FORM FACTORS and WIDE-ANGLE EXCLUSIVE SCATTERING at LARGE MOMENTUM TRANSFER

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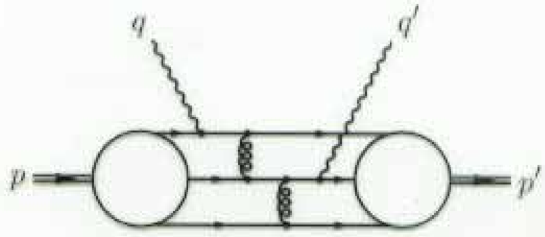
Pittsburgh, October, 12, 2002

Outline:

- **Leading-twist vs. handbag factorization**
- **GPDs**
- **Wide-angle Compton scattering**
- **Soft form factors**
- **Comparison with experiment**
- **Excited nucleons**
- **Summary**

Leading-twist factorization



$-t\,(-u)$ large

all valence quarks participate in hard scattering

quarks are moving collinear with their parent hadron

hard process: $\gamma qqq \to \gamma qqq$

soft physics:

distribution amplitudes $\Phi_p = \Phi_p(x_1, x_2, x_3)$

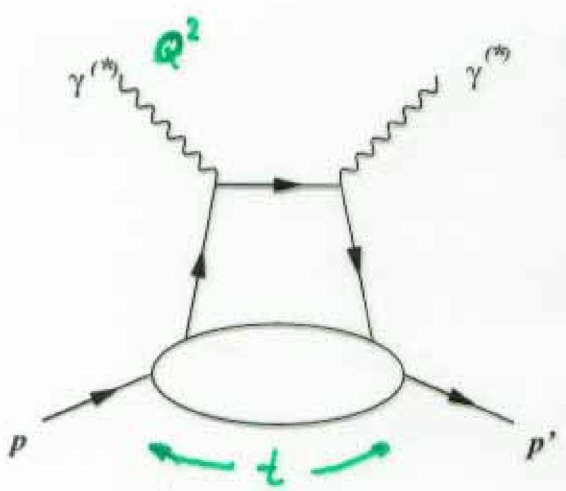
$$\mathcal{M} \sim \Phi_p \otimes \mathcal{H} \otimes \Phi_p$$

high resolution is required to see three quarks in a proton

onset of this region probably for $-t(-u) > 100\,\mathrm{GeV}^2$

Handbag factorization

(e.g. Compton scattering and corresponding processes)



only one active parton
(others are spectators)

hard process: $\gamma^{(*)} q \to \gamma q$

soft physics: GPDs

two classes of hard exclusive reactions:

DEEP VIRTUAL $\quad Q^2$ large $\quad -t/Q^2 \ll 1$

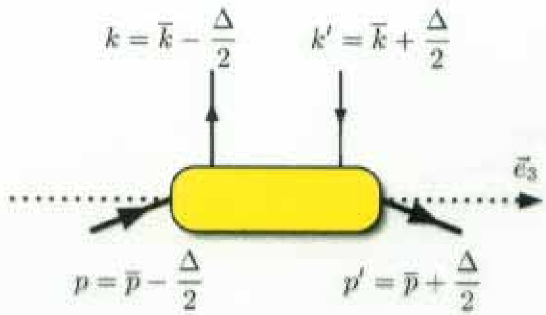
WIDE-ANGLE $\quad -t(-u)$ large $\quad Q^2/(-t) \ll 1$

formally a power correction to leading twist

which scheme dominates at $-t \simeq 10$ GeV^2?

Generalized Parton Distributions

D. Müller et al (9[illegible]), Ji(97), Radyushkin (97)



$$\xi = \frac{(p - p')^+}{(p + p')^+} \qquad \overline{x} = \frac{\overline{k}^+}{\overline{p}^+}$$

$$x = \frac{\overline{x} + \xi}{1 + \xi} \qquad x' = \frac{\overline{x} - \xi}{1 - \xi}$$

$$\overline{p}^+ \int \frac{dz^-}{2\pi} e^{i\overline{x}\,\overline{p}^+ z^-} \langle p' | \overline{\psi}_q(-\overline{z}/2)\, \gamma^+ \psi_q(\overline{z}/2)\, |p\rangle =$$

$$\overline{u}(p')\gamma^+ u(p) H^q(\overline{x}, \xi; t) + \overline{u}(p') i\sigma^{+\alpha} \frac{\Delta_\alpha}{2m} u(p)\, E^q(\overline{x}, \xi; t)$$

