Survey of Lattice Results for Baryon States.

David Richards (LHPC and Jefferson Laboratory)

- Lattice QCD spectrum recipe
- Quenched approximation
- The quenched baryon spectrum - so far
- Full QCD, light quarks. . .
- Decays and transitions
- Further and outlook. . .

In principle, the recipe is to determine the mass of particle P is straightforward:

- Choose an *interpolating operator* $\mathcal{O}_P$ that has large overlap:

$$\langle 0|\mathcal{O}_P|P\rangle \neq 0$$

- Construct the time-sliced correlator

$$C(t) = \sum_{\vec{x}} \langle \mathcal{O}(\vec{x},t)\mathcal{O}^\dagger(\vec{0},0)\rangle$$

- Insert a complete set of states

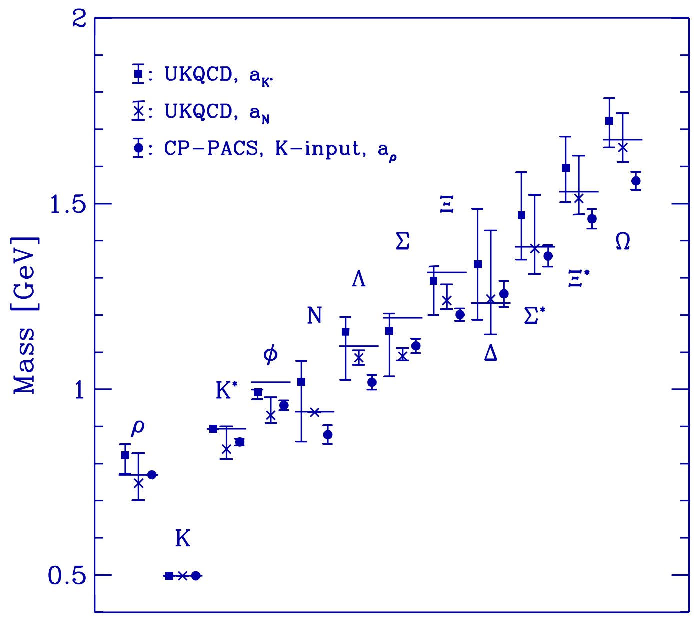
$$\begin{aligned} C(t) &= \sum_{\vec{x}} \sum_{P} \int \frac{d^3k}{(2\pi)^3 2E(\vec{k})} \langle 0|\mathcal{O}(\vec{x},t)|P(\vec{k})\rangle\langle P(\vec{k})|\mathcal{O}^\dagger(\vec{0},0)|0\rangle \\ &\longrightarrow \sum_P \frac{|\langle 0|\mathcal{O}|P\rangle|^2}{2m_P} e^{iM_P t} \longrightarrow \sum_P \frac{|\langle 0|\mathcal{O}|P\rangle|^2}{2m_P} e^{-M_P t} \text{ Euclidean} \end{aligned}$$

- time-slicing puts the intermediate states at rest.
- At large times, correlator dominated by lightest state

Hadron Spectrum - Benchmark of LQCD

It is the most precise and extensively pursued lattice calculation.

Final Quenched Spectrum UKQCD, PRD62 (2000), 054506



Quenched Spectrum Agrees with Experiment to 10%

Much work behind the scenes...

- Continuum extrapolation $a \longrightarrow 0$
- Extrapolation $V \longrightarrow \infty$
- Chiral extrapolation $M_{\text{PS}} \longrightarrow M_{\pi}$

Inconsistency in meson sector resolved in full QCD

Baryon Operators

- Flavour structure
- Parity
- Angular Momentum

Cubic group admits only three irreducible ray representations

Rep.	Spin cpts.
G_1	$1/2, 7/2, \ldots$
H	$3/2, 5/2, 7/2, \ldots$
G_2	$5/2, 7/2, \ldots$

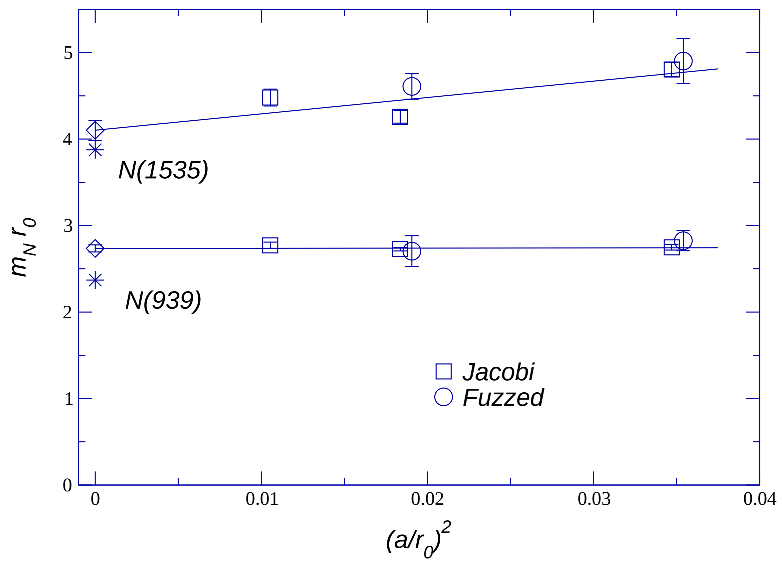
Measure local interpolating operators for the nucleon

