Recent Advances in Dyson-Schwinger Studies

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Dyson-Schwinger Studies of QCD

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  Hadrons are composites of quarks and gluons.

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- For $\alpha_S(Q^2) \gg 1$, incorporates the exclusively non-perturbative aspects of QCD.
- Quark and gluon degrees of freedom.
- Explore implications on hadron observables:
  dynamical chiral symmetry breaking,
  confinement of quarks and gluons,
  IR behavior of $\alpha_S(Q^2)$, ...

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The simplest DSE gives *dressed* quark propagator,

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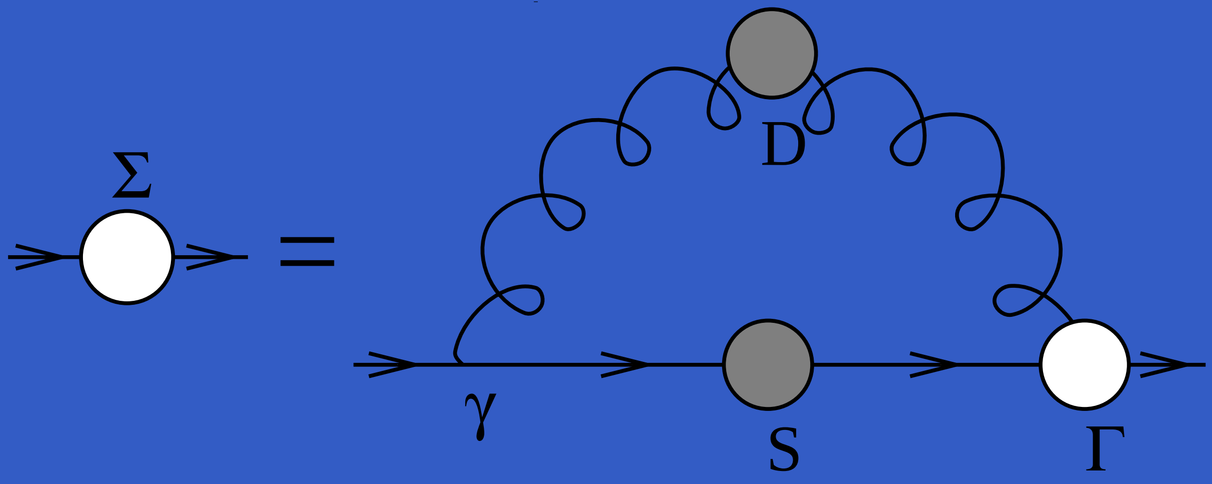
$$S(p) = S_0(p) \;+\; S_0(p)\;\; \Sigma(p)\;\; S(p),$$

where the quark *self-energy* $\Sigma(p)$ is

$$\Sigma(p) = \frac{4}{3}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\;\gamma_\mu S(k)\Gamma_\nu(k,p)\;g^2 D_{\mu\nu}(kp).$$



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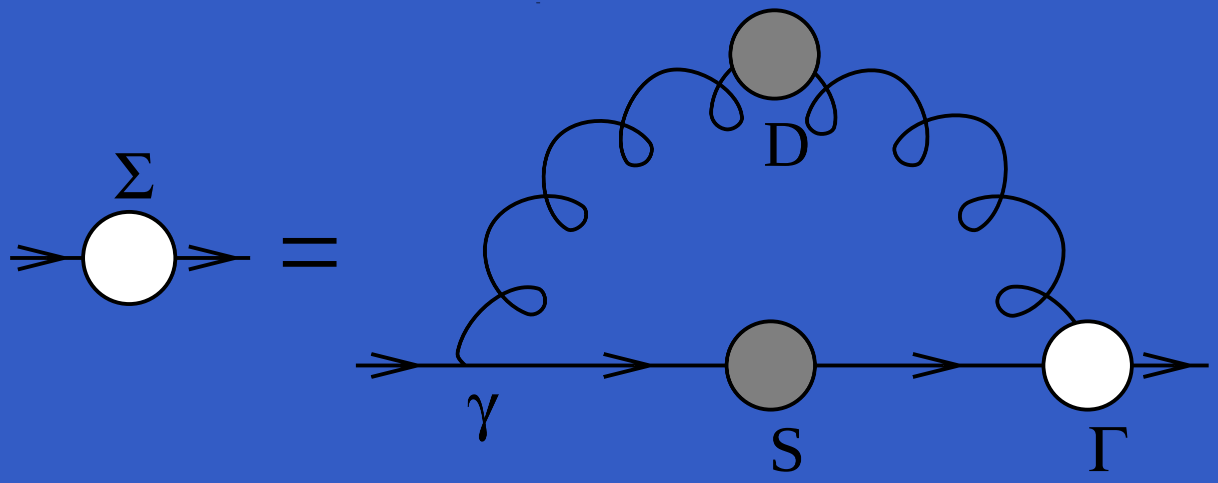
$Z(p^2)$ is the “wave-function renormalization.”

Five Aspects of Solving the Quark DSE

1. Integral is divergent, must be *renormalized.*



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$$\Sigma'(p,\Lambda^2) = Z(\Lambda^2,\mu^2)\frac{4}{3}\int_k^{\Lambda}\gamma_\mu S(k)\Gamma_\nu(k,p)\; g^2 D_{\mu\nu}(k-p).$$

1) Regulate integrals w/ Poincaré-invariant Λ.
2) Theory depends on scale Λ: e.g., $m(\Lambda)$.
2) Introduce counter-terms $\delta m(\Lambda)$.
3) Combine terms so we have finite, Λ-independent “renormalized” $m(\mu)$.
example: $\Sigma(p,\mu^2) = \Sigma'(p,\Lambda^2) - \Sigma'(\mu,\Lambda^2)$.
4) Renormalized parameters fixed at $p^2 = \mu^2$.

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Resolved years ago.

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Analytic solutions are very difficult.
No uniqueness of solutions.

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$$g^2 \text{ small: } M(p^2) = m(1 + \cdots) \xrightarrow{m\to 0} 0$$

$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)} \xrightarrow{g\to 0} \frac{1}{i\gamma\cdot p + m}.$$

One recovers usual perturbative-QCD results.

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   Multiple solutions for $M(p^2)$ when $m = 0$.
   1.) Wigner-Weyl mode: $M(p^2) \propto m = 0$.
   2.) Nambu-Goldstone mode: $M(p^2) \neq 0$.
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Well understood.

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3. Weak coupling $\Rightarrow$ perturbative QCD. $\surd$
4. Strong $\Rightarrow$ dynamical χ symmetry breaking. $\surd$
5. DSE depends on *unknown* $D_{\mu\nu}(k)$ and $\Gamma_\mu(k,p)$.

$$\Sigma(p) = \frac{4}{3}\int\frac{d^4k}{(2\pi)^4}\,\gamma_\mu S(k)\Gamma_\nu(k,p)\;g^2 D_{\mu\nu}(kp).$$

These are solutions of other DSEs!
Once we have them, we solve quark DSE.

Coupled-nature of Dyson-Schwinger Equations

To obtain dressed quark propagator $S(p)$, one needs

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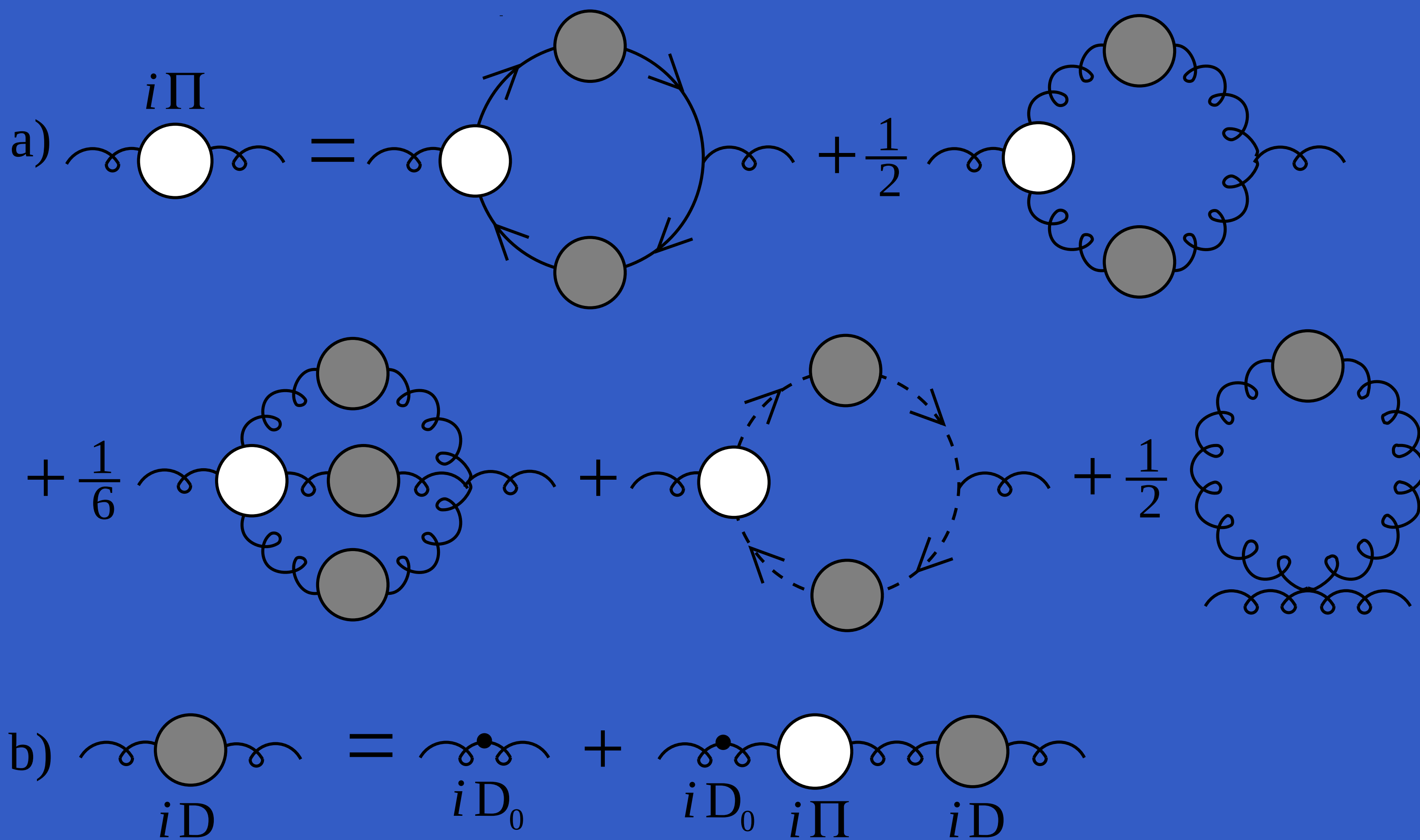

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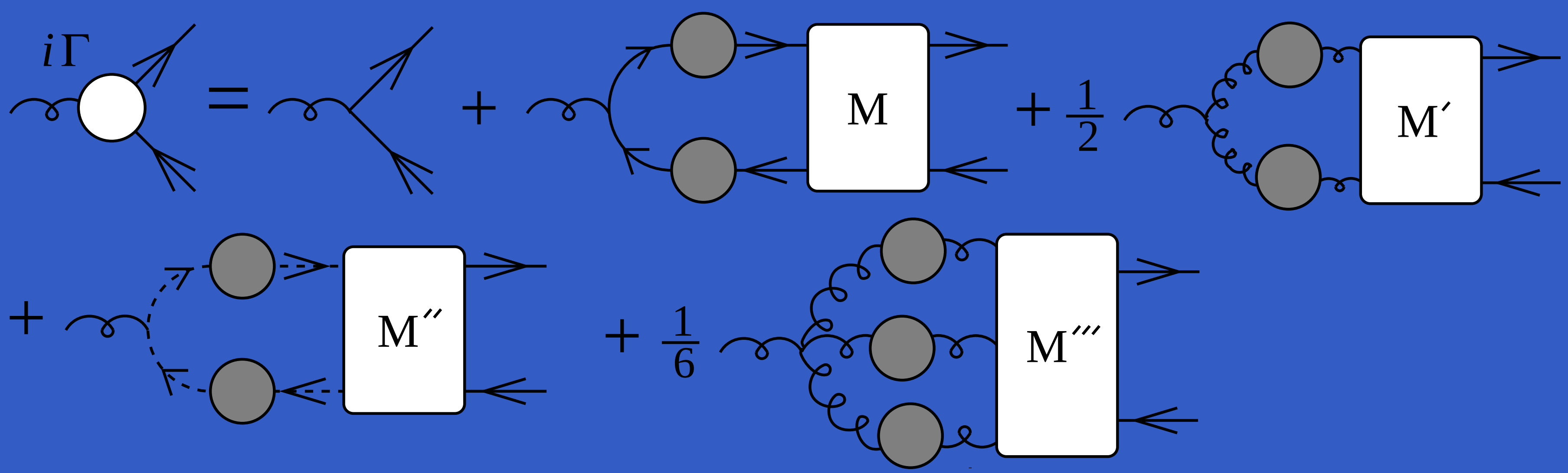

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These functions depend on higher n-point functions, which in turn depend on still higher n-point functions.

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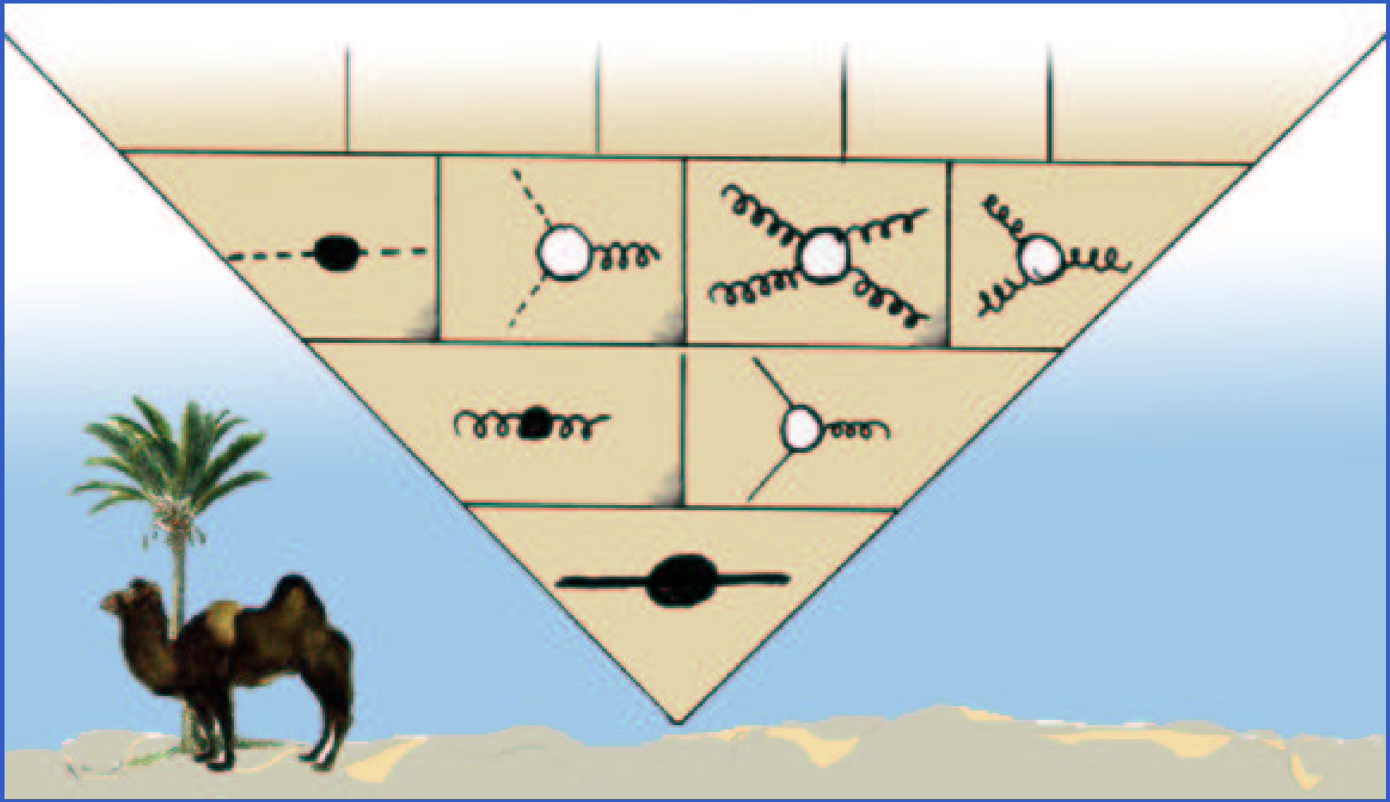
Proceed by truncating the system of equations.

