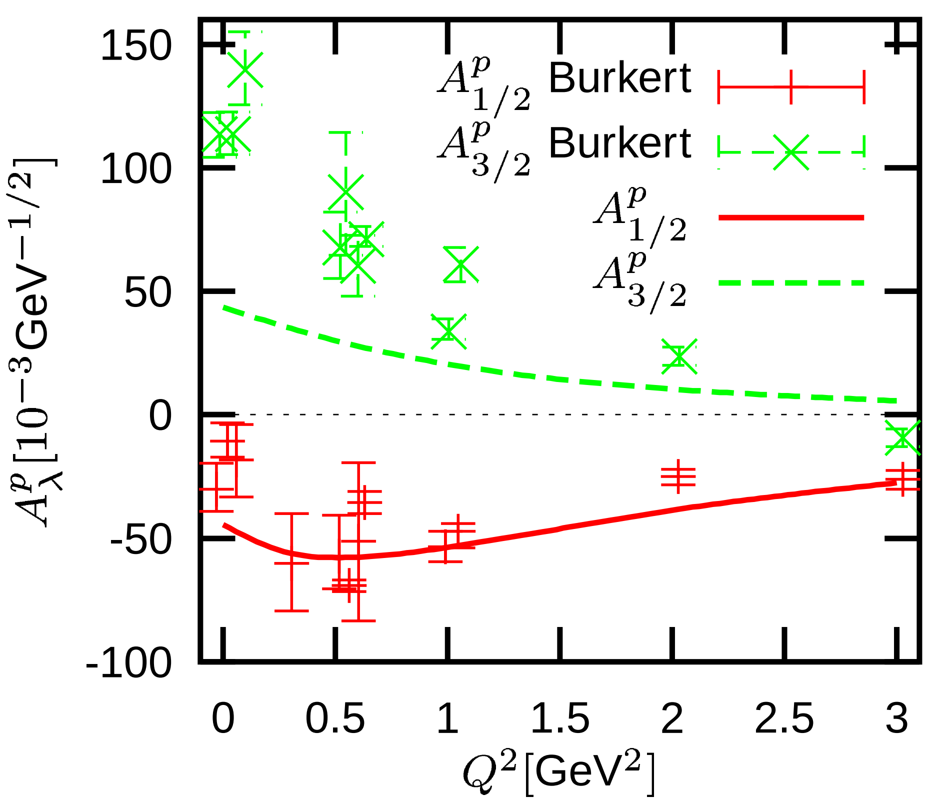
Transition form factors

Nucleon Delta transition

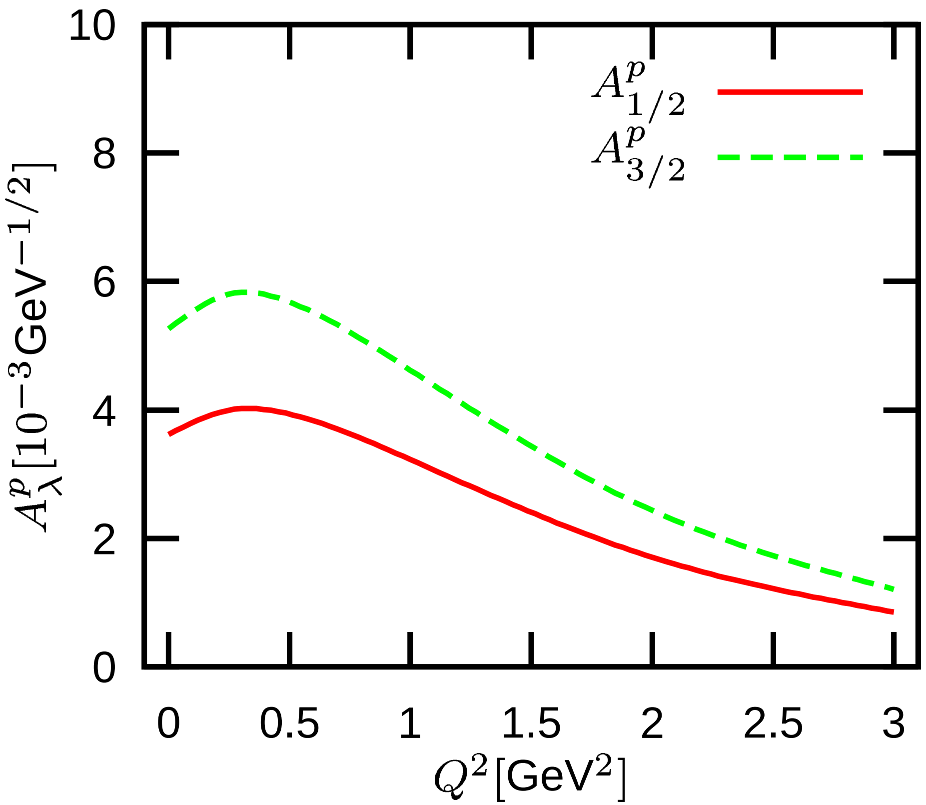


Helicity amplitudes

F15(1680) Nucleon transition



D15(1675) Nucleon transition



Strong decay widths

$N\pi$ decay widths Γ[MeV] — $\Delta\pi$ decay widths Γ[MeV]

Decay	Calc	$^3P_0$	PDG	Decay	Calc	$^3P_0$	PDG
$S_{11}(1535) \to N\pi$	33	216	$(68 \pm 15)^{+45}_{-23}$	$\to \Delta\pi$	1	2	< 2
$S_{11}(1650) \to N\pi$	3	149	$(109 \pm 26)^{+29}_{-4}$	$\to \Delta\pi$	5	13	$(6 \pm 5)^{+2}_{0}$
$D_{13}(1520) \to N\pi$	38	74	$(66 \pm 6)^{+8}_{-5}$	$\to \Delta\pi$	35	35	$(24 \pm 6)^{+3}_{-2}$
$D_{13}(1700) \to N\pi$	0.1	34	$(10 \pm 5)^{+5}_{-5}$	$\to \Delta\pi$	88	778	seen
$D_{15}(1675) \to N\pi$	4	28	$(68 \pm 7)^{+14}_{-5}$	$\to \Delta\pi$	30	32	$(83 \pm 7)^{+17}_{-6}$
$P_{11}(1440) \to N\pi$	38	412	$(228 \pm 18)^{+65}_{-65}$	$\to \Delta\pi$	35	11	$(88 \pm 18)^{+25}_{-25}$
$P_{33}(1232) \to N\pi$	62	108	$(119 \pm 0)^{+5}_{-5}$				
$S_{31}(1620) \to N\pi$	4	26	$(38 \pm 7)^{+8}_{-8}$	$\to \Delta\pi$	72	18	$(68 \pm 23)^{+14}_{-14}$
$D_{33}(1700) \to N\pi$	2	24	$(45 \pm 15)^{+15}_{-15}$	$\to \Delta\pi$	52	262	$(135 \pm 45)^{+45}_{-45}$

$^3P_0$: S. Capstick, W. Roberts, Phys.Rev. D49 (1994) 4570-4586