$$\left\{ \begin{array}{lcll} N_1^{1/2+} & = & \epsilon_{ijk}(u_i^T C\gamma_5 d_j)u_k & \text{Nucleon} \\ N_2^{1/2+} & = & \epsilon_{ijk}(u_i^T C d_j)\gamma_5 u_k & \text{Roper} \end{array} \right.$$

$N_1 (N_2)$ connects upper (lower) spinor components in diquark piece - N_2 vanishes in NR limit.

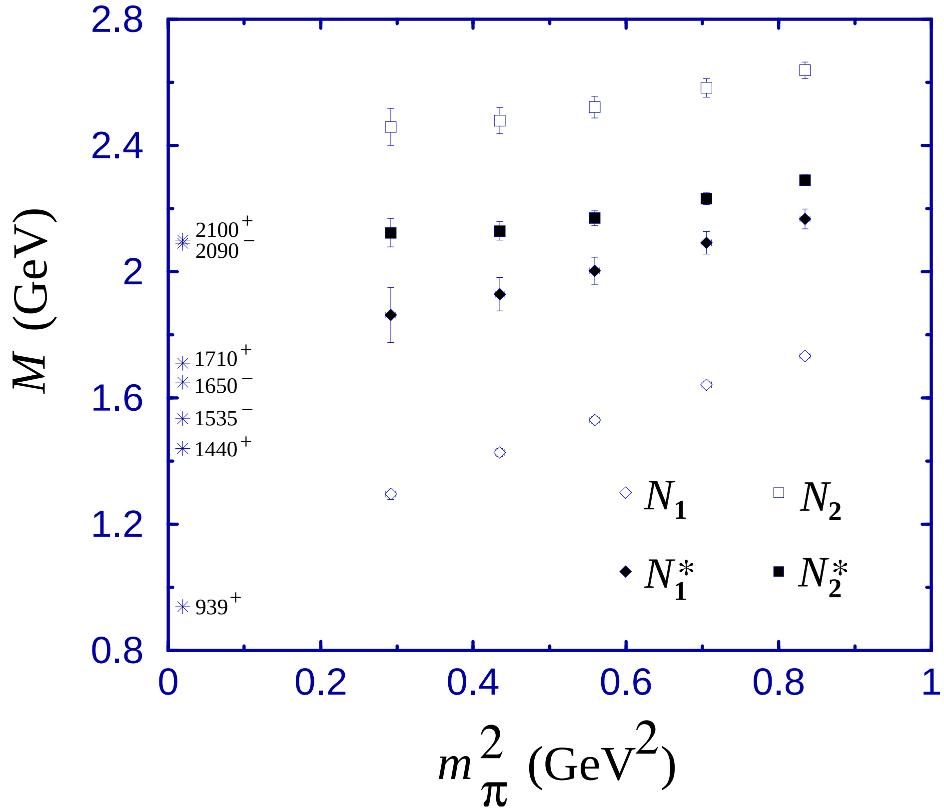
Flurry of lattice activity aimed at extracting lightest states of both parities in each channel:-

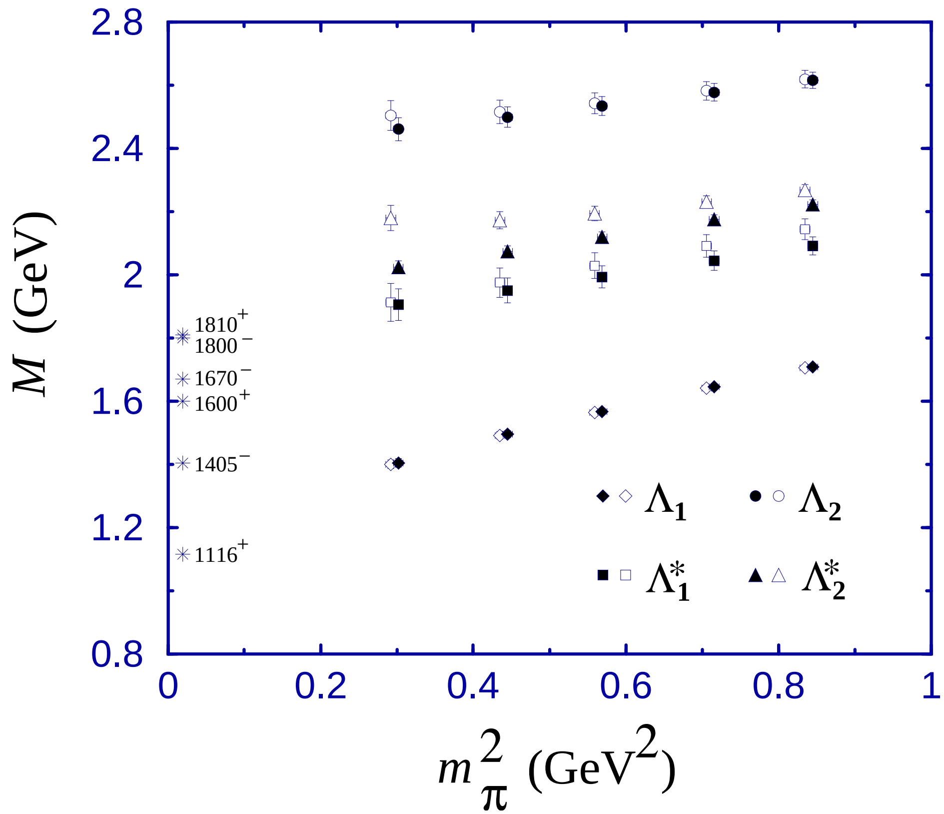
- D_{234} action - Frank Lee and Derek Leinweber
- Domain-wall fermions - BNL/Riken
- Clover - LHPC/QCDSF/UKQCD; anisotropic clover - LHPC
- Improved gauge/FLIC actions - Adelaide Group