Truncation of Dyson-Schwinger Equations



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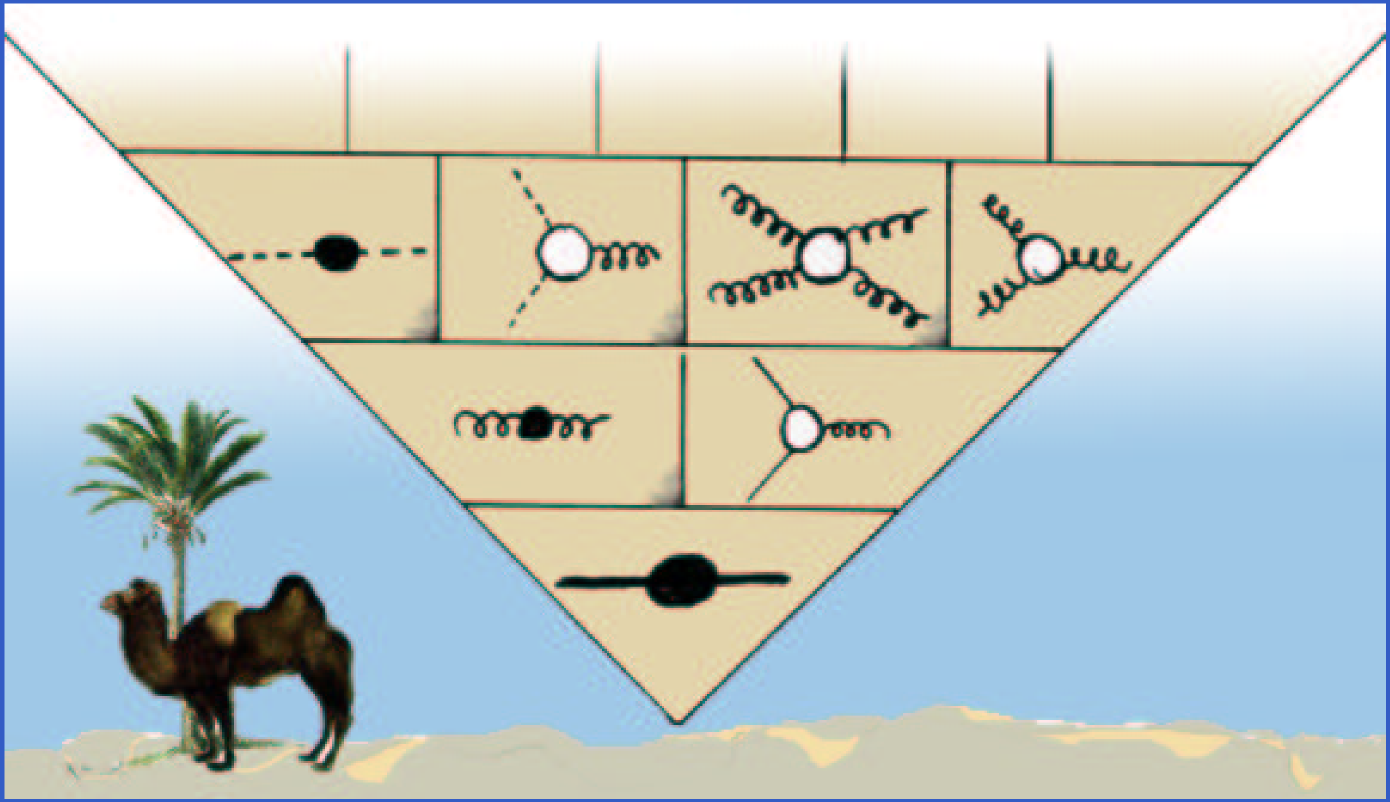

"DSEs form an *infinite* set of *coupled* integral equations"

⋆ Truncation schemes reduce the number of DSEs. ⋆

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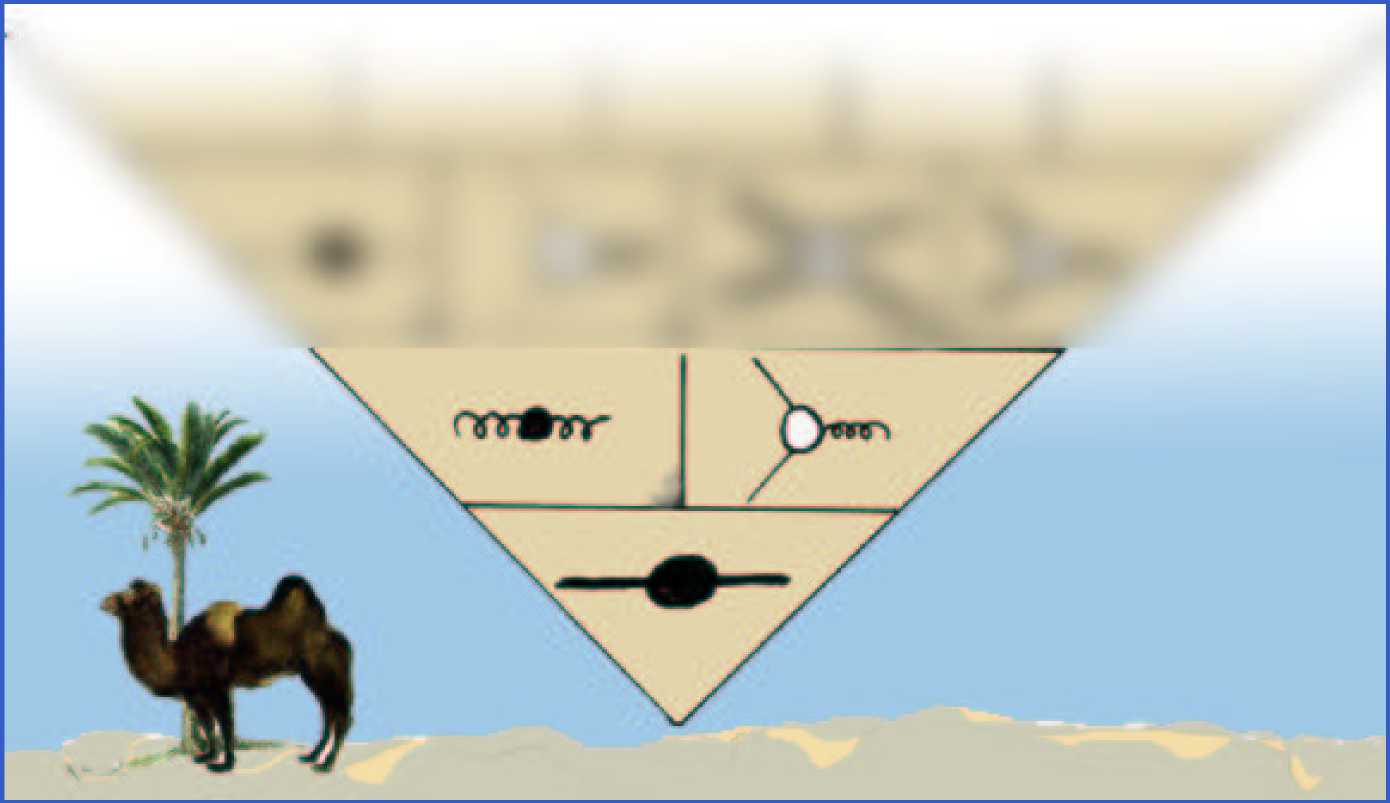

Simplest scheme: Ansätze for $D_{\mu\nu}(k)$ and $\Gamma_\mu(k,p)$.

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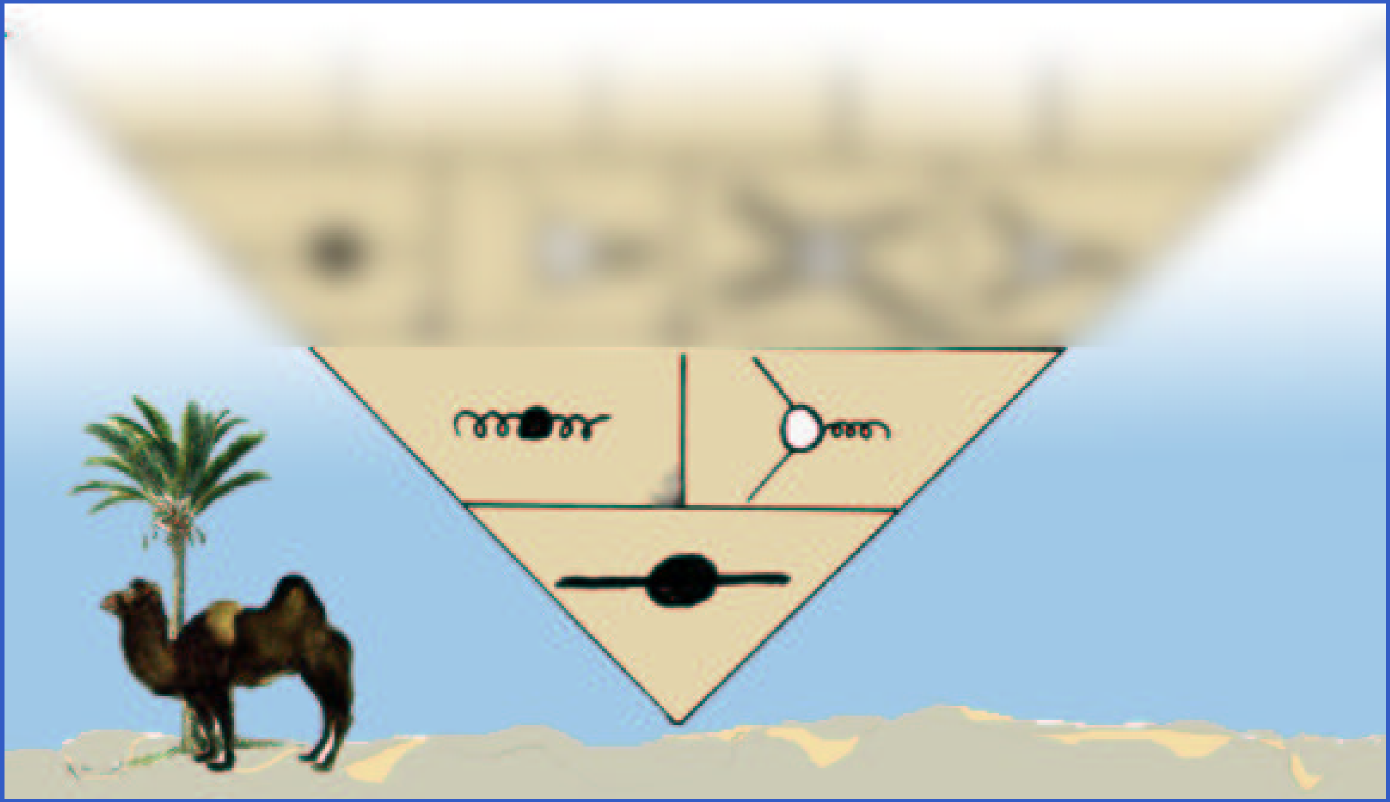

Simplest scheme: Ansätze for $D_{\mu\nu}(k)$ and $\Gamma_{\mu}(k,p)$.
This is the "rainbow approximation."

Rainbow Truncation

The simplest truncation scheme is

$$g^2 D_{\mu\nu}(q) = \left(\delta_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \Delta(q^2),$$
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Here, $\Delta(q^2)$ is phenomenological input.

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\begin{aligned}
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\Gamma_\mu(k,p) &= \gamma_\mu
\end{aligned}
$$

Insert picture of rainbow expansion here.

$$
\Sigma(p) = \frac{4}{3} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \left[\gamma_\mu \, S(k) \, \gamma_\mu - \gamma \cdot q \, S(k) \, \gamma \cdot q\right] \Delta(q^2)
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Example: Maris-Tandy Model

PRC60 055214 (1999)

$$\Delta(q^2) = \frac{4\pi^2 D}{\omega^6}\, e^{-q^2/\omega^2} + \frac{4\pi^2 \gamma_m \mathcal{F}(q^2)}{\frac{1}{2}\ln[\tau + (1 + q^2/\Lambda^2_{\rm QCD})^2]}.$$

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Non-perturbative enhancement near $q^2 \approx 0$.
Perturbative-gluon tail: $1/q^2$ + log corrections.

Solution of $S(k)$ in Rainbow Approximation

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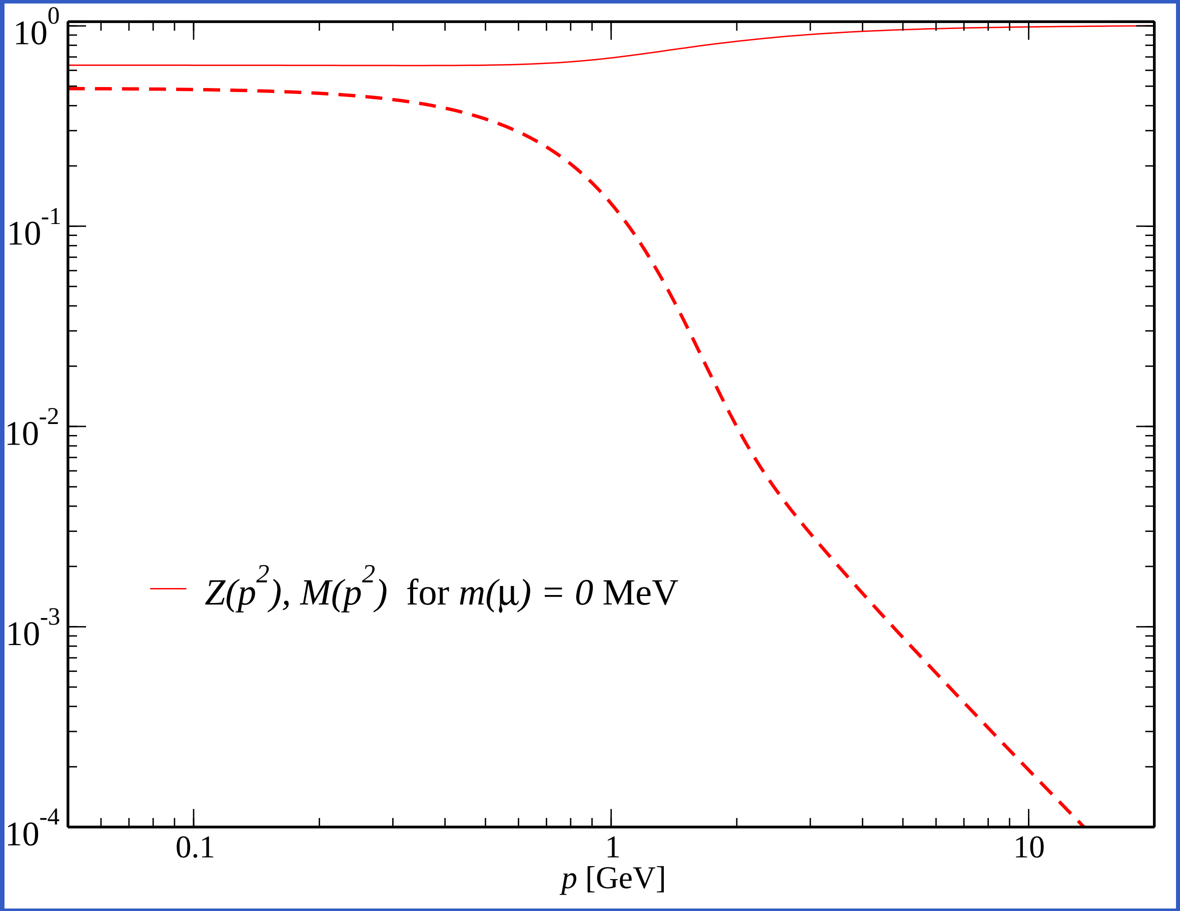

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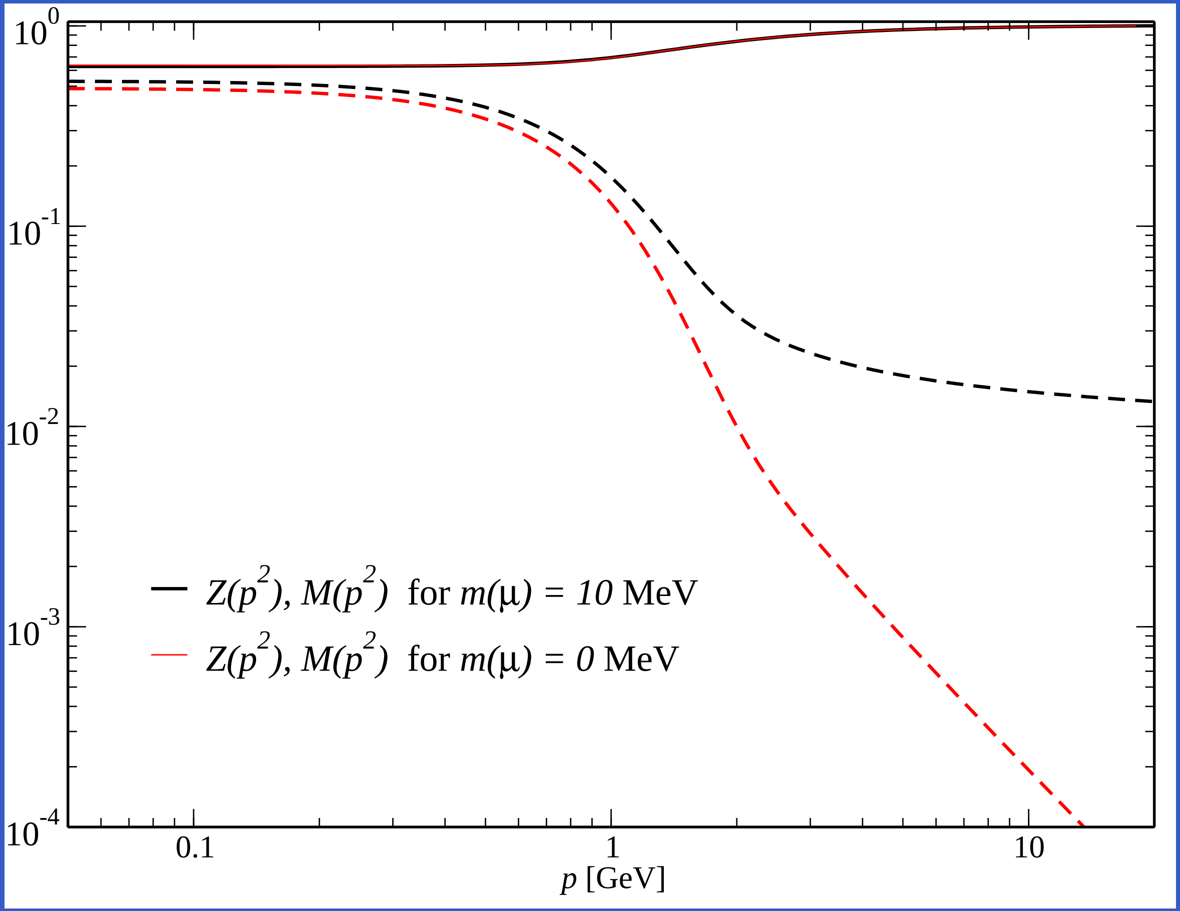

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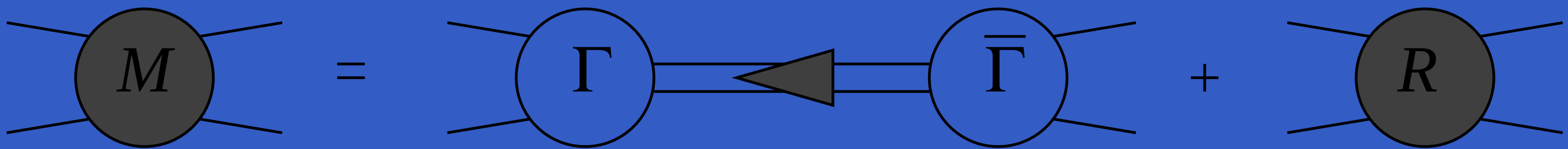
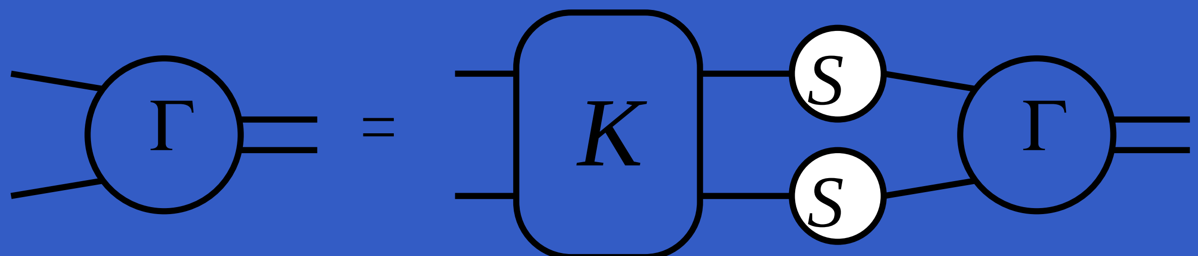
Having $S(k)$ is only the beginning.
Turn now to calculating $\bar{q}q$ (meson) and
qqq baryon bound states using our Green functions.

Mesons are $\bar{q}q$ Bound States

Mesons appear as poles in $q\bar{q}$ Bethe-Salpeter amp M,



Meson wave-functions are solutions of homogenous BSE.

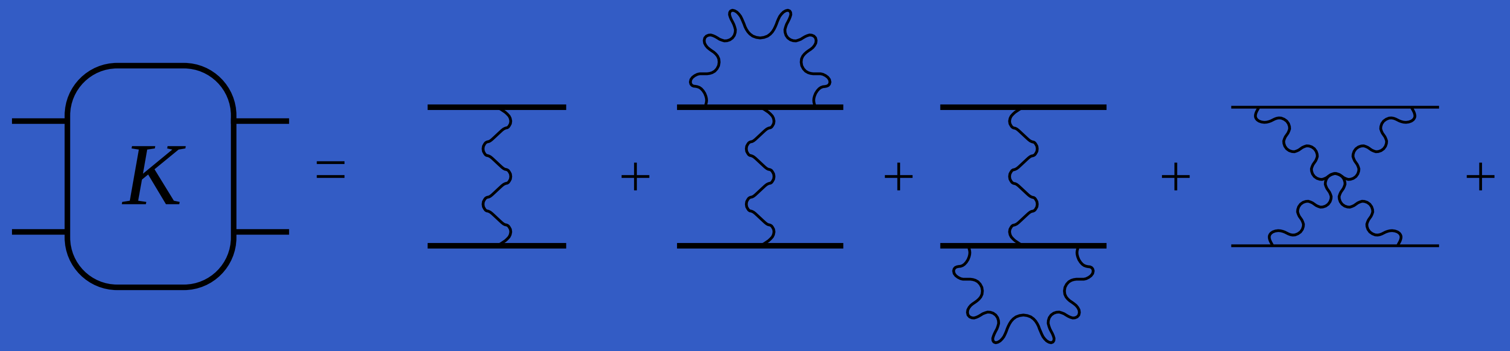


$$\Gamma(p_1, p_2) = \int_k \Gamma(k_1, k_2) S(k_2) K(k_1, k_2; p_1, p_2) S(k_2)$$

BS kernel K contains *all* 2PI $q\bar{q}$ scattering diagrams.