(gauge $A^+ = 0$; $\overline{z} = [0, z^-, \mathbf{0}_\perp]$) $\qquad \gamma^+\gamma_5 \longrightarrow \widetilde{H}^q, \widetilde{E}^q$

$\xi < \overline{x} \leq 1$ $(x > 0,\ x' > 0)$	emission and absorption of quarks
$-\xi < \overline{x} < \xi$ $(x > 0,\ x' < 0)$	emission of a $q\overline{q}$ pair
$-1 \leq \overline{x} \leq -\xi$ $(x < 0,\ x' < 0)$	emission and absorption of antiquarks

gluons analogously

Properties of GPDs

- reduction formulas

  $H^q(\overline{x},0;0) = q(\overline{x}) \qquad \widetilde{H}^q(\overline{x},0;0) = \Delta q(\overline{x})$
- sum rules

  $F_1^q = \int_{-1}^{1} d\overline{x} H^q(\overline{x},\xi,t) \qquad F_1 = \sum_q e_q F_1^q$

  $E^q \to F_2^q \qquad \widetilde{H}^q \to F_A^q \qquad \widetilde{E}^q \to F_P^q$
- polynomiality
- universality
- evolution ($\overline{x} < \xi$ ERBL; $\overline{x} > \xi$ DGLAP)
- Ji's sum rule

  $J_q = \int_{-1}^{1} d\overline{x}\, \overline{x}\, [H^q(\overline{x},\xi,t=0) + E^q(\overline{x},\xi,t=0)]$
- positivity constraints
- overlap representation

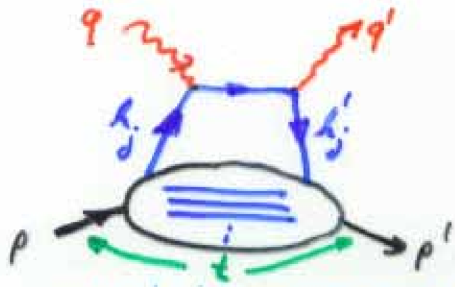
  $$H^q(\overline{x},\xi,t) \sim \sum_N (1-\xi^2)^{(1-N)/2} \int [dx]_N [d^2k_\perp]_N \Psi_N^* \Psi_N$$

  $-\xi < \overline{x} < \xi \qquad N+1 \to N-1$ overlaps

but we are not able to calculate them as yet

- models
- extraction from experiment
- lattice QCD in the long run

The handbag contribution to WACS



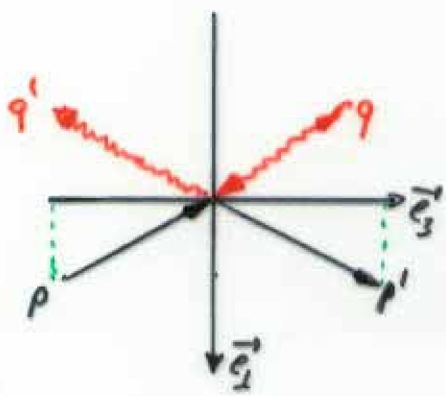
$s, -t, -u \gg \Lambda^2$

$\Lambda \sim \mathcal{O}(1\text{GeV})$

typical hadronic scale

(VCS: $Q^2 << -t$, fixed)

- work in a symmetric frame



$$p^{(\prime)} = [p^+, \frac{m^2 - t/4}{2p^+}, \pm \mathbf{\Delta}_\perp]$$

$$\xi = \frac{(p-p')^+}{(p+p')^+} = 0 \quad t = -\Delta_\perp^2$$

- definition of soft contribution
  - parton virtualities $k_i^2 < \Lambda^2$
  - intrinsic transverse momenta $k_{\perp i}^2 / x_i < \Lambda^2$

- consequences

  $(k_j + q)^2 \simeq (p+q)^2 = s$ propagators poles avoided

  $(k_j - q')^2 \simeq (p-q')^2 = u$ (in contrast to DVCS)

  active partons approximately on-shell, collinear with their parent hadrons and with $x_j, x_j' \simeq 1$