LHPC/QCDSF/UKQCD

FLIC action, Melnitchouk *et al*





All calculations quenched, at large pion mass, use simple, three-quark operators.

Ordering of states ↔ quark model

$$N^{1/2+} < N^{1/2-} < N'^{1/2+}.$$

Roper resonance

- Is Roper lurking as excited state in N_1 correlator - different interpolating operator? - matrix analysis suggests not (BNL, PRD 65, 074503).
- Full QCD?
- Light pion mass?

Bayesian analysis of correlators:

- Wilson - Sasaki *et al* - `hep-lat/0209059`
- Clover/dynamical - C. Marnard, DGR - `hep-lat/0209165`
- Overlap fermions - - Kentucky, F.X. Lee *et al*, `hep-lat/0208070`

c. 1900 dentists discover Nitrous Oxide - **painless tooth extraction**.

c. 2000 lattice theorists discover Bayesian Statistics - **painless mass extraction**

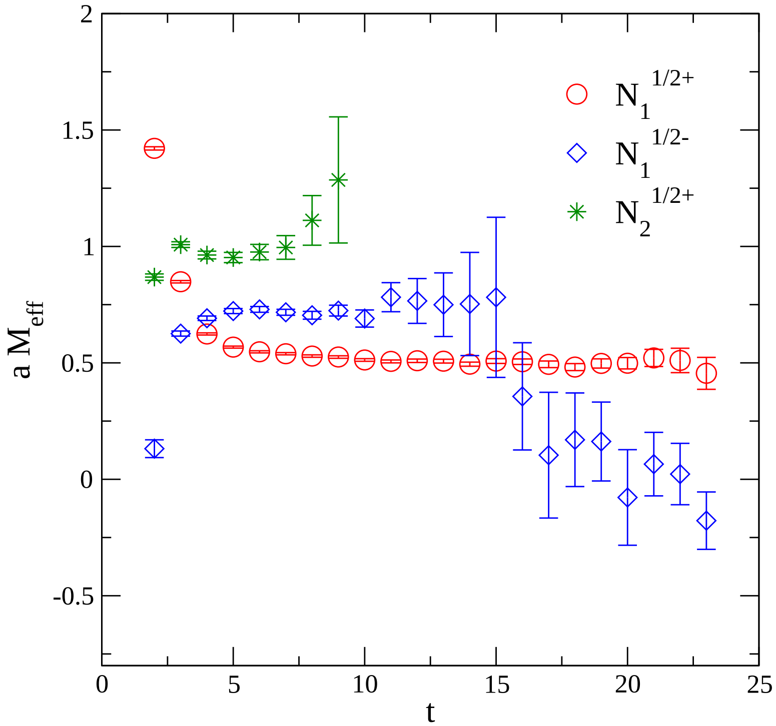
Bayes's Theorem

$$P(M|D \cap I) = \frac{P(D|M \cap I)P(M|I)}{\int dM\, P(D|M \cap I)P(M|I)}$$

- D is data
- M is model
- I is our prior knowledge

Why?