Mesons are $\bar{q}q$ Bound States

K has infinite interactions of differing topologies



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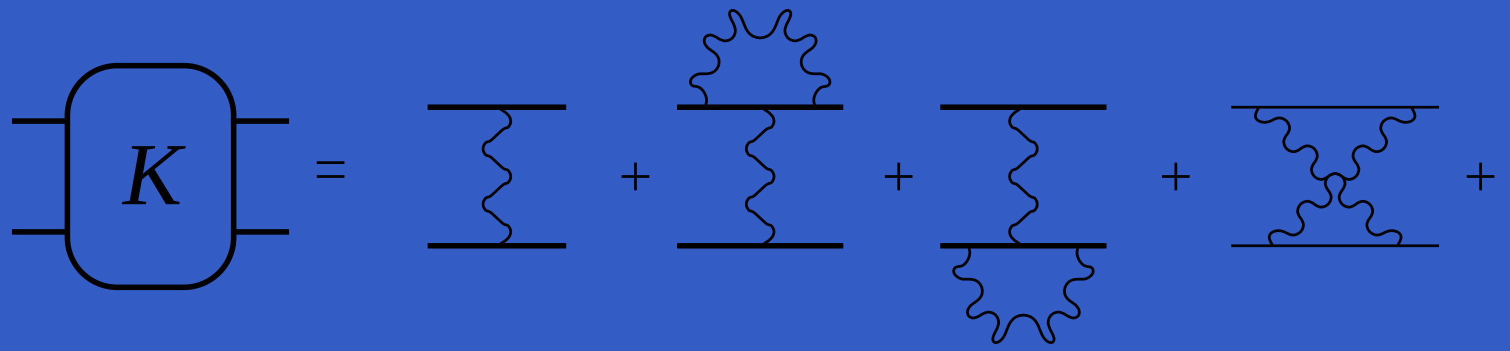
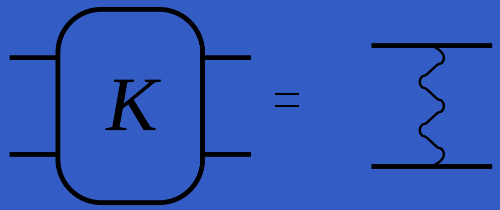
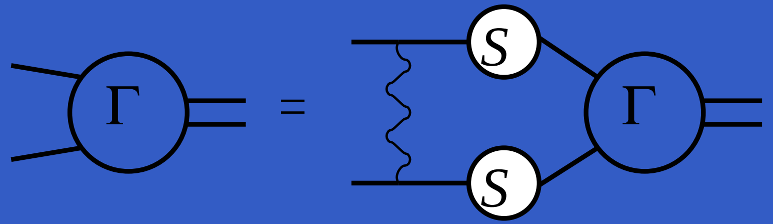


Simplest truncation for K is called “ladder”:





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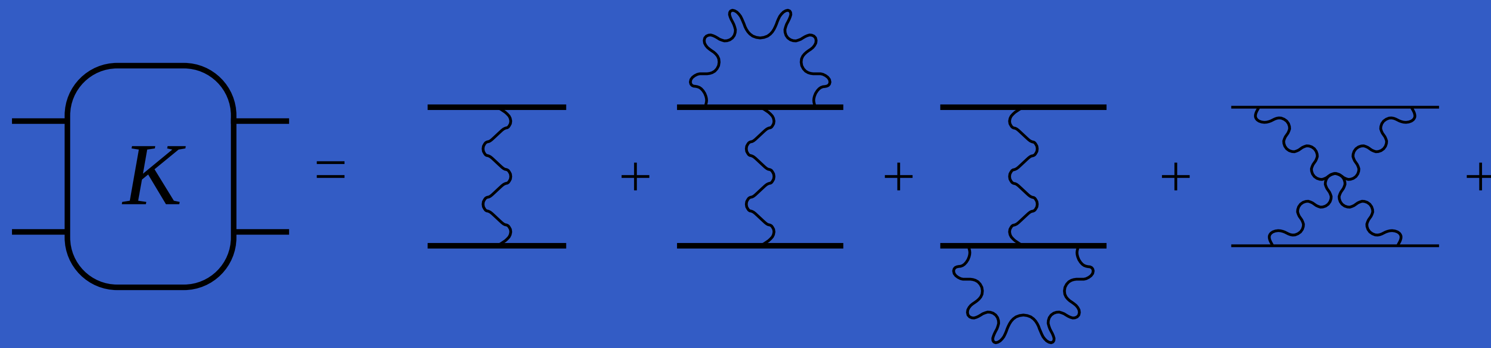

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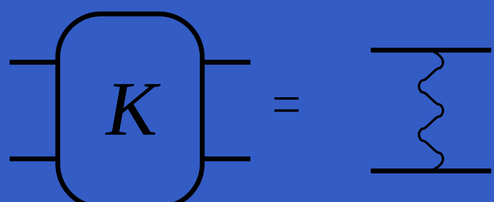
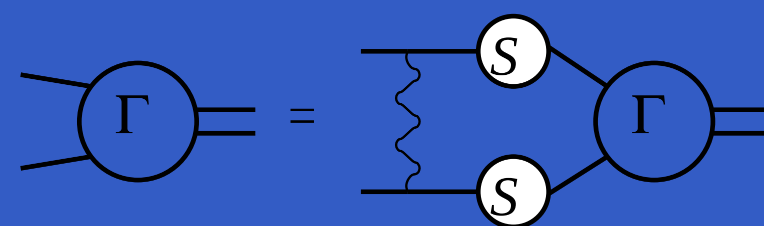
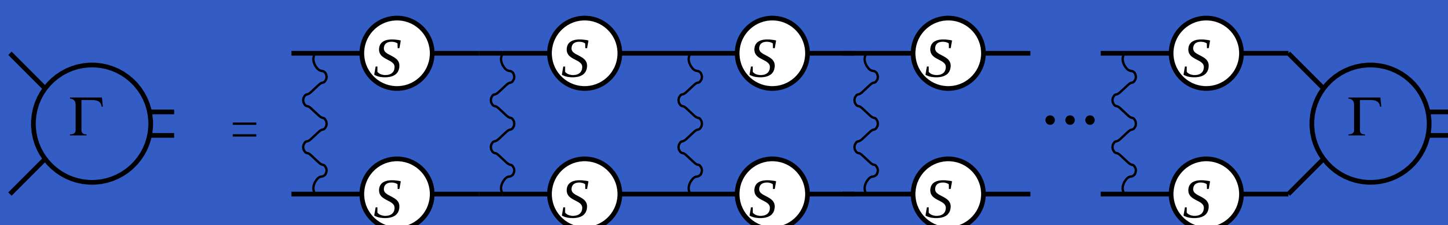


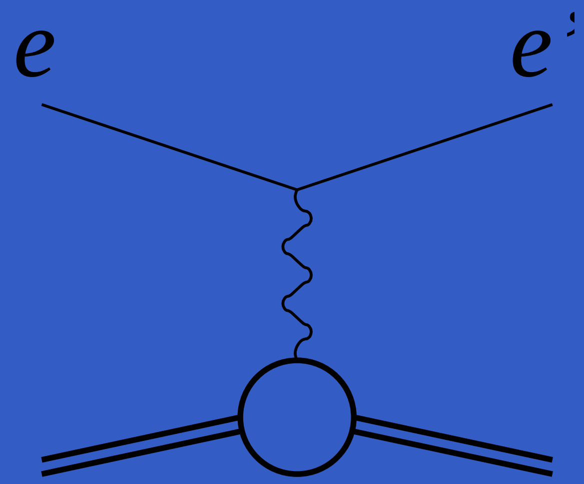




Calculation of Observables

Having propagators and
BS amplitudes, one may calculate
meson observables.

Example: EM form factor of meson
is probed by scattered electrons.



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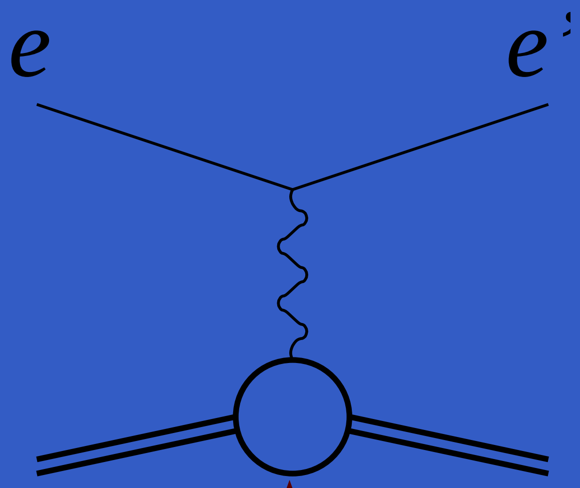

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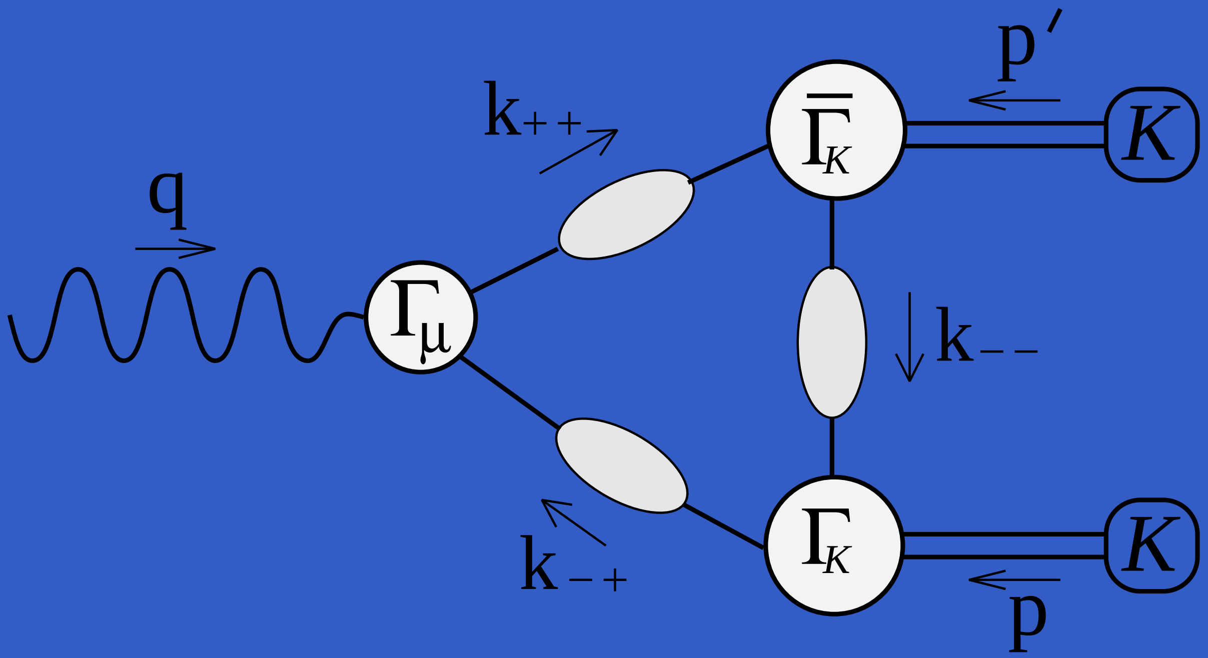
Form factor calculated from quark structure of meson. Use dressed-quark propagators $S(k)$, meson amps $\Gamma_\pi(k, P)$.

Meson form factor requires evaluating the loop integral



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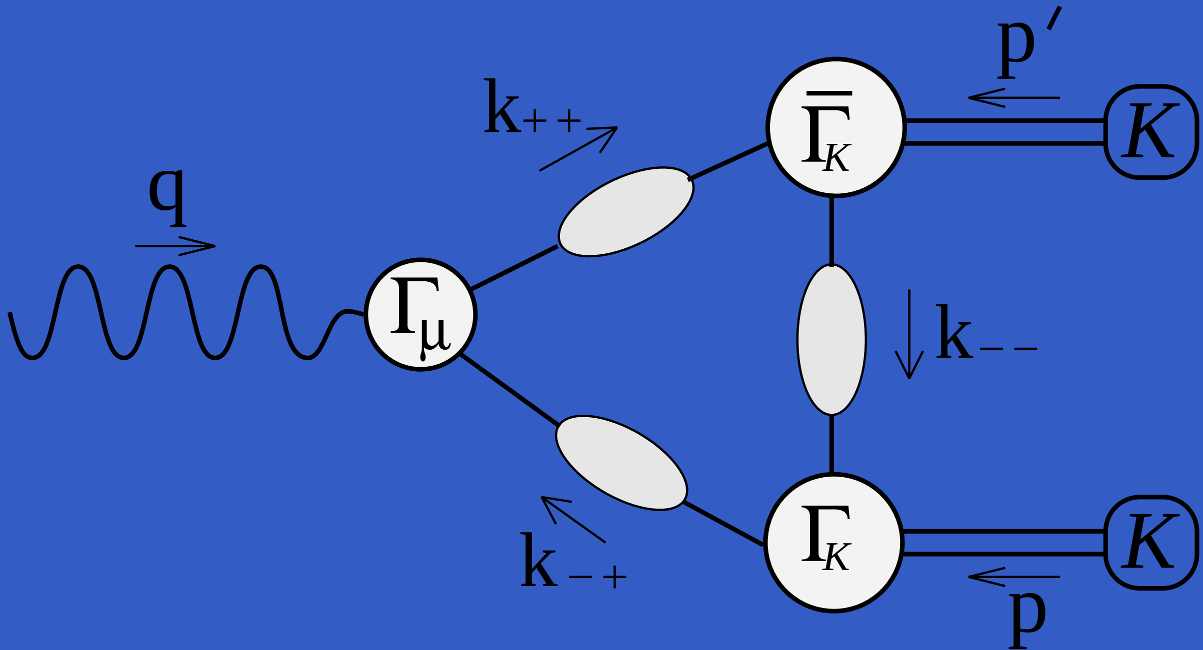
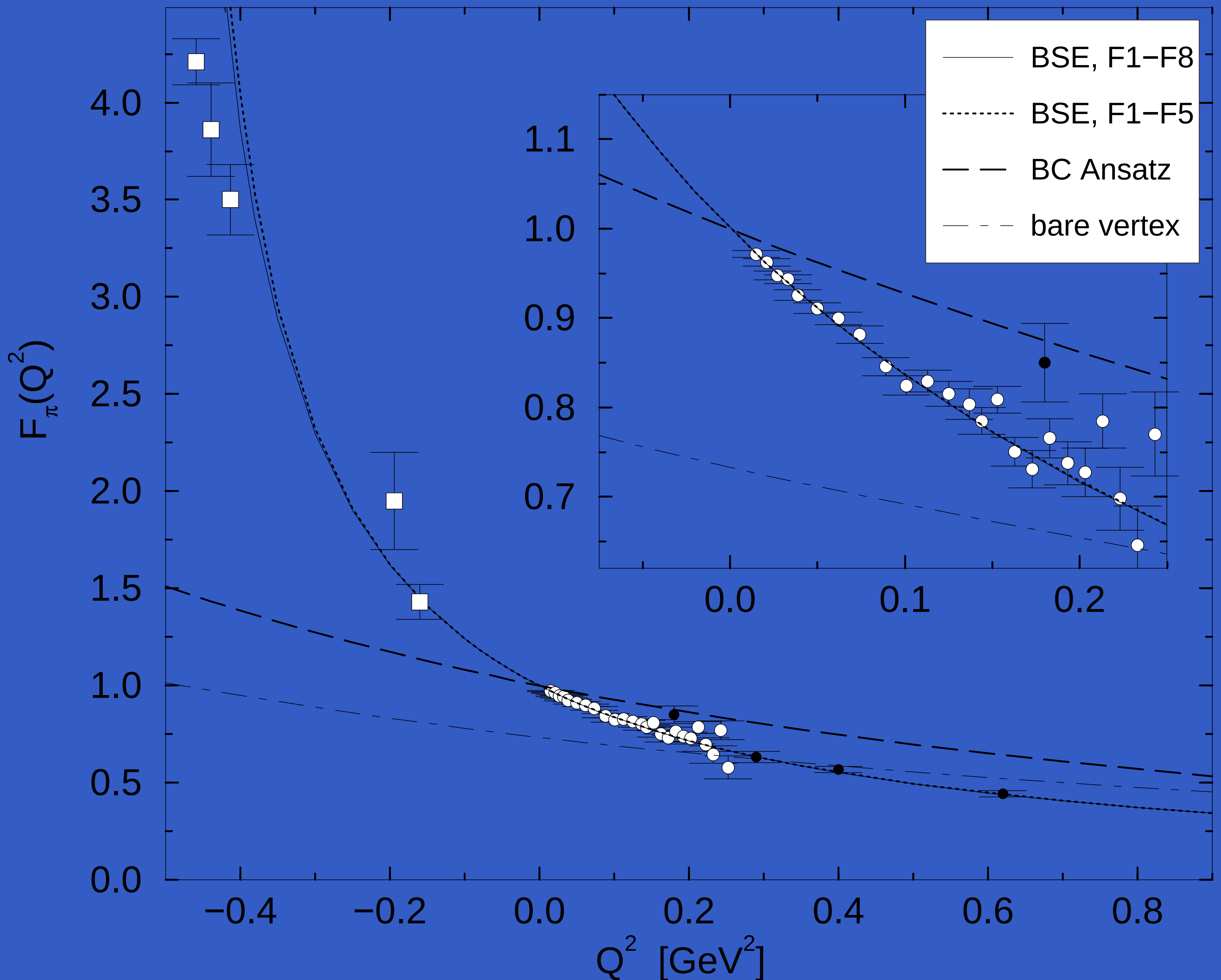


⋆All of these elements are known.⋆

Pion EM Form Factor

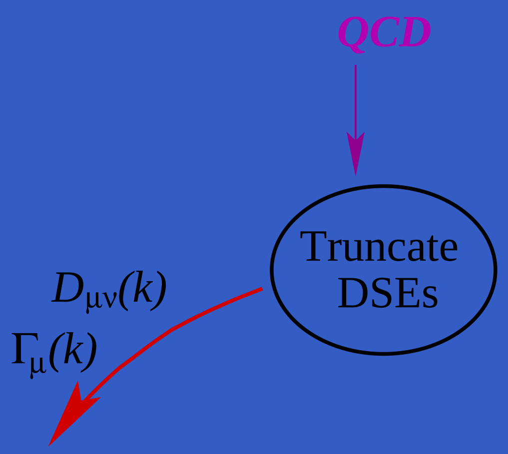
Maris, Tandy PRC61 045202 (2000)



Other Observables

- Strong, weak and EM decays of mesons.
  $\pi \to \gamma\gamma$, $\pi \to \mu e \nu$, $\rho \to \pi\pi$, etc.
- EM form factors of mesons and baryons.
  π, K, ρ, K^*, and the nucleon.
- Deep inelastic scattering from pions.
- Meson spectroscopy.
  masses and properties of the light mesons,
  heavy mesons and exotic mesons.
- Diffractive meson scattering πp, Kp, ρp, etc.