- physical situation: hard photon-parton scattering and soft emission and reabsorbtion of partons by hadrons (similar to DVCS)

The Compton amplitudes

$$\mathcal{M}_{\mu'+,\mu+} = 2\pi\alpha_{elm}\Big\{\mathcal{H}_{\mu'+,\mu+}\big[R_V + R_A\big] + \mathcal{H}_{\mu'-,\mu-}\big[R_V - R_A\big]\Big\}$$

$$\mathcal{M}_{\mu'-,\mu+} = -\pi\alpha_{elm}\frac{\sqrt{-t}}{m}\Big\{\mathcal{H}_{\mu'+,\mu+} + \mathcal{H}_{\mu'-,\mu-}\Big\}R_T$$

$R_i(t)$: $1/\bar{x}$-moments of GPDs (at $\xi=0$) $\qquad$ $\mathcal{H}(s,t)$: $\gamma q \to \gamma q$ amplitudes

only $+$ components appear

(it is a dynamical feature in contrast to DVCS and DIS where p^+ is large)

Radyushkin, hep-ph/9803316; DFJK, hep-ph/9811253

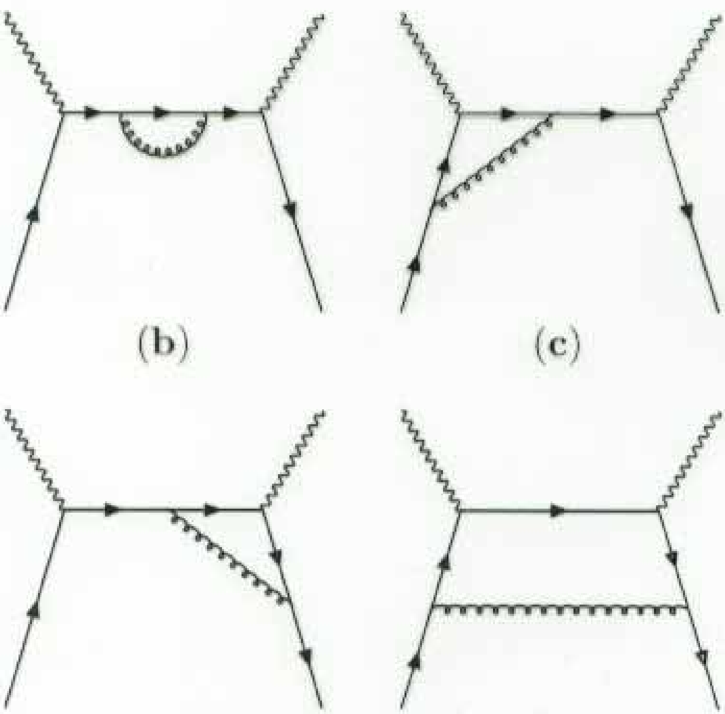
inclusion of R_T: Huang et al, hep-ph/0110208

$\gamma q \to \gamma q$ to NLO

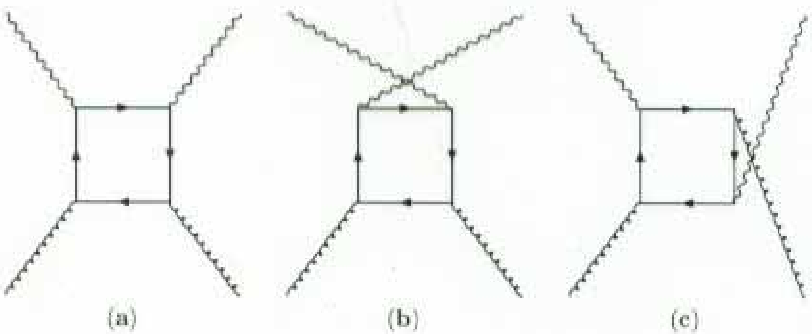
(a)

$$\mathcal{H}^{LO}_{++++} = 2\sqrt{-s/u}$$

$$\mathcal{H}^{LO}_{-+-+} = 2\sqrt{-u/s}$$



NLO provide
photon helicity flip
and imaginary parts



requires
gluonic GPDs
and
form factors

$\gamma q \to \gamma q$ to NLO

Feynman gauge, $n = 4 + \epsilon$

UV divergencies cancel

$$\mathcal{H}^{IR}_{\mu+,\mu+} = \frac{\alpha_s}{4\pi} C_F C_{IR}(\mu_F) \mathcal{H}^{LO}_{\mu+,\mu+} + \mathcal{H}^{NLO}_{\mu+,\mu+}$$