- Can we obtain more information from the lattice simulations?
- How do we incorporate prior knowledge, e.g. $m > 0$



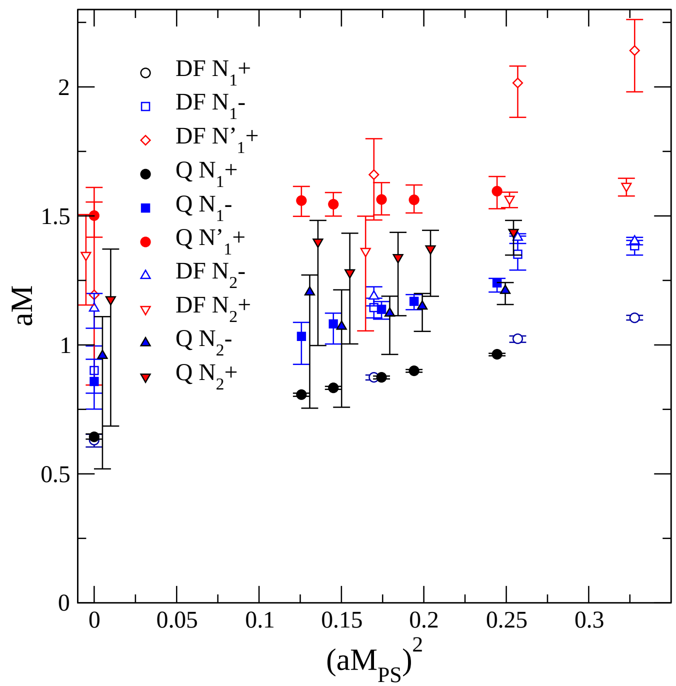
$$m_{\text{eff}} = \ln C(t)/C(t+1)$$

Bayesian Priors

What prior knowledge can we incorporate?

- Masses are positive
- Masses are ordered:
- Splitting?

Comparison of ordering between full and quenched QCD - C. Maynard, DGR (LHPC/UKQCD)



Observed Nucleon Spectrum has ordering $M^+ < M^- < M'^+$ for pion masses larger than around $500 MeV$ in full and quenched QCD.

Extension to light pion masses

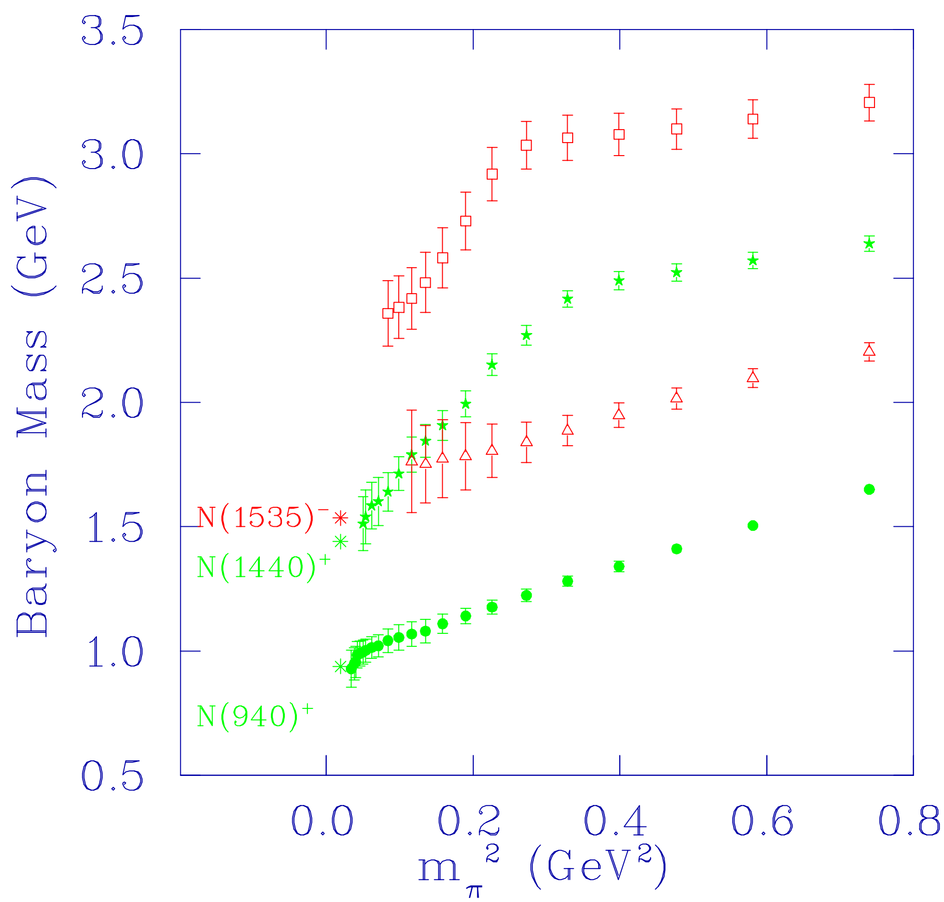
- Wilson fermions limited to $m_\pi \simeq 400$MeV
- "Smoothed" actions to $m_\pi \simeq 300$MeV - see FLIC action.
- Chiral symmetry restored only in continuum limit.

Overlap or Domain-wall Fermions - Neuberger, Narayanan, Ginsparg/Wilson

- Possess exact analogue of chiral symmetry at finite lattice spacing.
- No additive mass renormalisation
- Improved operator mixing
- Automatically $\mathcal{O}(a)$-improved.