Dyson-Schwinger Philosophy 101



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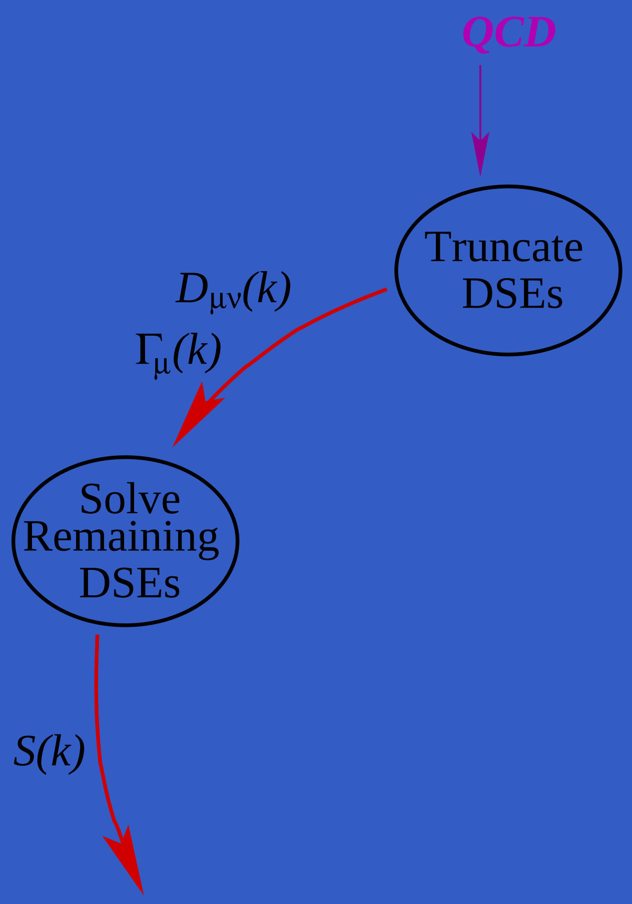

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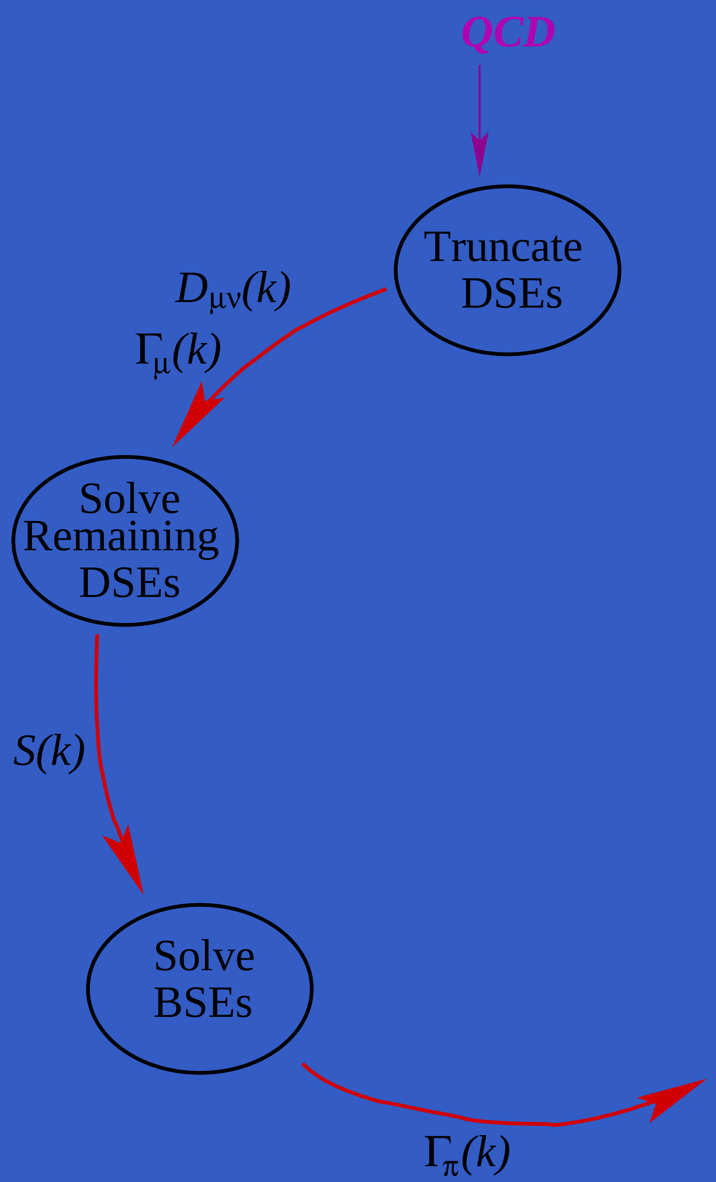

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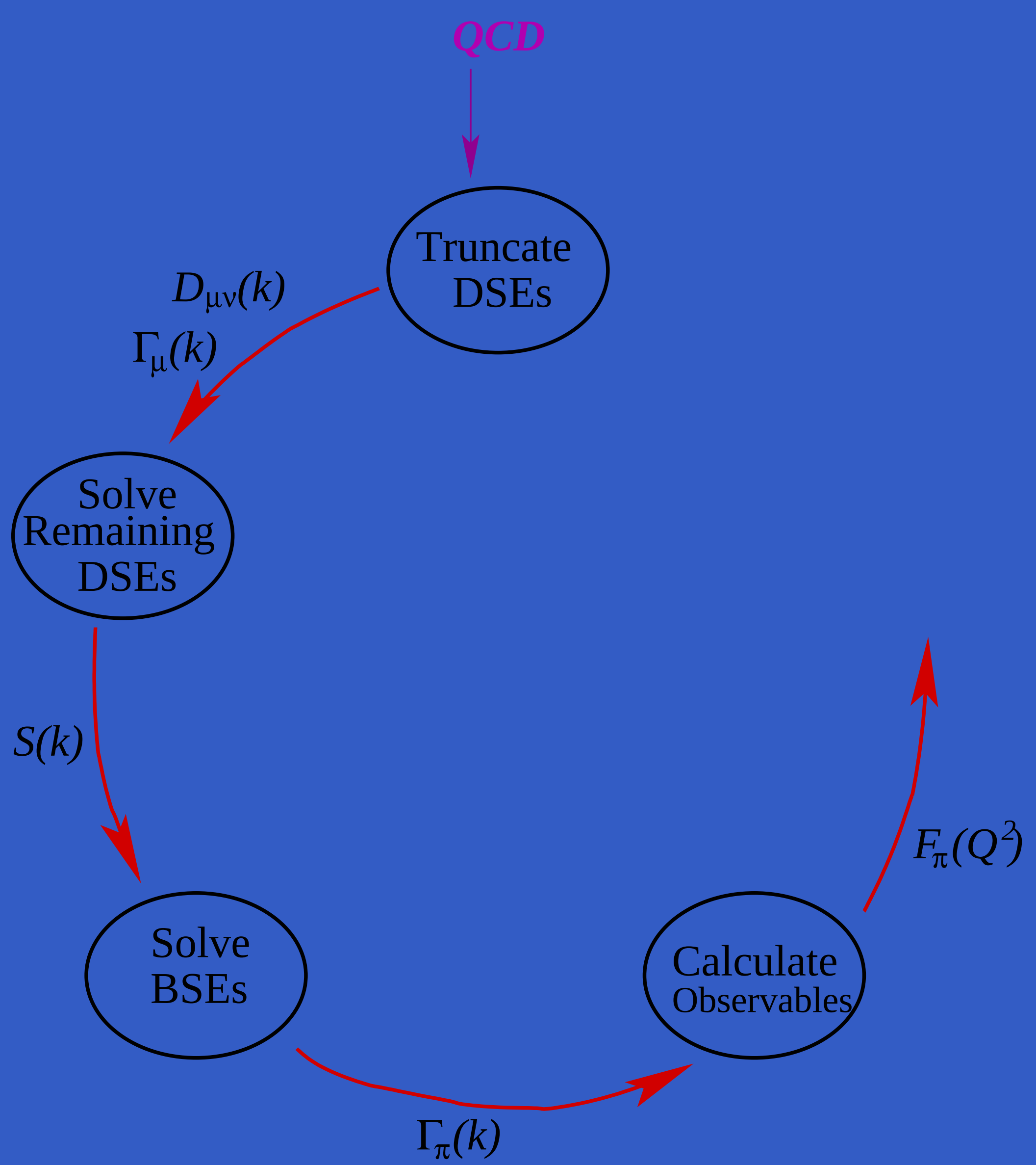

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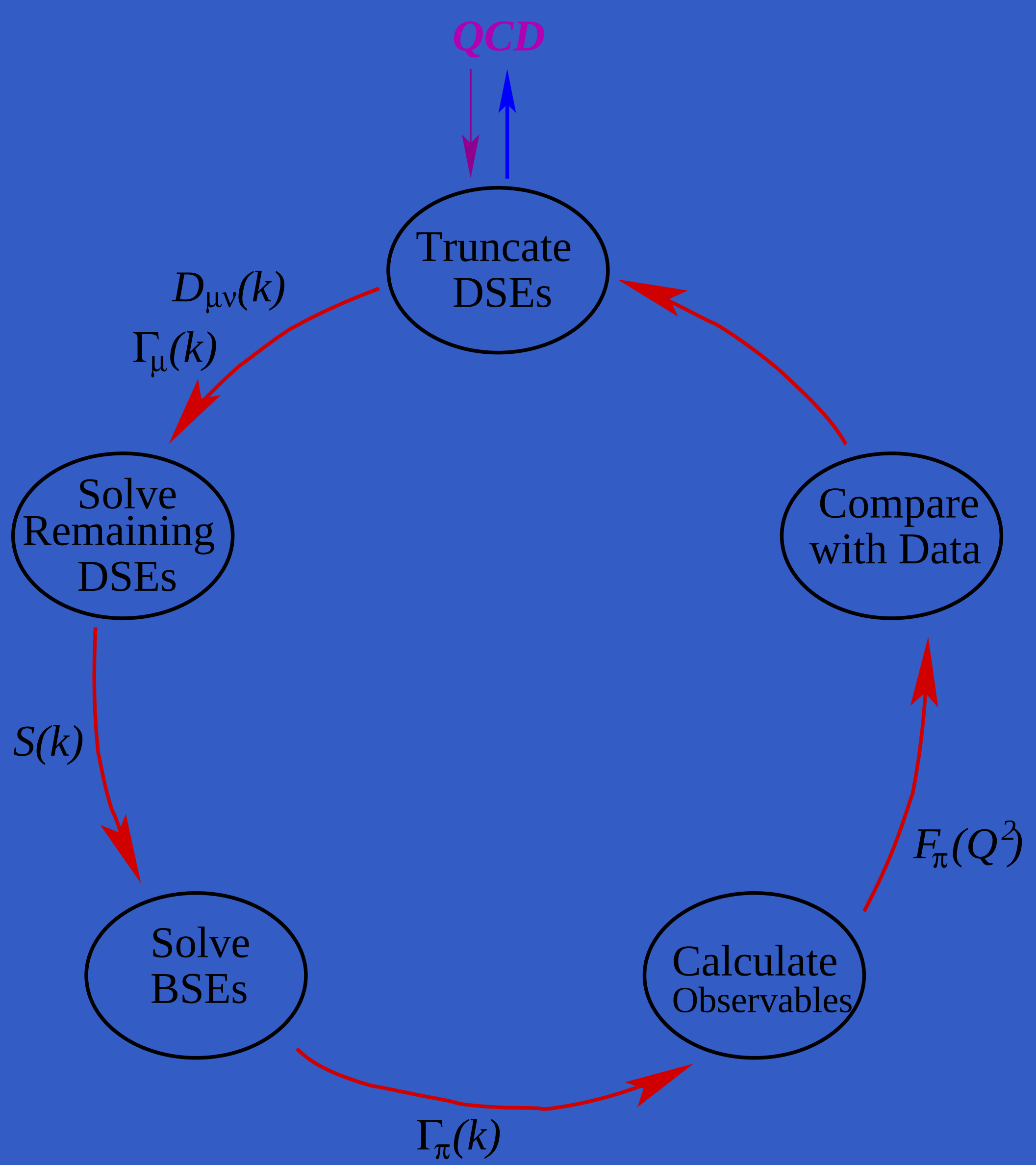

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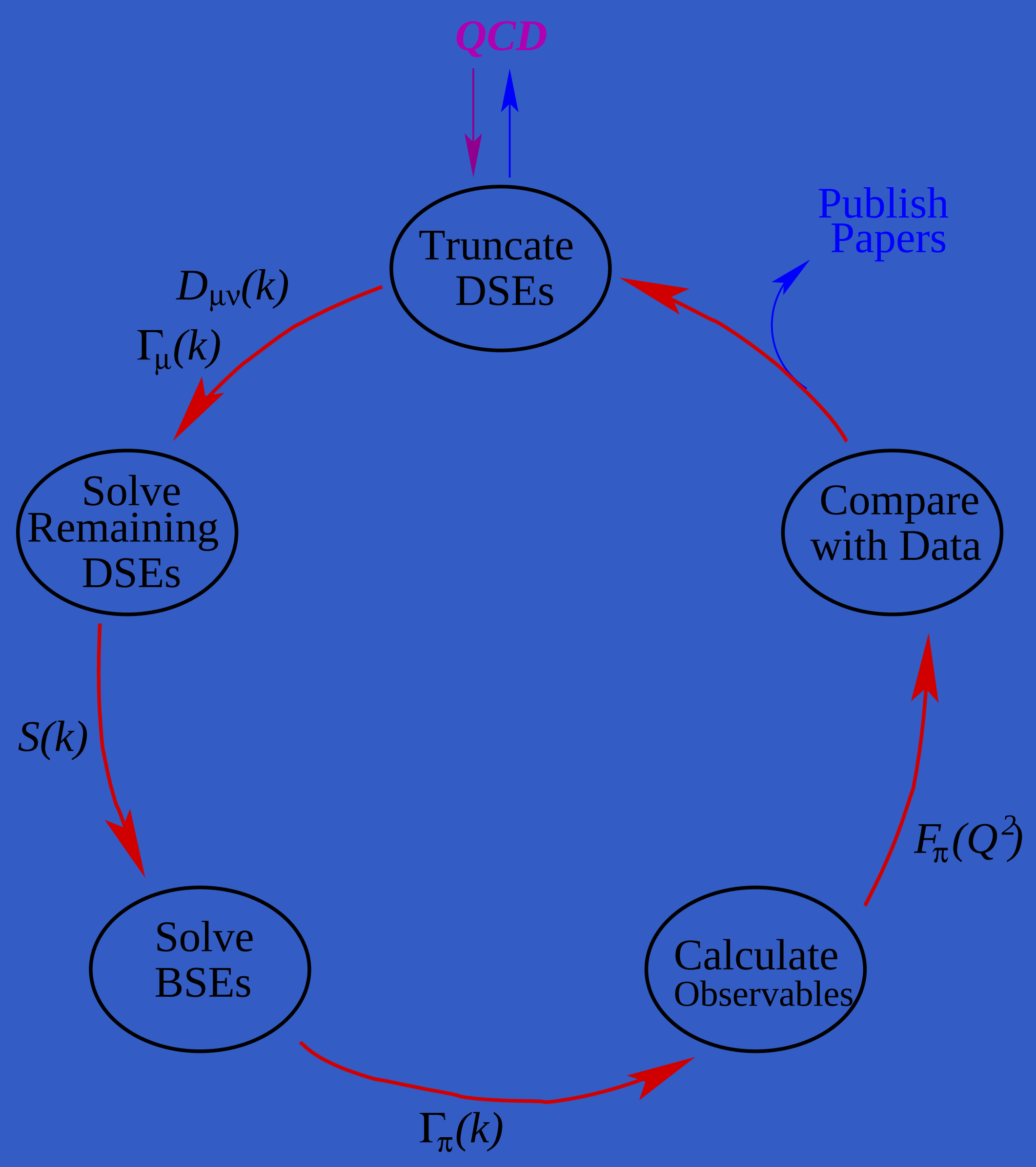

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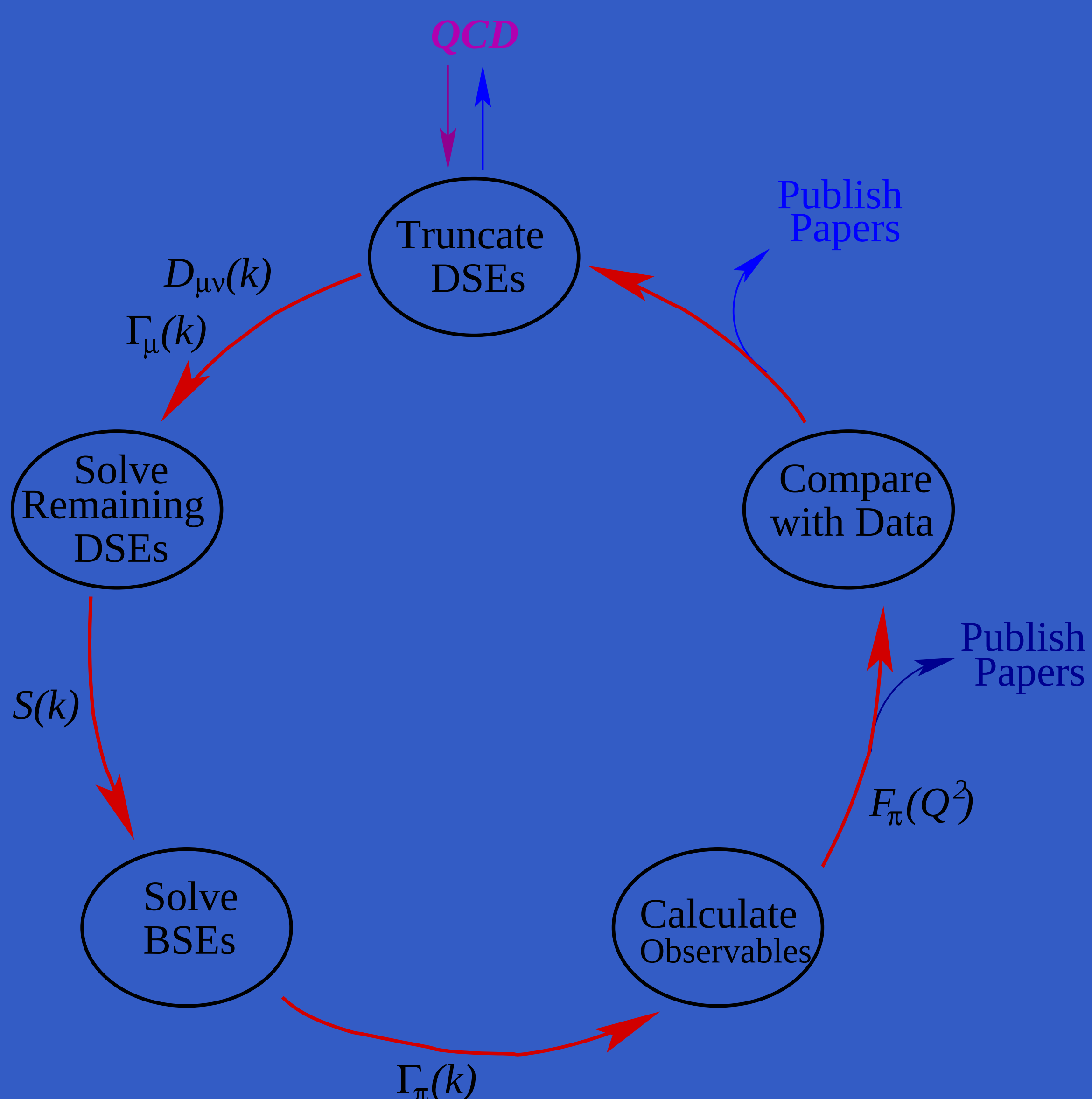

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   Can approach provide useful predictions?