C_{IR} embodies IR singularities

$$\begin{aligned}
&\mathcal{H}_{\mu+,\mu+}(s,t)\, R_i(t) \\
&= \left[\mathcal{H}^{LO}(1 + \frac{\alpha_s}{4\pi} C_F C_{IR}(\mu_F)) + \mathcal{H}^{NLO}\right] R_i(t) \\
&= \left[\mathcal{H}^{LO} + \mathcal{H}^{NLO}\right] R_i(t, \mu_F) + \mathcal{O}(\alpha_s^2)
\end{aligned}$$

$$C_{IR} = -\left(\frac{-t}{4\pi\mu_F^2}\right)^{\epsilon/2} \Gamma(1 - \epsilon/2)\,(8/\epsilon^2 - 6/\epsilon)$$

$\sim 1/\epsilon^2$ overlapping soft and collinear divergencies

$\Longrightarrow \quad [\alpha_s \ln^2(-t/\mu_F^2)]^n$

Sudakov factor (Collins; Musatov-Radyushkin)

absorbed into form factors (as in elm. case)

NLO corrections in $\gamma e \to \gamma e$: (Brown-Feynman)

IR divergencies cancel against those of $\gamma e \to \gamma\gamma e$

Model GPDs

for large $-t$ and x

$$H^q(x,0;t) = \exp\left[a^2 t \frac{1-x}{2x}\right] q(x)$$

$$\widetilde{H}^q(x,0;t) = \exp\left[a^2 t \frac{1-x}{2x}\right] \Delta q(x)$$

gluons analogue; limit $t \to 0$ OK

$a \simeq 1 \pm 0.2\mathrm{GeV}^{-1}$ transverse size of proton

(only free parameter)

based on overlaps of light-cone wave functions (generalized Drell-Yan formulas for GPDs)

$$\Psi_N(x, \mathbf{k}_\perp) = \Phi_N(x_1, ...x_N) \exp[-a_N^2 \sum_{i=1}^{N} k_{\perp i}^2 / x_i]$$

in line with central assumption of handbag approach (restricted $k_{\perp i}^2 / x_i$)

symplifying assumption $a_N = a$ for all N

Improvements:
- explicit treatment of lowest Fock states
- inclusion of evolution

  $a \to a(t)$, see Vogt hep-ph/0007277
- .....

Form factors in the handbag approach

$$
\begin{aligned}
F_1(t) &= \sum_q e_q \int_{-1}^{1} d\bar{x} H^q(\bar{x},0;t) & \sim q(\bar{x}) - \bar{q}(\bar{x}) \\
R_V(t) &= \sum_q e_q^2 \int_{-1}^{1} \frac{d\bar{x}}{\bar{x}} H^q(\bar{x},0;t) & \sim q(\bar{x}) + \bar{q}(\bar{x}) \\
F_A(t) &= \int_{-1}^{1} d\bar{x} [\widetilde{H}^u(\bar{x},0;t) - \widetilde{H}^d(\bar{x},0;t)] & \\
& & \sim \Delta q(\bar{x}) + \Delta\bar{q}(\bar{x}) \\
R_A(t) &= \sum_q e_q^2 \int_{-1}^{1} \frac{d\bar{x}}{\bar{x}} \widetilde{H}^q(\bar{x},0;t) & \sim \Delta q(\bar{x}) + \Delta\bar{q}(\bar{x})
\end{aligned}
$$