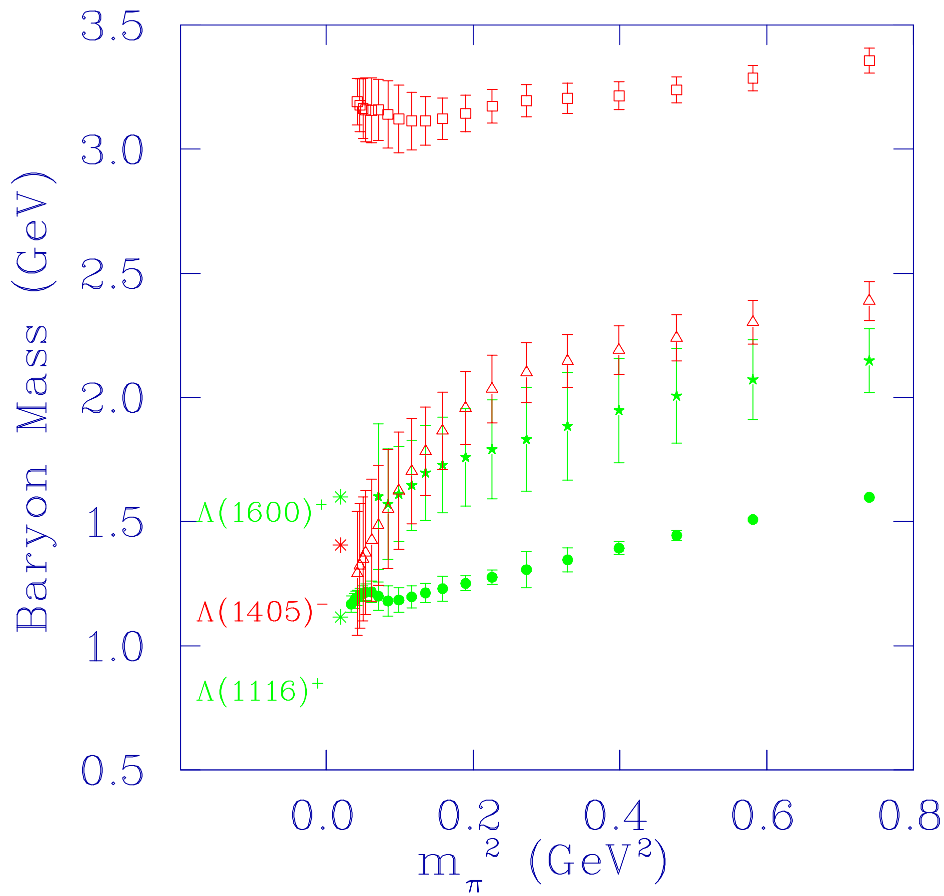
BUT . . . around 30 times as computationally demanding. . .

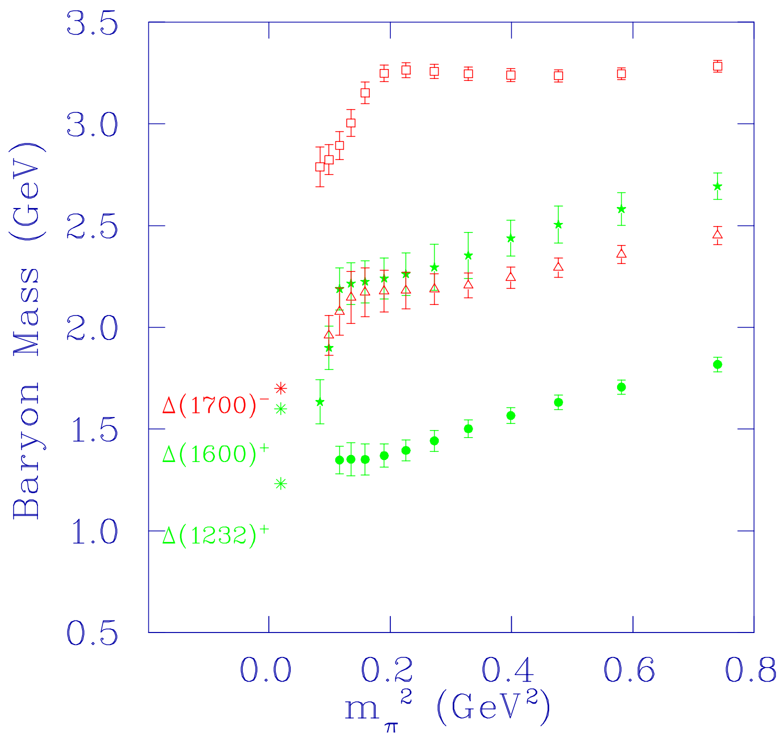
F.X. Lee *et al.*



3.5
3.0
2.5
2.0
1.5
1.0
0.5
Baryon Mass (GeV)
N(1535)⁻
N(1440)⁺
N(940)⁺
0.0
0.2
0.4
0.6
0.8
m_π^2 (GeV^2)