Goldstone's Theorem

"If $m = 0$ and $M(p^2) \neq 0$ then $m_\pi = 0$."

Dynamical quark-mass generation is closely linked to small pion mass.

$\star$ Important aspect of nuclear/hadron physics $\star$

In the chiral limit ($m = 0$) the quark DSE is related to pseudo-scalar BSE by an fundamental identity of quantum field theory.

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The axial-vector Ward-Takahashi identity:

$$2m\Gamma_5(k;P) - iP_\mu\, \Gamma_{5\mu}(k;P) = S^{-1}(k_+)\gamma_5 + \gamma_5\, S^{-1}(k_-)$$

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$\star$Rainbow/Ladder truncation preserves Goldstone's theorem$\star$

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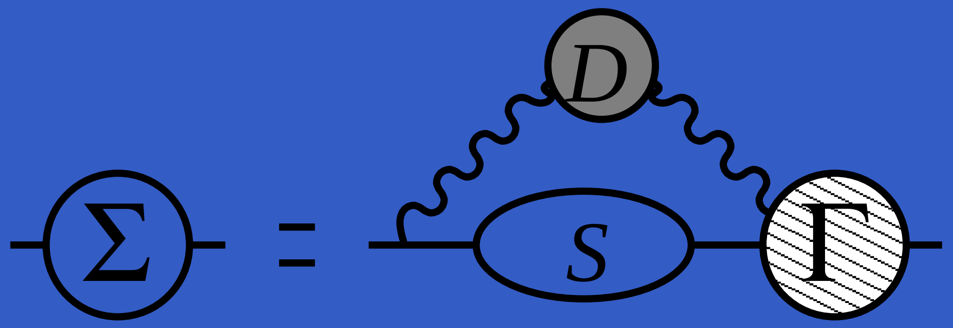

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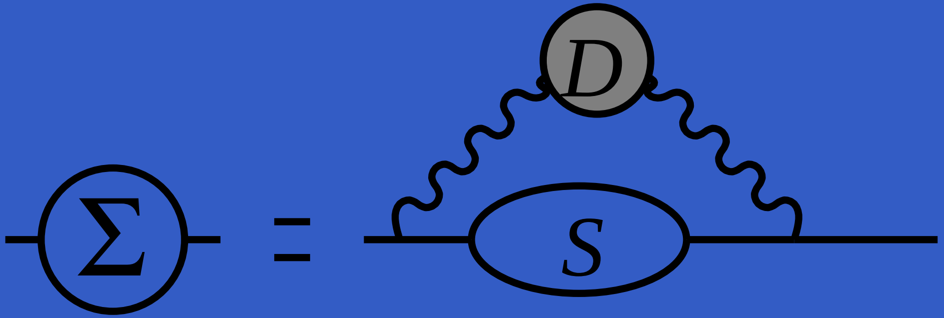
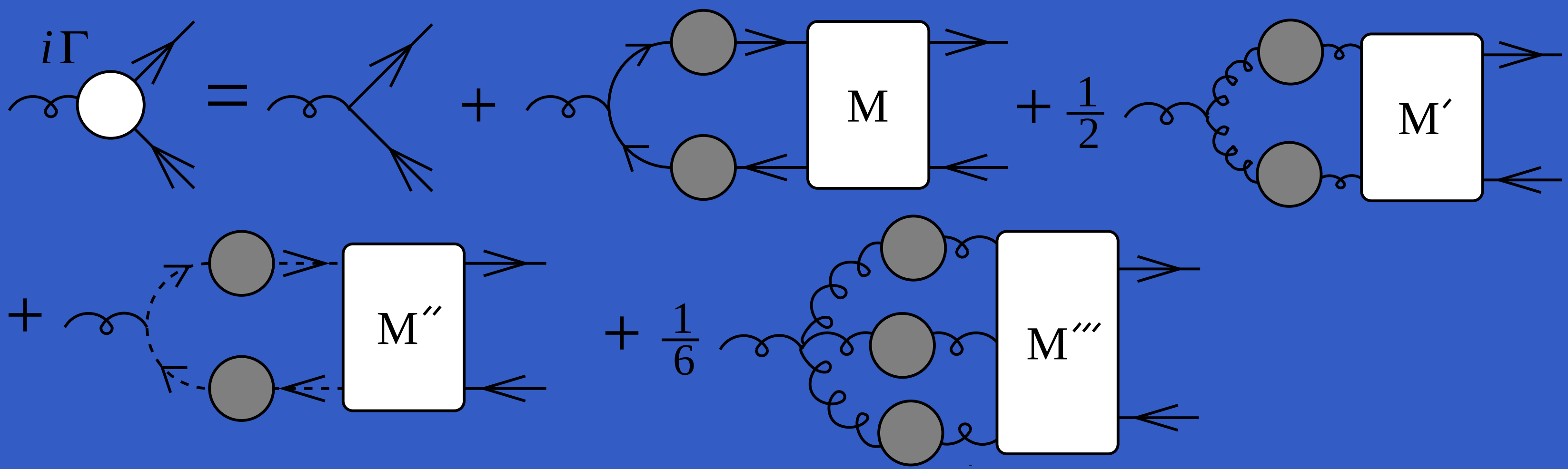


Corresponds to the ladder kernel in BSE.

A first step towards including diagrams beyond rainbow is to consider g^4 corrections to quark-gluon vertex:



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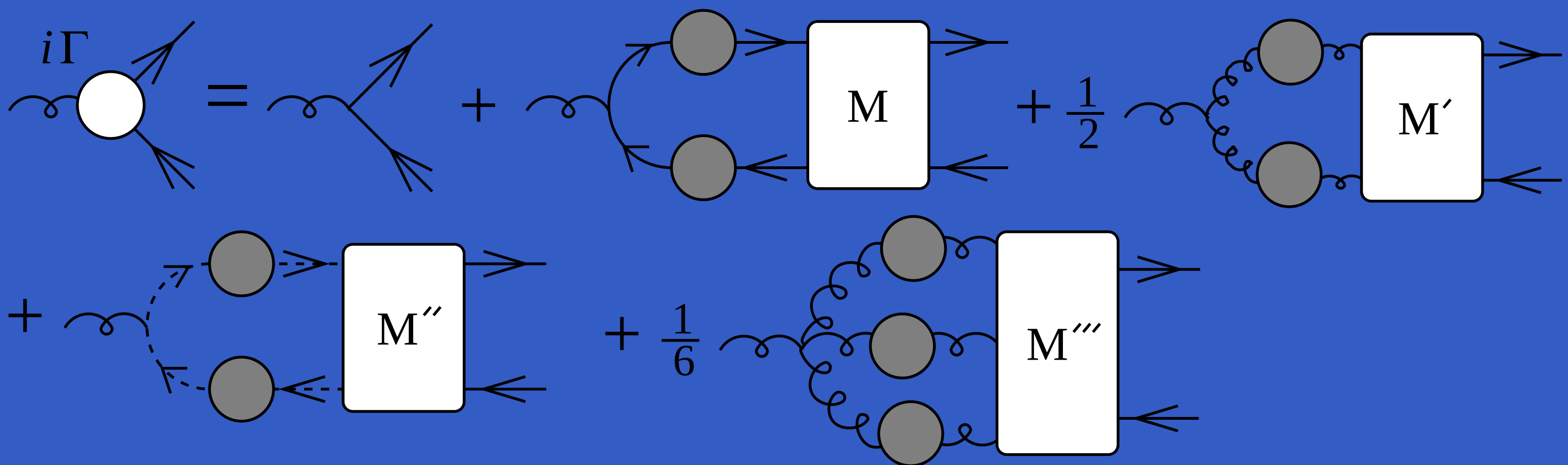
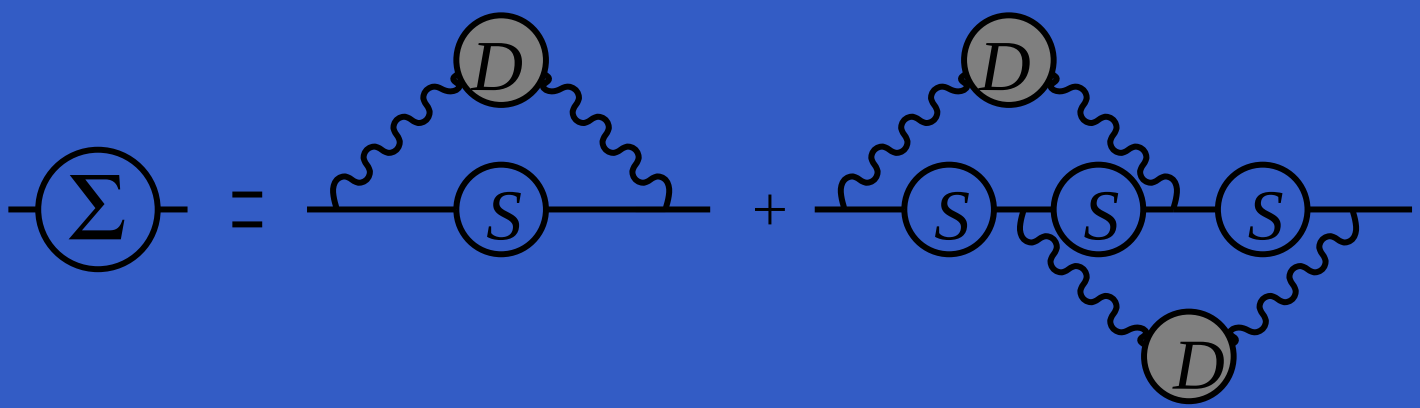


The DSE kernel is a bare vertex plus one-gluon dressing:



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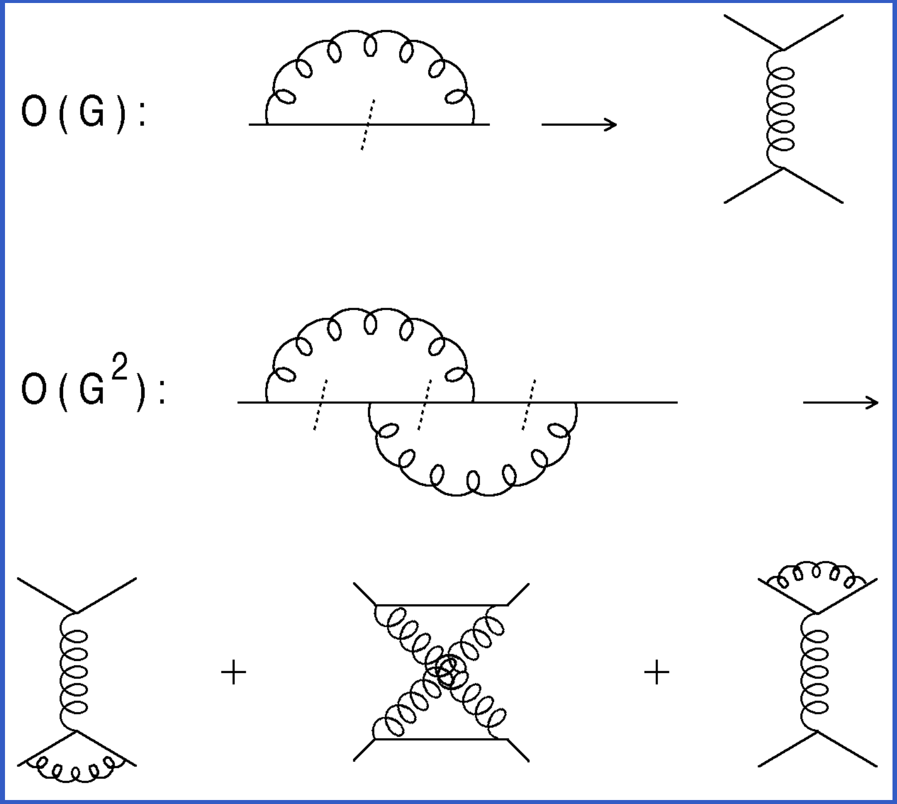

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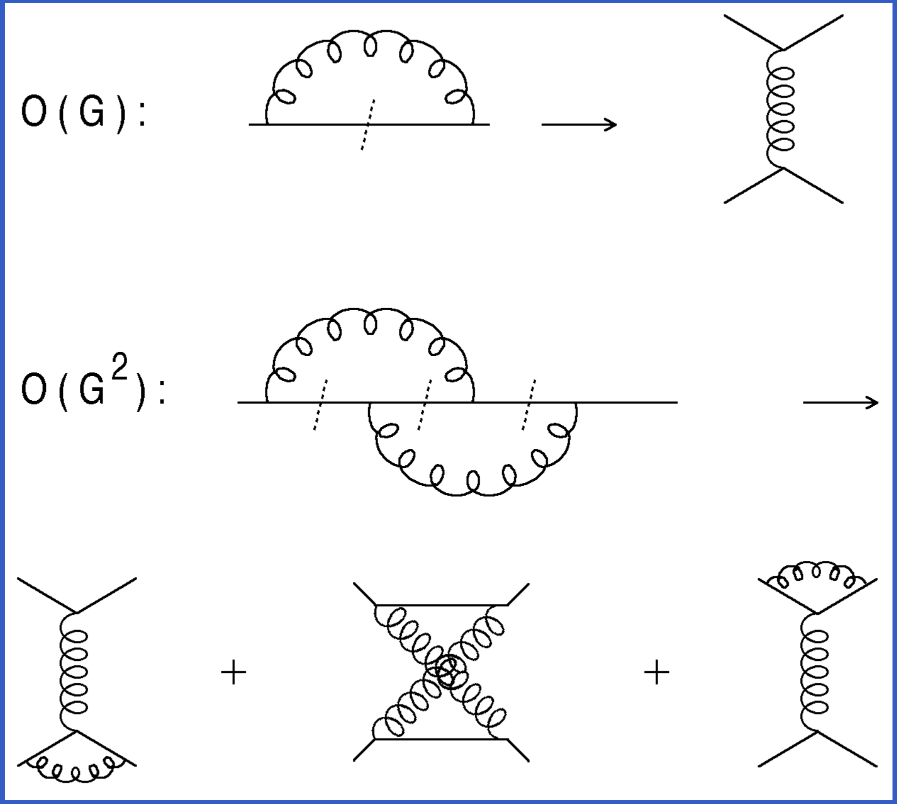

$\star$ Systematic method: BSE $K \Leftrightarrow$ DSE kernel. $\star$

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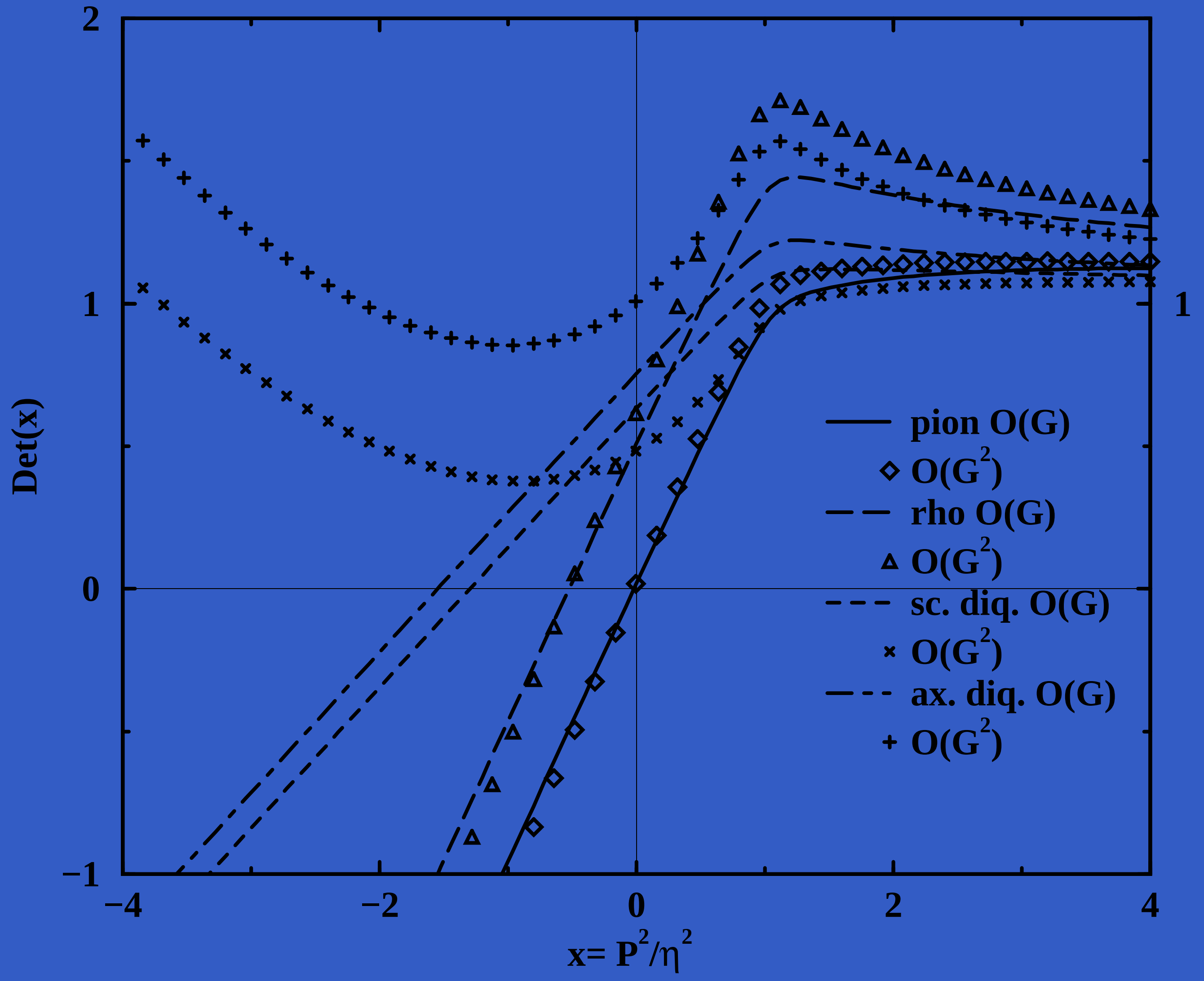
If eigenvalue $\lambda(P^2) = 1$, bound state has $M = \sqrt{|P^2|}$.