exclusive $\Longleftrightarrow$ inclusive

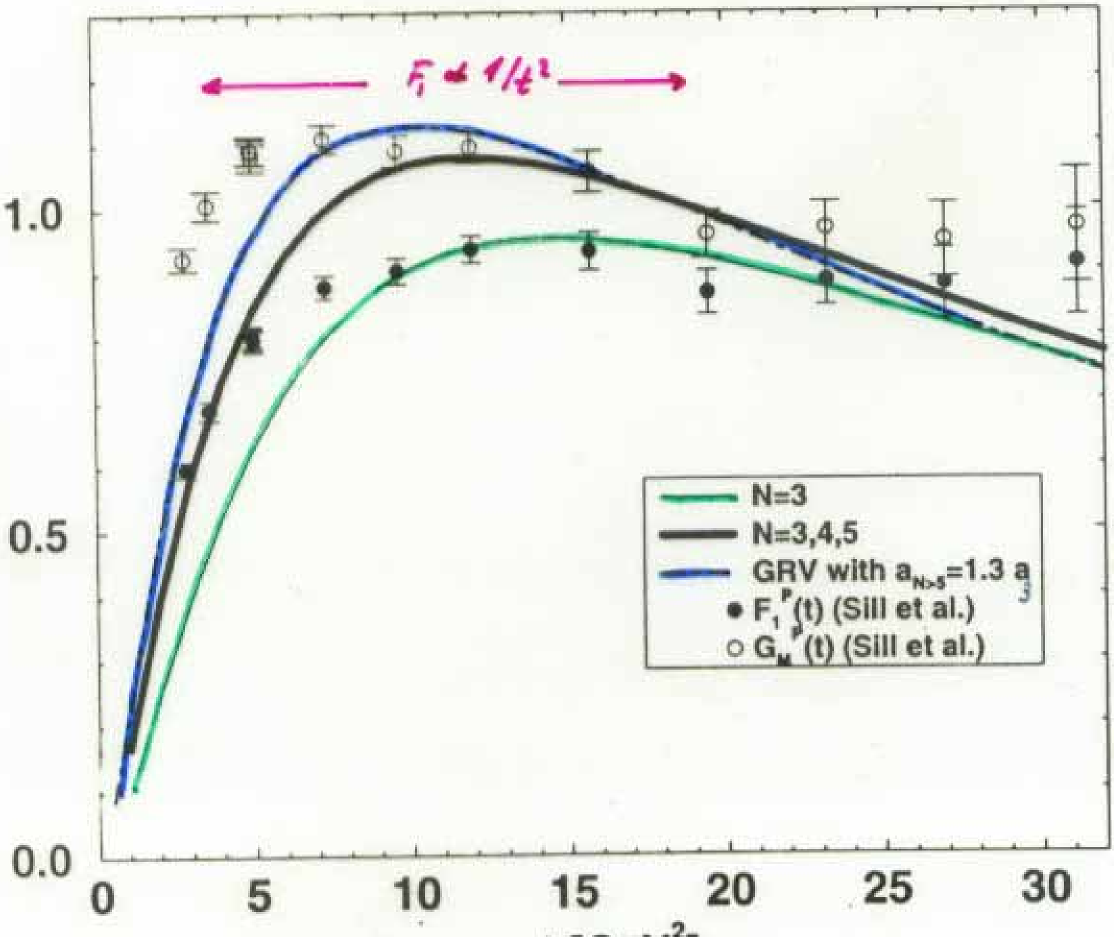
DFJK hep-ph/09811253

F_1, R_V Radyushkin hep-ph/9803316

F_1 Barone et al. (93), Afanasev hep-ph/9910565

DFJKII(99) (similar Radyushkin (98), Afanasev (99), Barone et al (93))
Stoler (91), ...