F.X. Lee *et al.*


3.5
3.0
2.5
2.0
1.5
1.0
0.5
Baryon Mass (GeV)
Λ(1600)⁺
Λ(1405)⁻
Λ(1116)⁺
0.0
0.2
0.4
0.6
0.8
m_π² (GeV²)

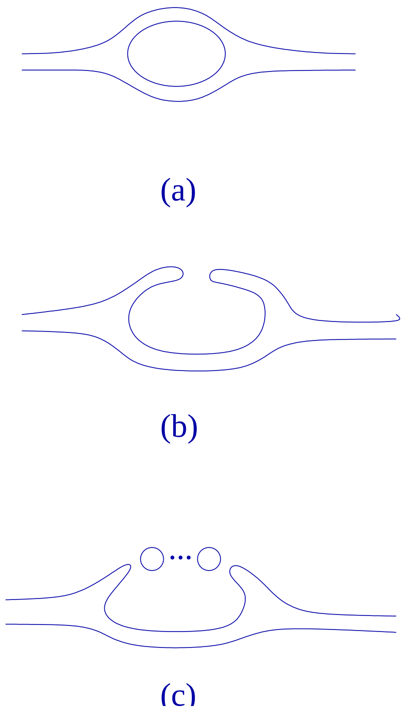


Dramatic reordering of spectrum at light pion mass.

Physics at physical pion mass different to that at heavy quark masses - chiral behaviour/importance of non-analytic terms.

Decays and Mixing

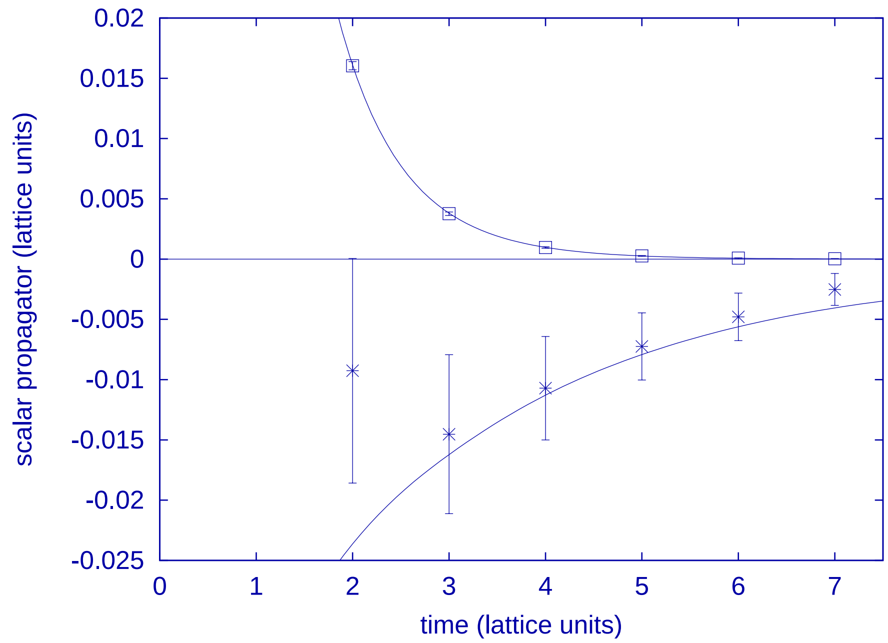
Great myths of lattice QCD - "Particles don't decay in the quenched approximation"



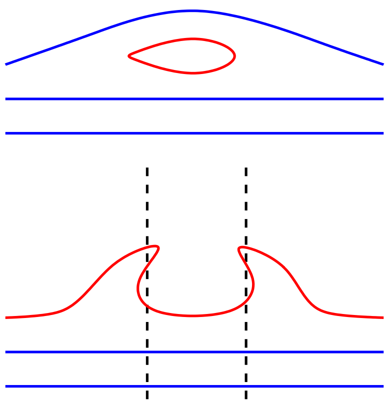
Contributions to the $\eta'\pi$ intermediate state in scalar isovector propagator.

Manifest as non-unitary behaviour in the hadron correlator.