π, ρ and diquarks to $O(g^2)$ and $O(g^4)$

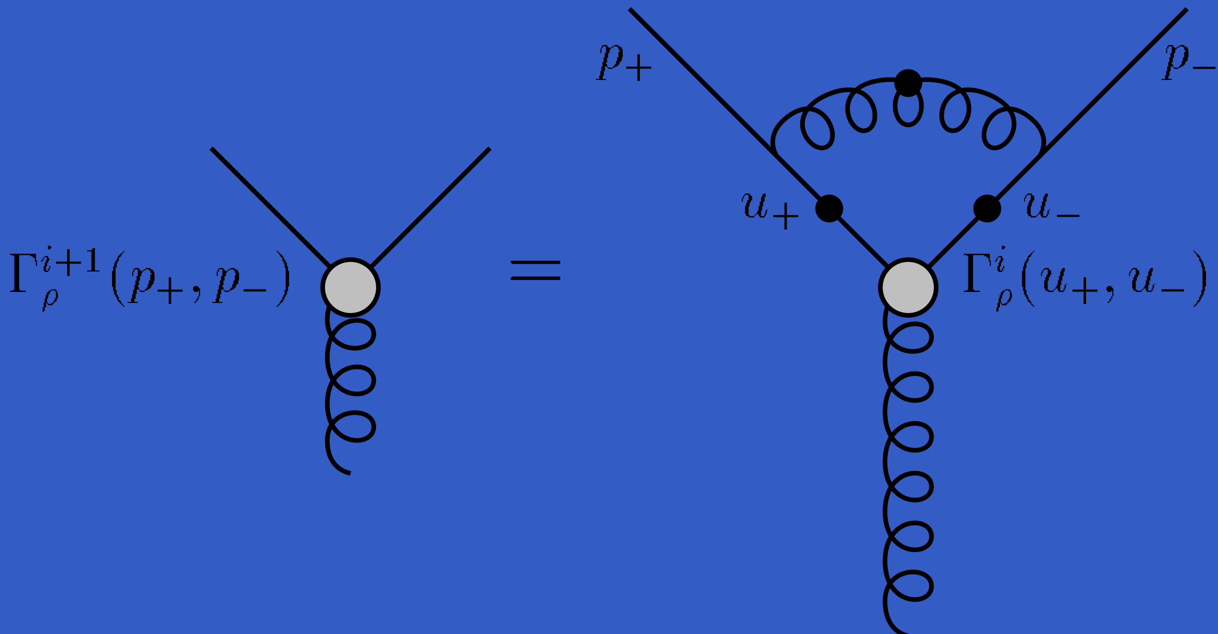
Bender, Roberts and Smekal, PLB 380 7 (1996)



Higher-Order Corrections in DSE

Recent work has extended this to *all* orders in g^{2n}:

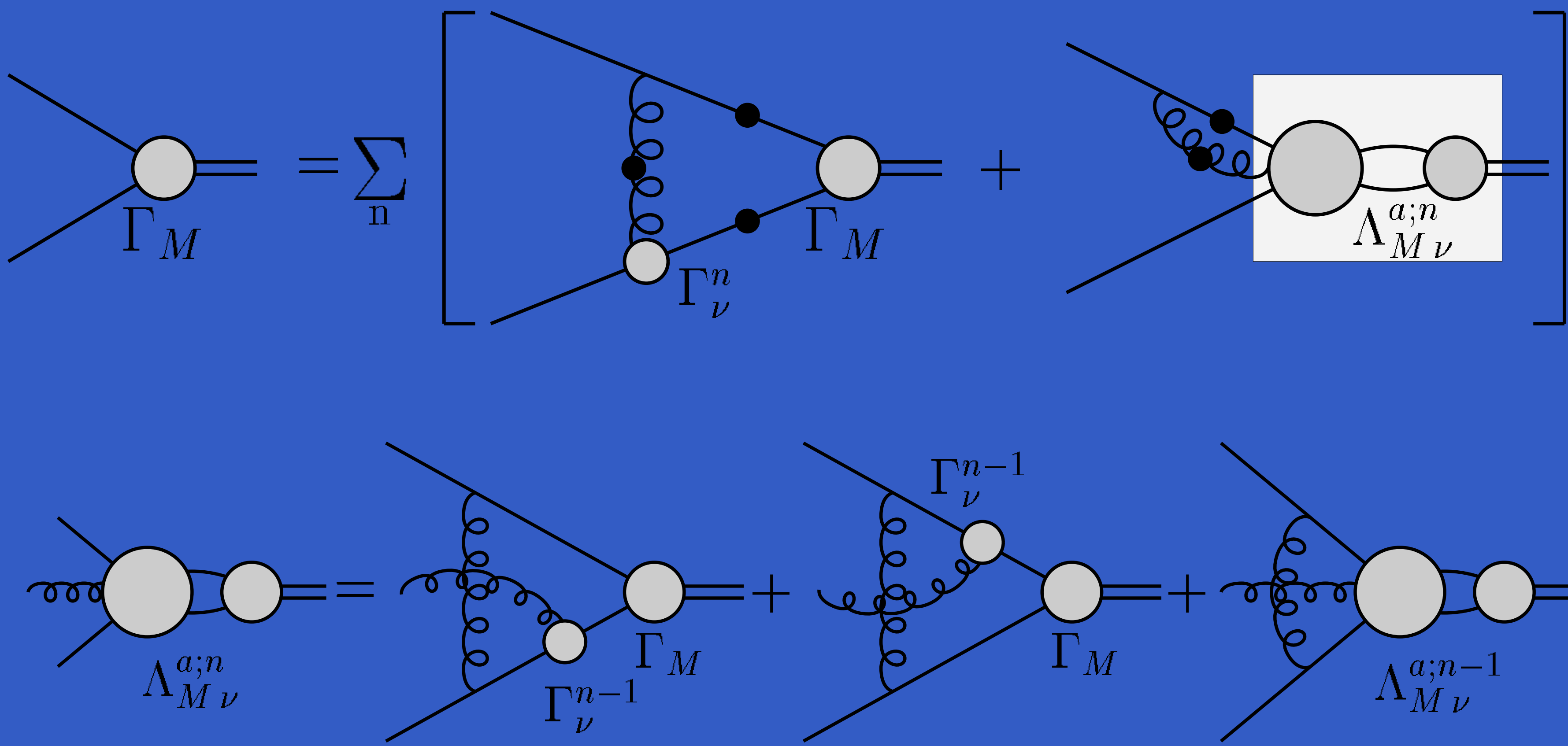
Bender, Detmold, Roberts and Thomas PRC65 065203 (2002)



This DSE gives ladder of gluon-exchange corrections.
Use “cutting procedure” to determine BS kernel K.

Higher-Order Corrections in BSE

The corresponding BSE kernel is very complicated!



π, ρ and diquarks to $O(q^2), \ldots, O(g^\infty)$

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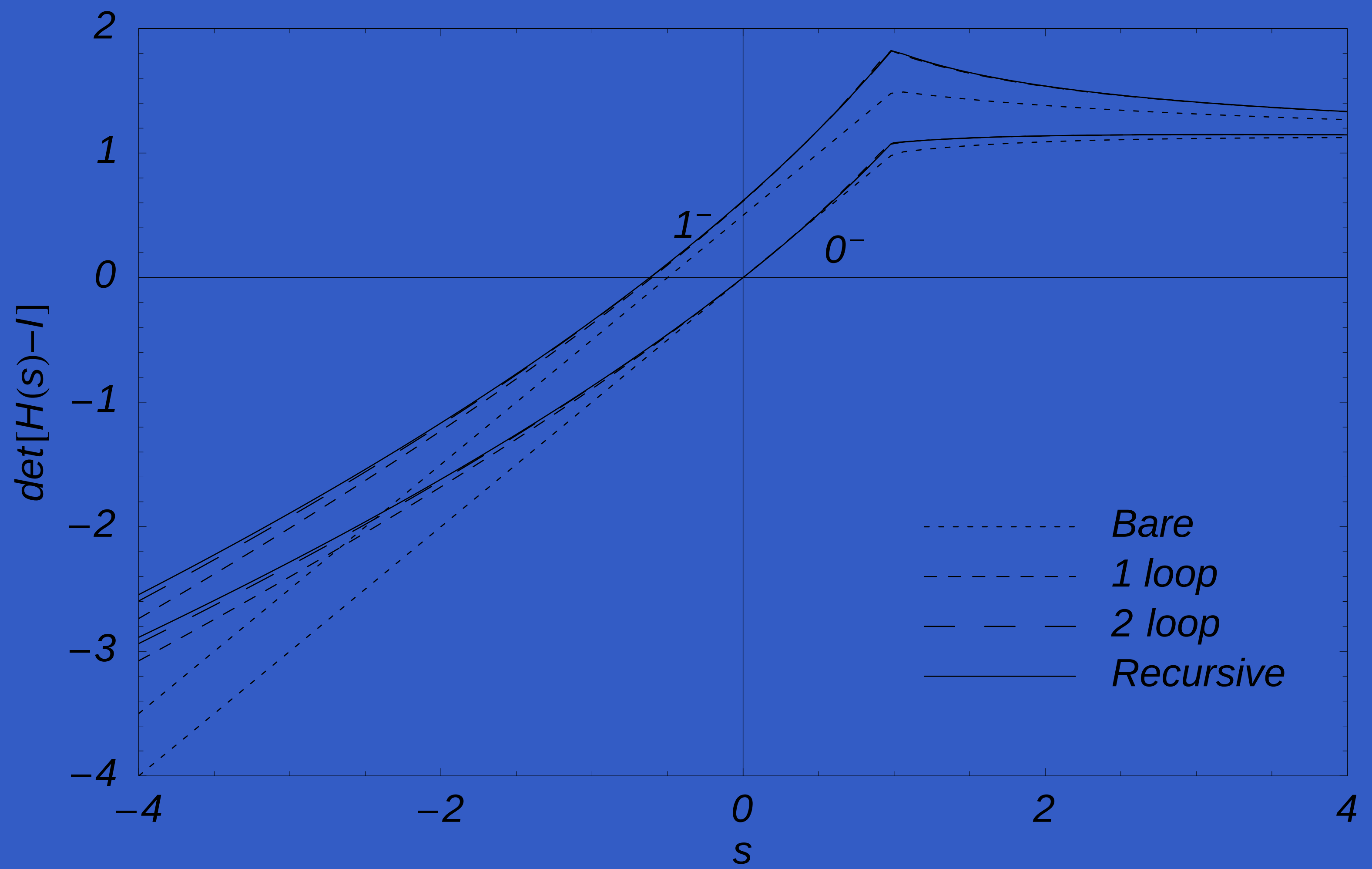

Mesons: $\pi = 0^-$ and $\rho = 1^-$.

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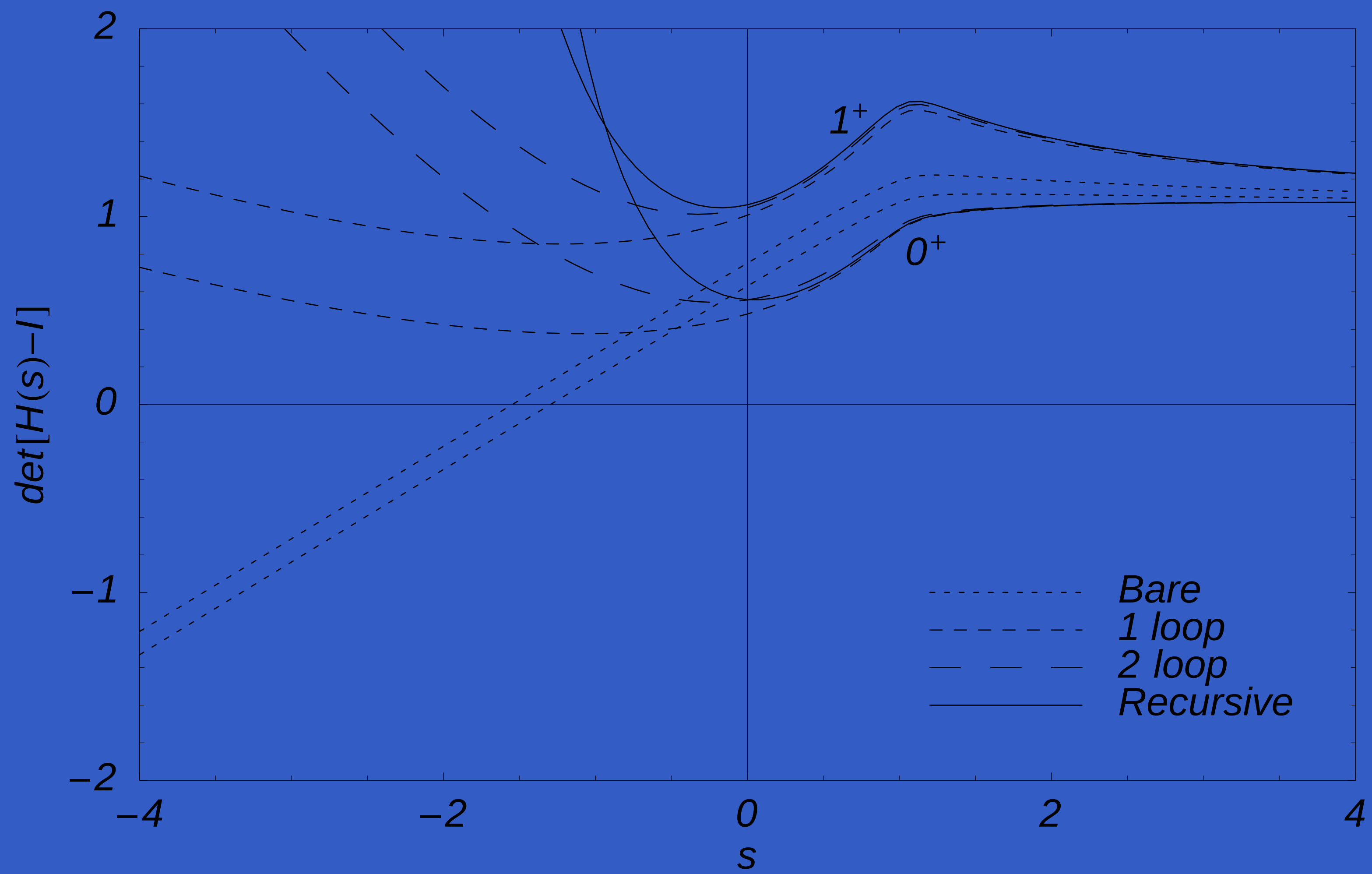

colored diquarks: sc$= 0^+$ and ax$= 1^+$.

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   Terms beyond rainbow/ladder $\rightarrow$ little impact.

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   Removes diquarks from hadron spectrum.

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Question of robustness of truncation is mostly resolved.

Issues for Dyson-Schwinger

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Formulation in Euclidean space

Consider a particle with 4-momentum,

$$\bar{p}_\mu = (\vec{p}, E) \qquad \bar{p}_\mu \bar{p}^\mu = \vec{p} \cdot \vec{p} - E^2 < 0 \quad \text{Minkowski}$$

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In Minkowski space, $\bar{p}_\mu \bar{p}^\mu = -m^2$,

$$E^2 = m^2 + \vec{p}\cdot\vec{p}$$

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This is achieved in Euclidean space ($p_\mu p_\mu > 0$) at the expense of reality of p_μ. Let $p_4 \to iE$, then

$$\vec{p}\cdot\vec{p} - E^2 = -m^2 < 0.$$

Dyson-Schwinger framework is based on Euclidean QFT.

“Analytic continuation” needed to make contact with reality.

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- Schwinger functions $S(k)$, $D_{\mu\nu}(k)$, $\cdots$ with $k^2 > 0$.
- Certain Schwinger functions may be analytically continued to Minkowski,

$$\lim_{z_i \to -x_i} W(z_1, z_2, z_3, z_4) = \mathrm{Observables}$$

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What type of behavior do we expect for $S(z)$?

Studies of QCD on a lattice find that quark propagator is easily parametrized as N pairs of complex-conjugate poles, $(z_n, m_n) \in \mathcal{C}$.

Bowman, Heller, Leinweber, Williams 2002

In general $m_n = m_{nR} + i m_{nI}$,

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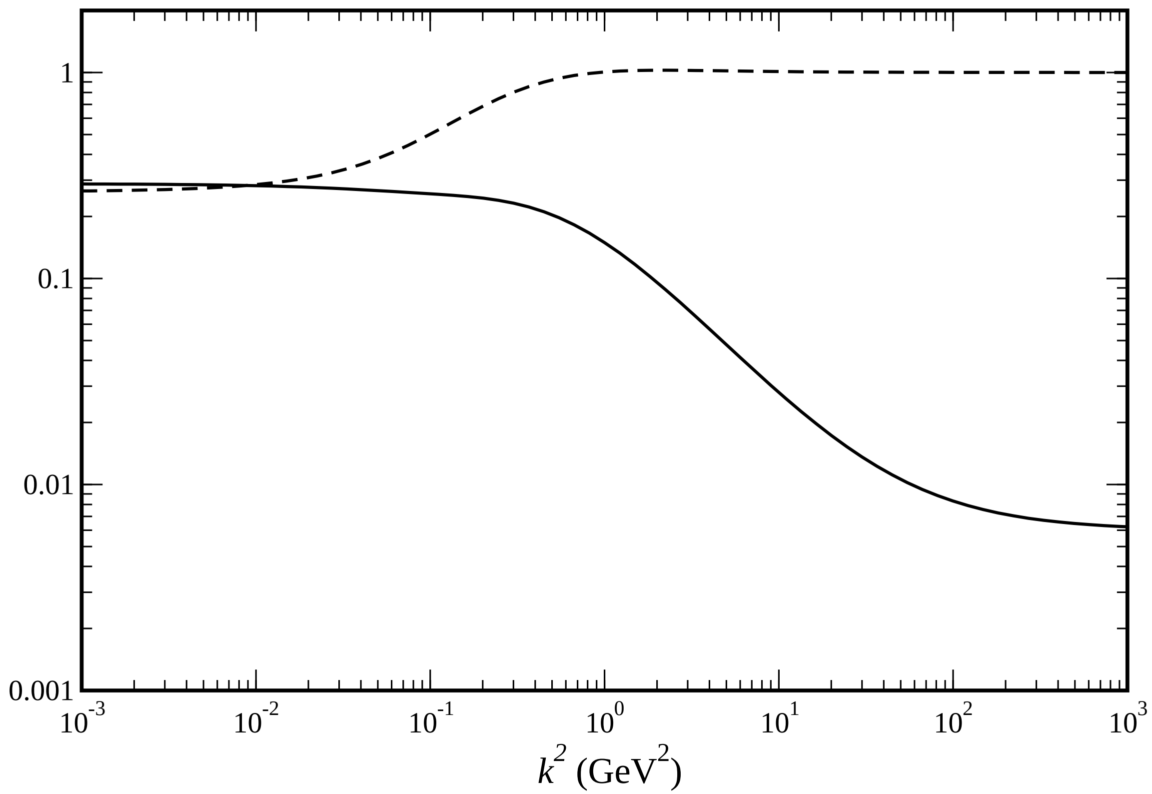
$\star$ No observable violation of unitarity $\star$

Bhagwat, Pichowsky & Tandy (2002)

Our quark propagator parametrization:



$m_1 = -0.40 + i0.20$, $z_1 = 0.15$, $m_2 = 0.550 + i0.350$, $z_2 = 0.35 + i0.37$ (GeV)

Poles in Bethe-Salpeter Equation

Consider impact of complex-conj quark poles in BSE

$$\lambda(P^2)\Gamma(p;P) = \int \frac{\mathrm{d}^4k}{(2\pi)^4} K(p,k;P) S(k_+) S(k_-) \Gamma(k;P),$$

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One cannot avoid singularities!

"*One of One*" says:

Singularities are futile.
You will be analytically continued.

Analytic Continuation Procedure

1. Replace integration over real k^2 with contour C,

$$\int_0^\infty \mathrm{d}k^2 \longrightarrow \int_{\mathcal{C}} \mathrm{d}k^2$$

This amounts to treating k^2 as a complex variable.