Bardeen *et al*, PRD 65, 014509 (2001)

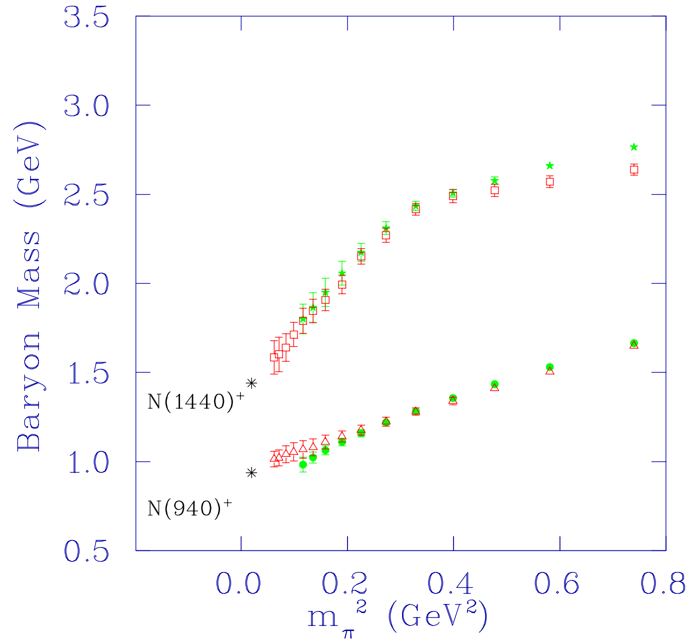


Behaviour well-described by quenched chiral perturbation theory.



Volume-behaviour of two-particle spectrum?

- $m_\pi L \simeq 3$
- Two-particle final states $\leftrightarrow$ strong L dependence



F.X. Lee *et al.* (preliminary)

Beyond the spectrum. . .

Transition form factors are straightforwardly accessible to lattice calculation - Gunnar Bali.

Decays $A \longrightarrow B + C$ complicated because phase information is obscured in Euclidean space - large time correlators dominated by lightest two-body state with minimum momentum - Maiani-Testa Theorem.

Lüscher relates shift in energies of two-particle system in finite box to extract phase-shifts.

Momenta on torus quantised: $\vec{q} = 2\pi n/L$

Thus, for two pions, say, total π-π energy

$$W = 2\sqrt{m_\pi^2 + p_i^2}$$

has discrete values.

Discrete energy eigenvalues of two-pion system shifted by finite size effect.

Luscher relates the shifts of discrete two-particle energies to the infinite-volume phase shifts.

For zero-momentum state, we have

$$\Delta E = -\frac{2\pi(m_1+m_2)a_0}{m_1 m_2 L^3}\left[1 - c_1\frac{a_0}{L} + c_2\left(\frac{a_0}{L}\right)^2\right]$$

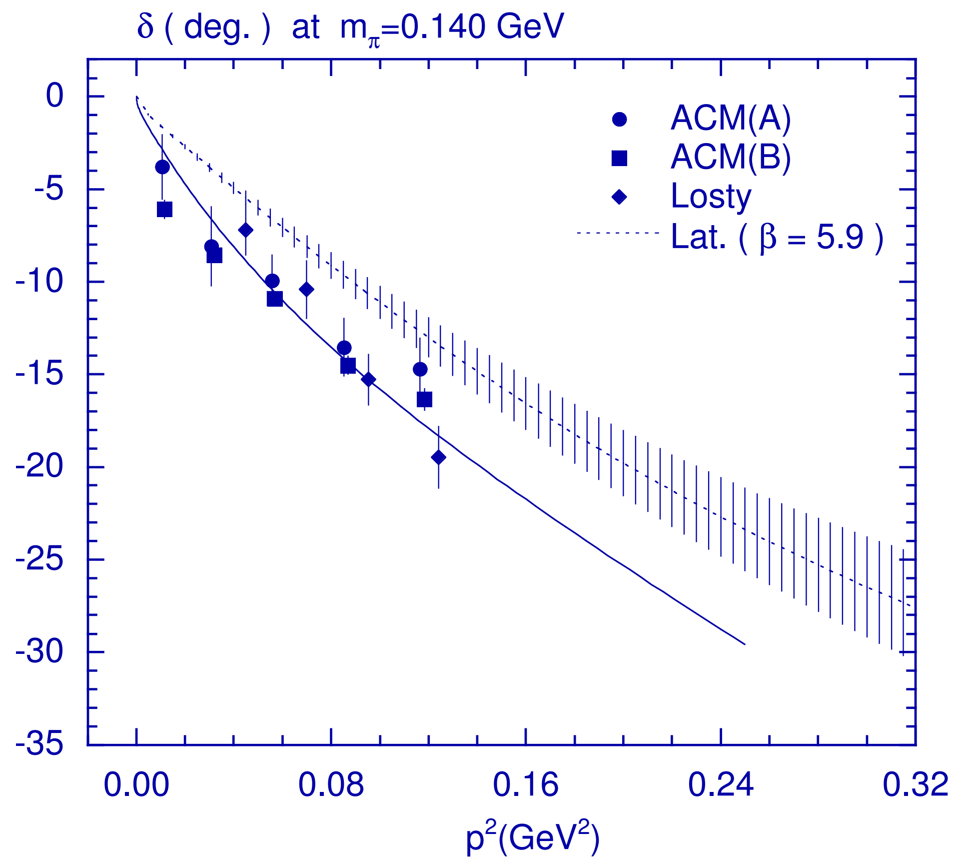
where a_0 is S-wave scattering length.

Need a_0/L small for this expansion to be useful.

The method require the extraction of energies from four-point function:

$$C_{h_1h_2}(t_1,t_2,t_3,t_4) = \sum_{\vec{x_4}}\mathcal{O}_{h2}(\vec{x_4},t_4)\sum_{\vec{x_3}}\mathcal{O}_{h1}(\vec{x_3},t_3)\sum_{\vec{x_2}}\mathcal{O}^{\dagger}_{h2}(\vec{x_2},t_2)\sum_{\vec{x_4}}\mathcal{O}^{\dagger}_{h2}(\vec{x_4},t_4)$$

$I = 2$ Pion scattering phase shift - Aoki *et al*, `hep-lat/0209124`

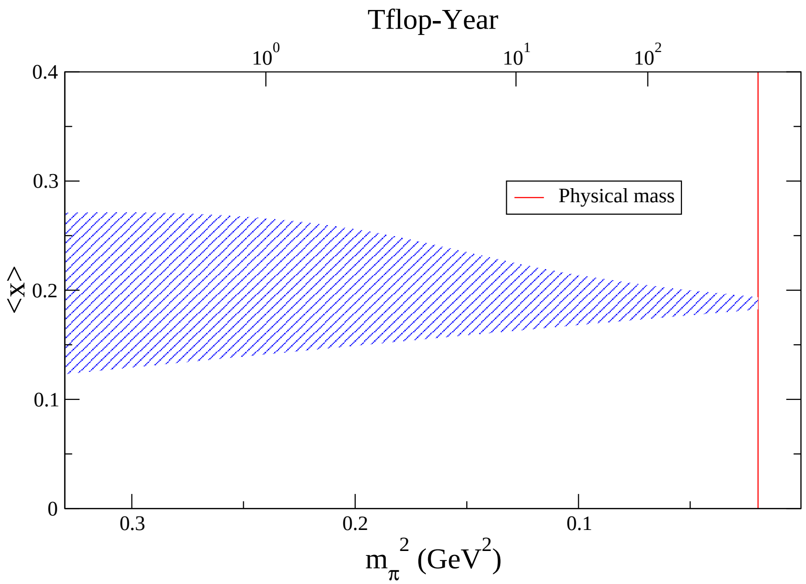


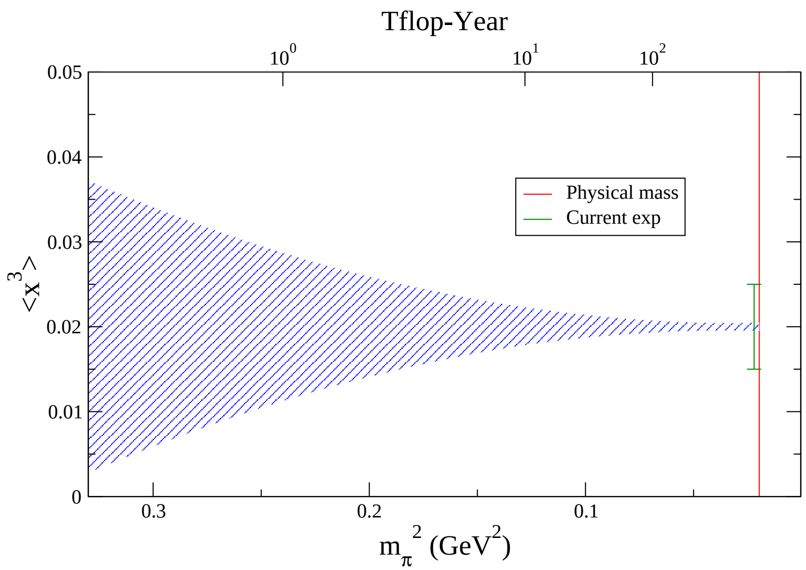
Summary

- Lattice baryon resonances exciting new application of lattice QCD
- At large masses, resonance spectrum described by quark model.
- Physics is different at light pion mass - domain-wall/chiral fermions.
- Lattice calculations must be matched by careful (quenched) chiral perturbation theory calculations.
- Need to understand finite-volume effects.
- Operators sensitive to "molecules" and excited glue - Feed back with models.
- Many operators $\leftrightarrow$ build up spectrum of higher states. *coupled-channel analysis*
- Enable us to test and verify models of hadronic physics and identiy degrees of freedom
- Future prospects. . .

Dominant cost: $m_\pi \longrightarrow 0$

- Straightforward to estimate for **moments of structure functions**
- "Synthetic data" assuming "CSSM" chiral expansion.


Tflop-Year
10^0
10^1
10^2
0.4
0.3
0.2
0.1
0
<x>
Physical mass
0.3
0.2
0.1
m_π^2 (GeV^2)


Tflop-Year
10^0
10^1
10^2
0.05
0.04
0.03
0.02
0.01
0
<x^3>
Physical mass
Current exp
0.3
0.2
0.1
m_π^2 (GeV^2)

Physics Roadmap at Jefferson Laboratory