We define coutour $C = (0, \infty)$, along real-k^2 axis.

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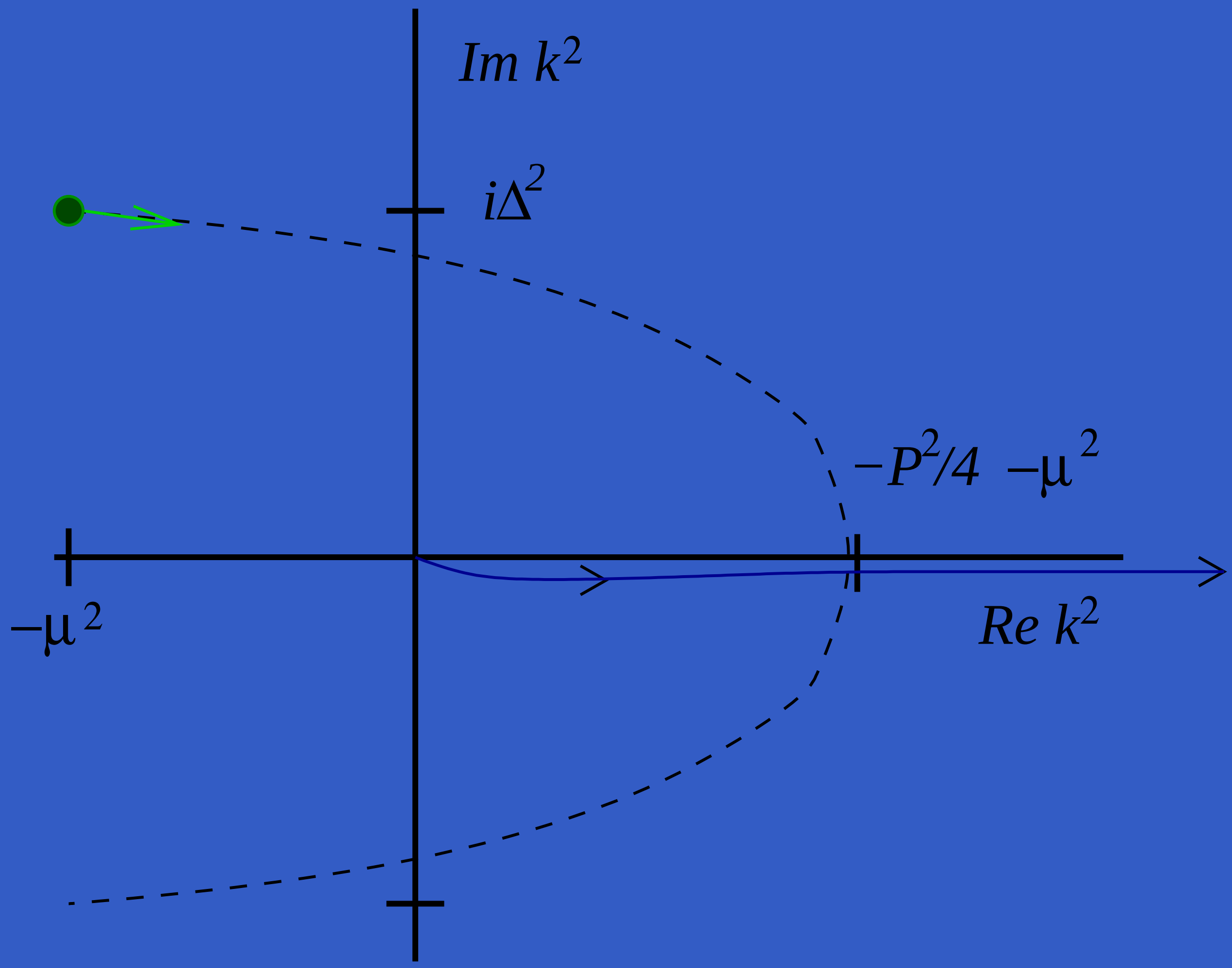

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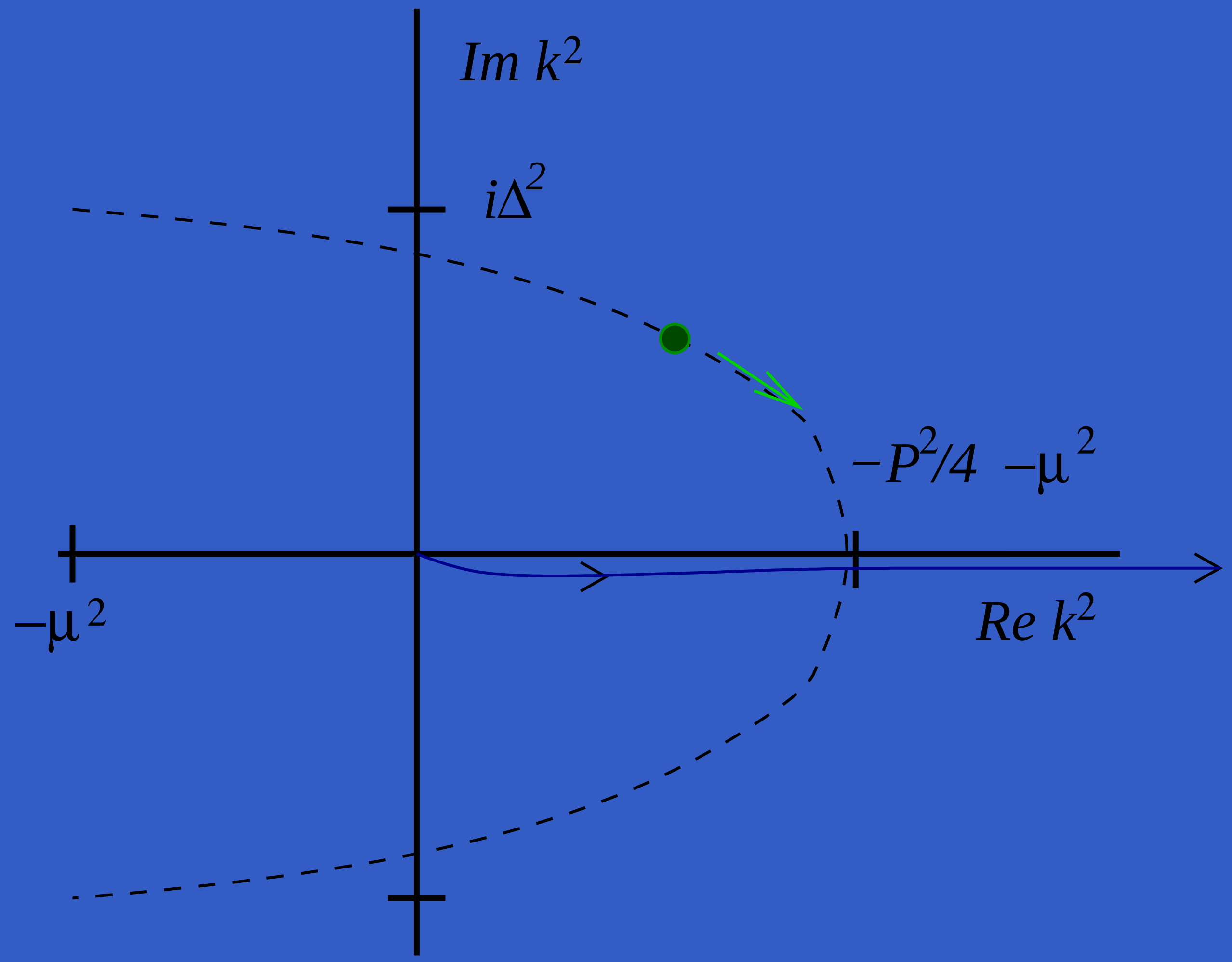

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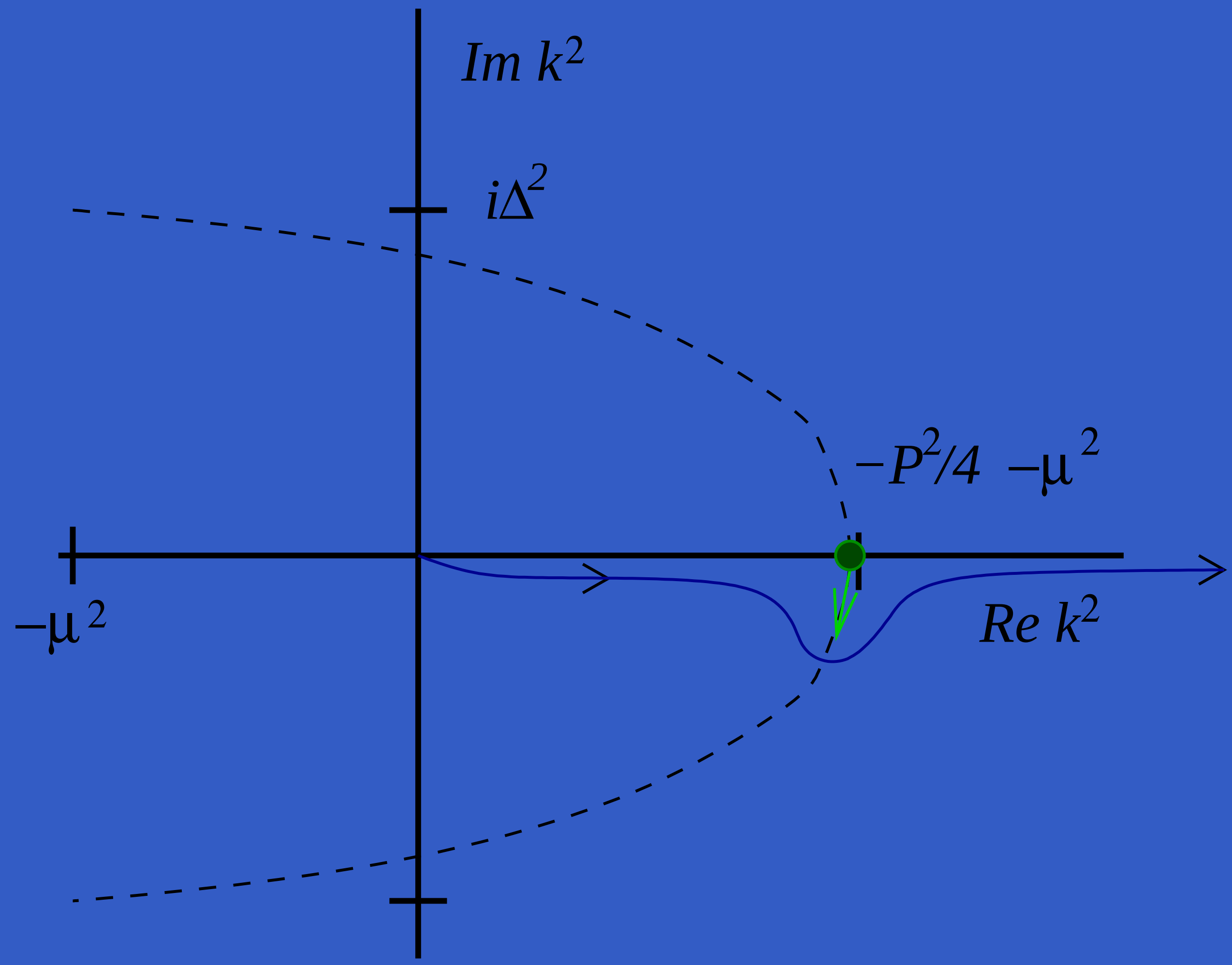

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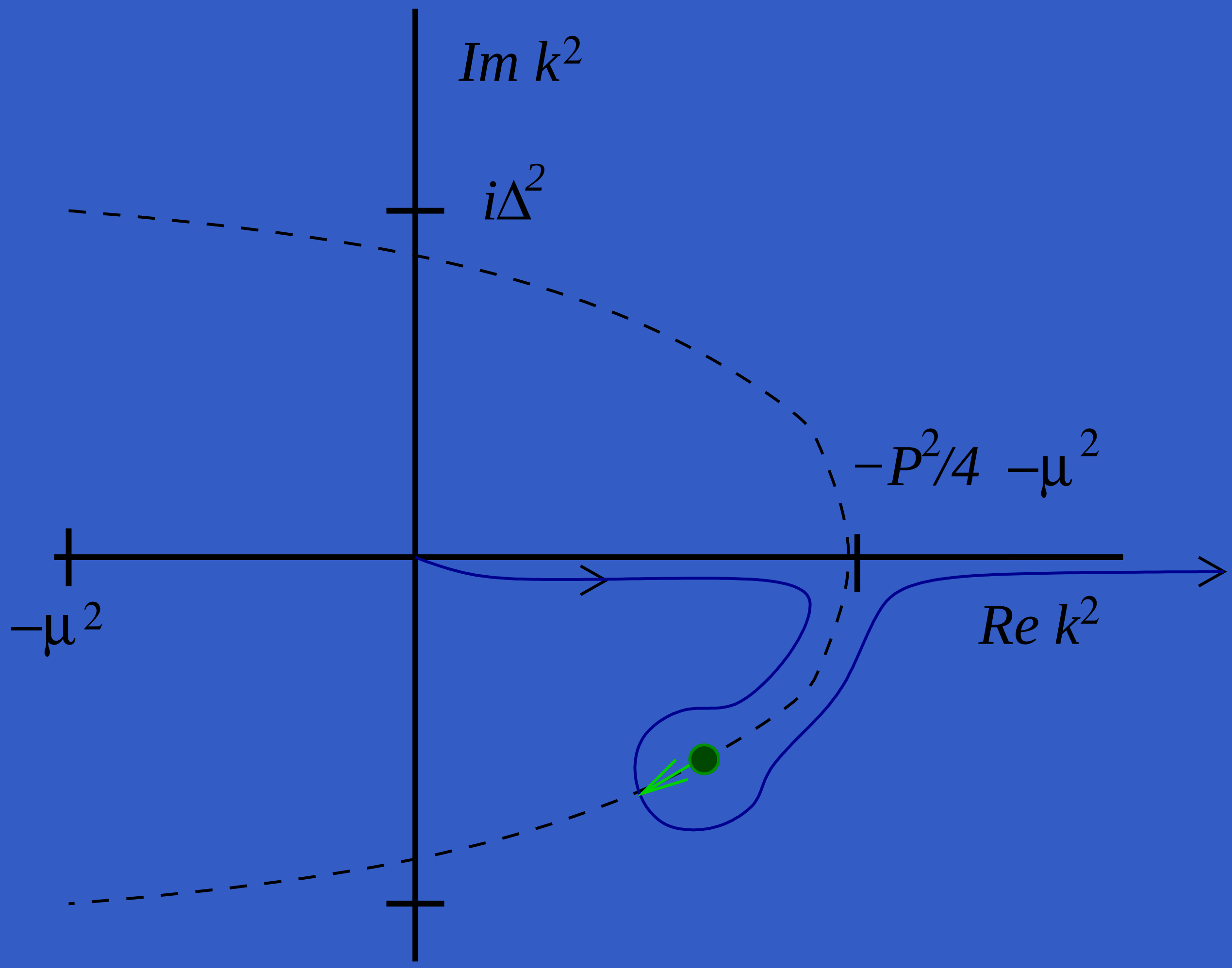

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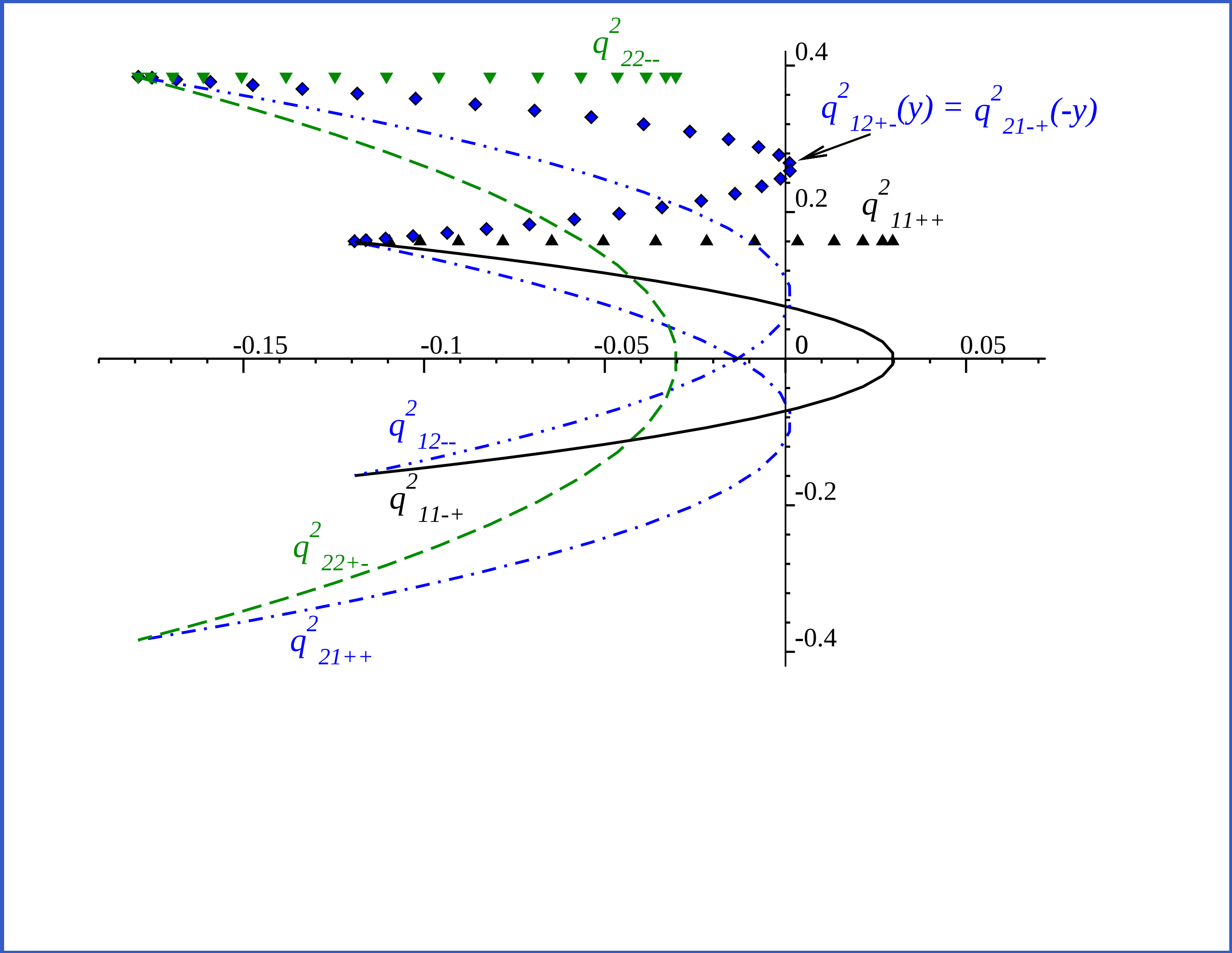

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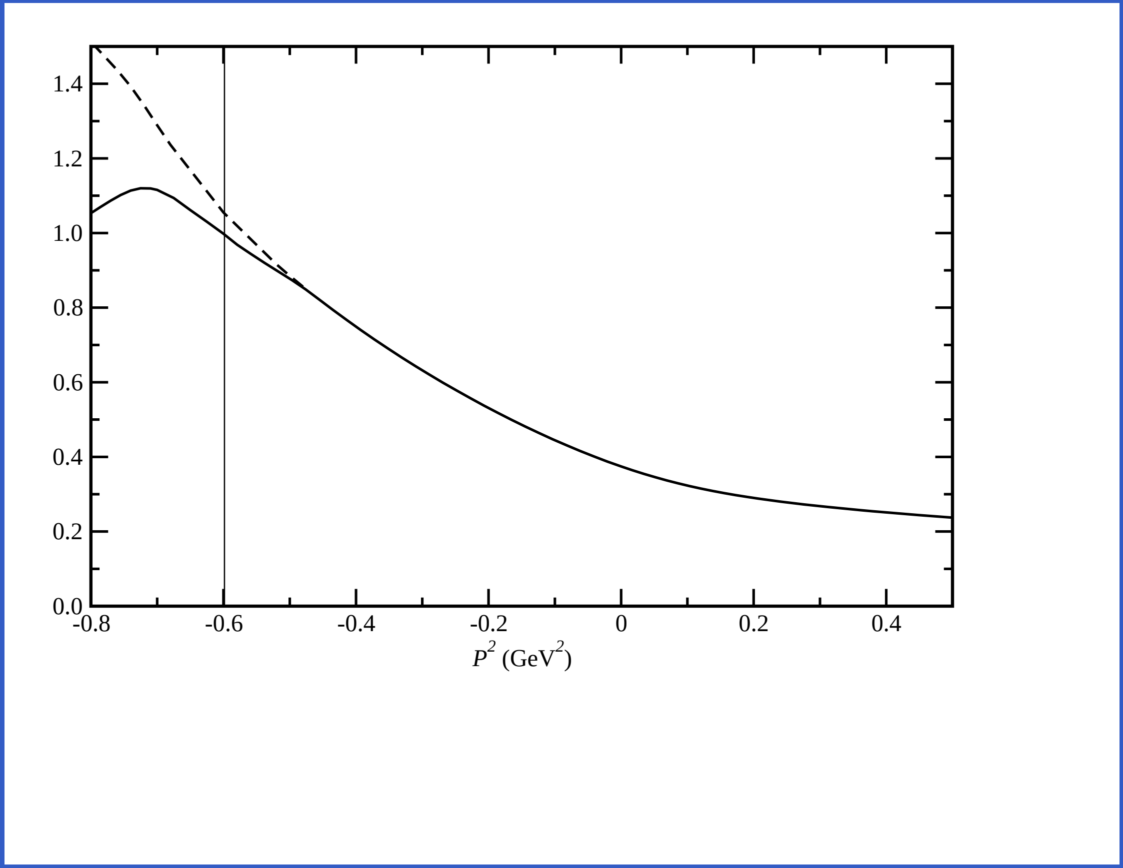

Resolution: Analytic Continuation

Studies of DSEs and Lattice-QCD find quark propagators may be sums of complex-conjugate poles *or cuts*.

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2. Is practical for models, can be computed!
3. Enables careful study of connection between Euclidean and Minkowski spaces.
4. Simple explanation of quark confinement.
   Quark loops are real, imaginary parts cancel term by term $\rightarrow$ no quark production thresholds!

Baryons in Dyson-Schwinger Framework

- qqq bound state $\Rightarrow$ covariant Faddeev Eq
  Very complicated.

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Maris, FBS 32, 41 (2002)

J^P	m_{ud}	m_{us}	m_{ss}
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$\star$ 0^+ and 1^+ diquarks are dominant $\star$

Faddeev Equation for Nucleon

$$\begin{bmatrix} \mathcal{S}(q;P)u(P) \\ \mathcal{A}^i_\mu(q;P)u(P) \end{bmatrix} = -4\int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathcal{K}(q,k;P)\begin{bmatrix} \mathcal{S}(k;P)u(P) \\ \mathcal{A}^j_\nu(k;P)u(P) \end{bmatrix}$$

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nucleon wave functions:

$$\begin{aligned}\mathcal{S}(k;P) &= s_1(k;P) + (i\gamma\cdot k - k\cdot P)\, s_2(k;P),\\ \vec{\mathcal{A}}_\mu(k;P) &= \vec{a}_1(k;P)\gamma_5\gamma_\mu + \vec{a}_2(k;P)\gamma_5\gamma\cdot k\; k_\mu.\end{aligned}$$

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and diquark correlations

$$\Delta^{0^+}(Q) = \frac{1}{m^2_{0+}}\,\mathcal{F}(Q^2/m^2_{0+}),$$

$$\Delta^{1^+}_{\mu\nu}(Q) = \left(\delta_{\mu\nu}+\frac{Q_\mu Q_\nu}{m^2_{1+}}\right)\frac{1}{m^2_{1+}}\mathcal{F}(Q^2/m^2_{1+}).$$

q and qq correlation lengths determine nucleon scales.

Results from Faddeev Equation

	M_N	M_Δ	R
0^+	1.590	–	1.28
0^+ & 1^+	0.940	1.230	0.25

Roberts, Hecht, et al.

- Including *only* scalar diquark:

  There is no Δ bound state.

$$R = \frac{s_2}{s_1} = 1.28$$

Nucleon has large spinor "lower component"

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Self-consistent, *natural* description of N, Δ

Nucleon EM Form Factors

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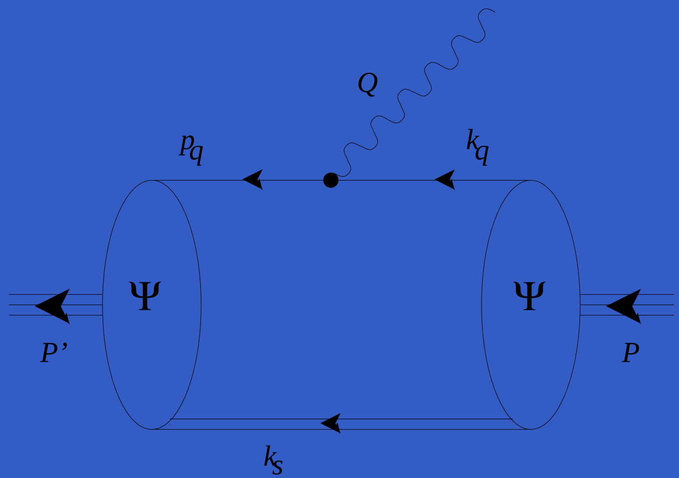
Be careful to maintain current conservation!

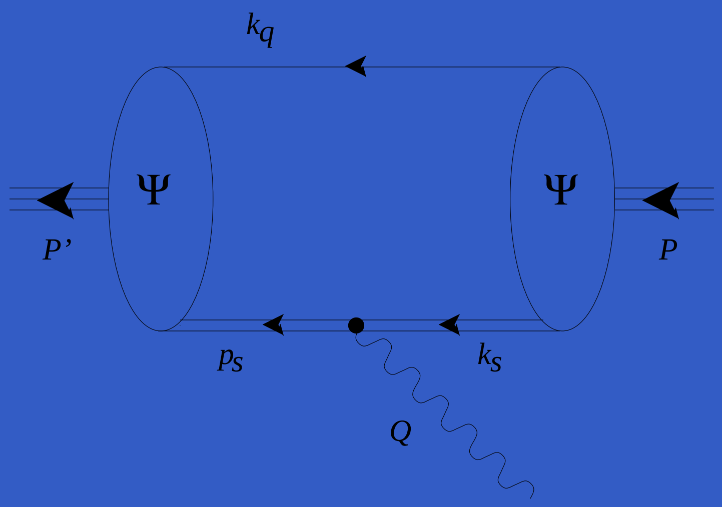
Current-Conservation in Nucleon

EM current conservation of nucleon is maintained when photon couples to *all* objects with a sub-structure.

Current-Conservation in Nucleon

impulse diagrams





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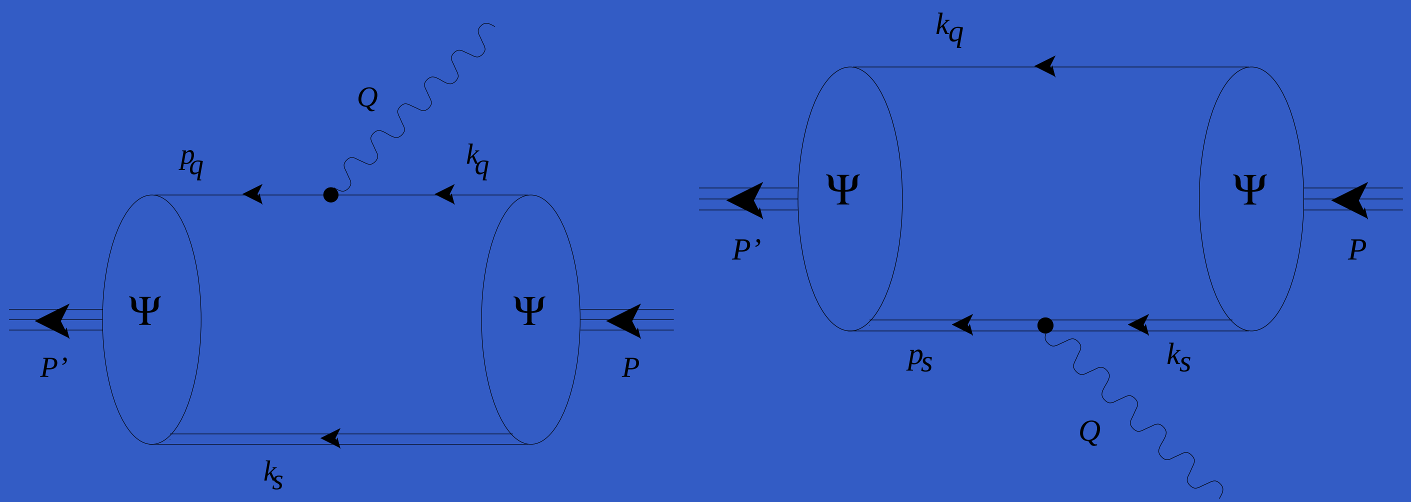

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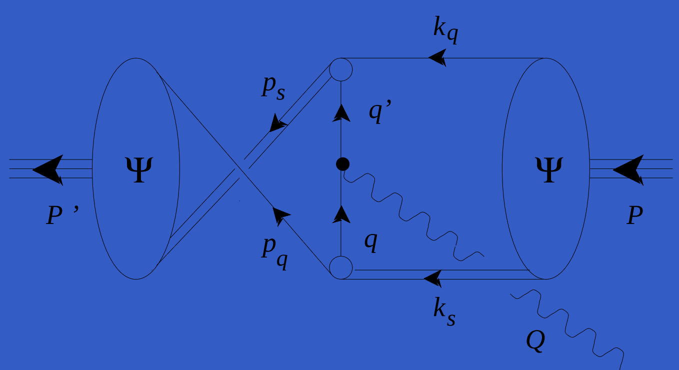

Oettel, Pichowsky & Smekal, Eur.Phys.J. A8 251 (2000)

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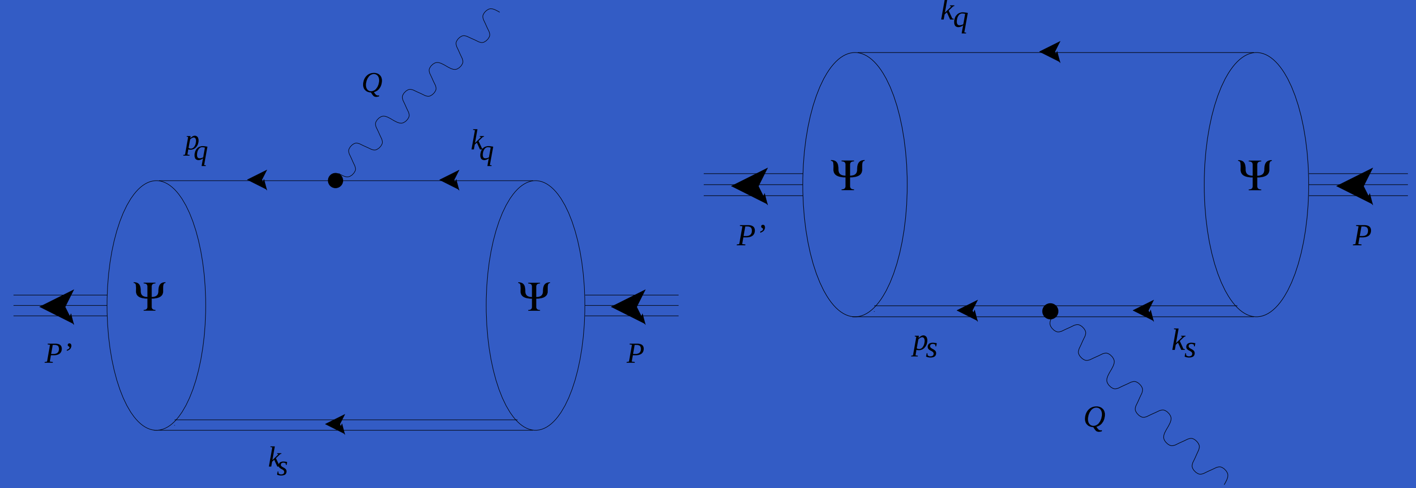

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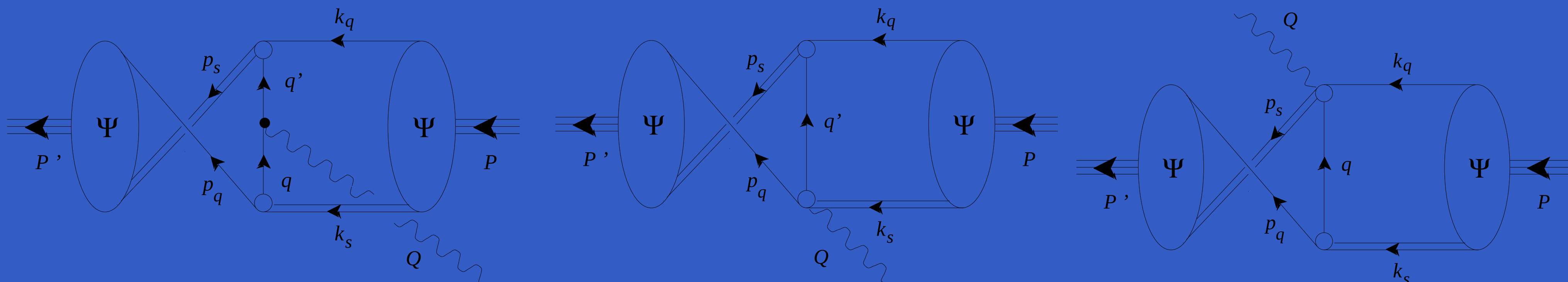

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Resulting Nucleon Form Factors $G_E(Q^2)$

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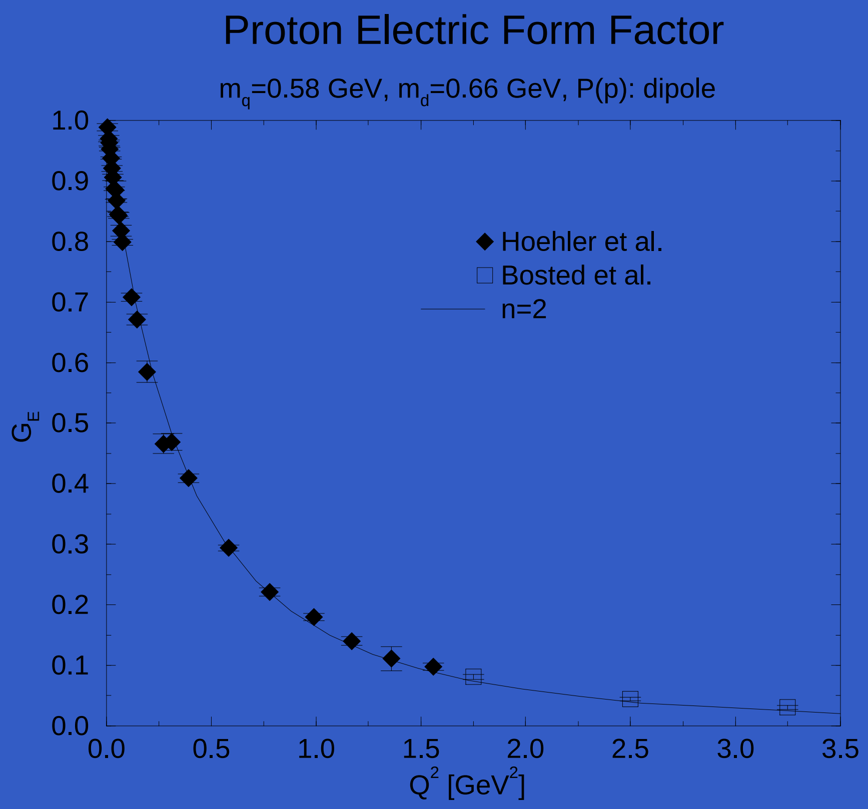

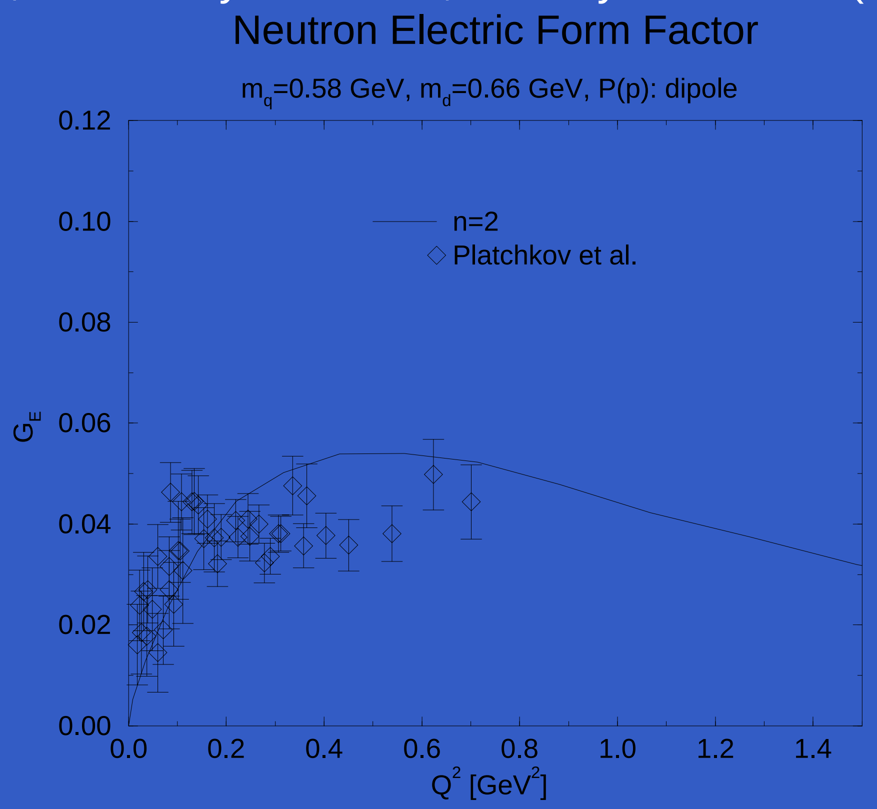

Summary of Baryon Studies within Dyson-Schwinger

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- Dyson-Schwinger: baryons $\Leftrightarrow$ mesons.
  necessary to provide real constraints on models
  approach can provide real predictive power

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  next-order corrections have little impact π, ρ
  next-order corrections make diquarks unbound.

  higher-order corrections... same conclusions.

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  2) How does *analytic continuation* really work?

  We have carried out continuation explicitly.
  Can carefully study Euclidean $\Leftrightarrow$ Minkowski.
  Complex-conj quark poles lead to confinement.

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  Baryon scales arise naturally from quark propagator and gluon interaction, like mesons.

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