Proton Form Factor



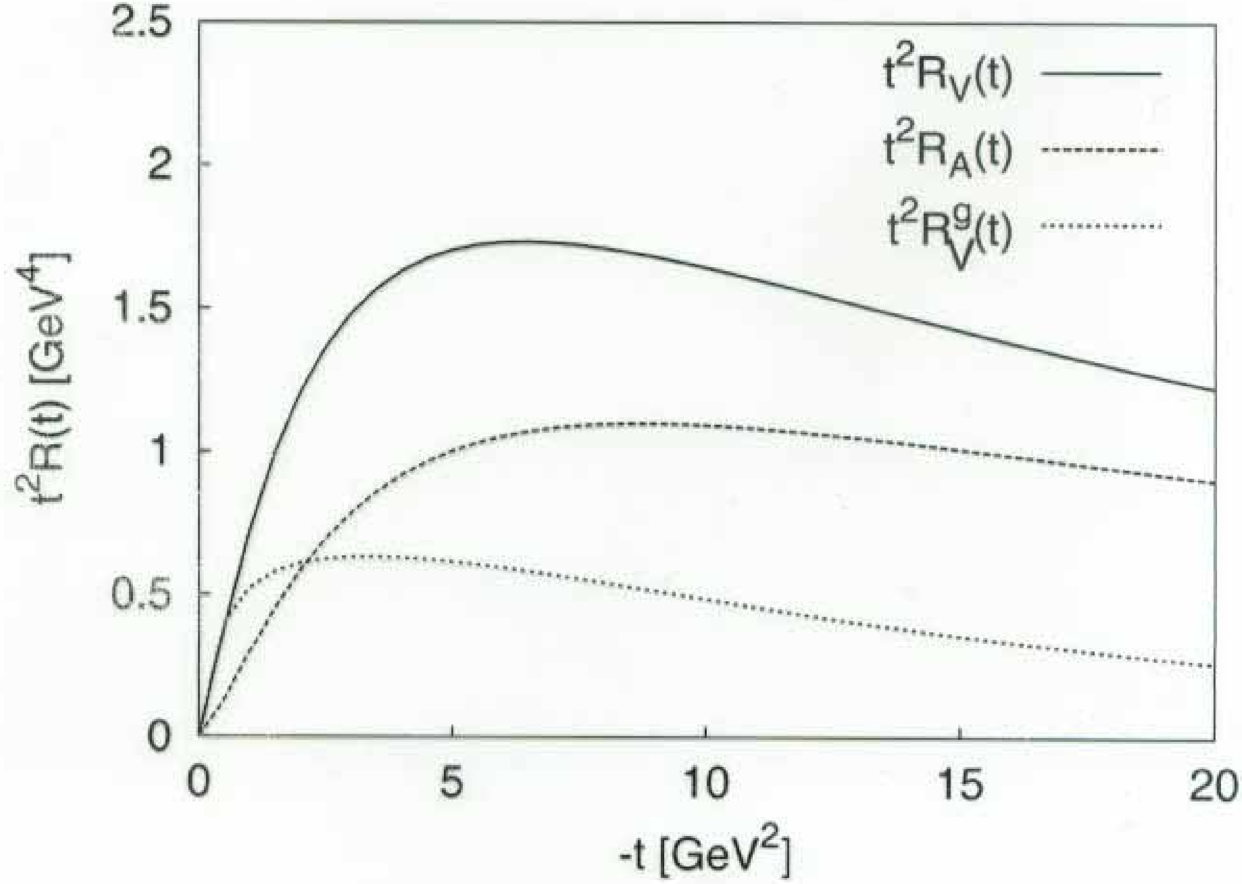
$F_1 \propto 1/t^4$
f. $t \to -\infty$
(soft physics asymptotics)

leading-twist contr. takes the lead

$a = 1\,\mathrm{GeV}^{-1}$ $a_3 = 0.75\,\mathrm{GeV}^{-1}$

valence Fock state dominates

Numerical results for model form factors



$t \to \infty \qquad R_i \propto 1/t^4$

Proton helicity flip

$F_2(t) = \sum e_q \int_{-1}^{1} d\bar{x} E^q(\bar{x}, 0; t), \; R_T(t) = \sum e_q^2 \int_{-1}^{1} \frac{d\bar{x}}{\bar{x}} E^q$

related to proton helicity flips

E^q involves parton orbital angular momentum

large $-t$: integrals dominated by large $\bar{x}$ and, hence,
by valence u-quarks, $1/\bar{x}$ unimportant

$$\Longrightarrow \qquad \boxed{R_T/R_V \simeq F_2/F_1}$$

SLAC(94)-data on $F_{1,2}$: $R_T/R_V \propto \Lambda^2/t$

R_T suppressed by $\Lambda/\sqrt{-t}$

Jlab Hall A data (2000): $R_T/R_V \propto \Lambda/\sqrt{-t}$

2002 R_T not suppressed

appears natural in overlap representation

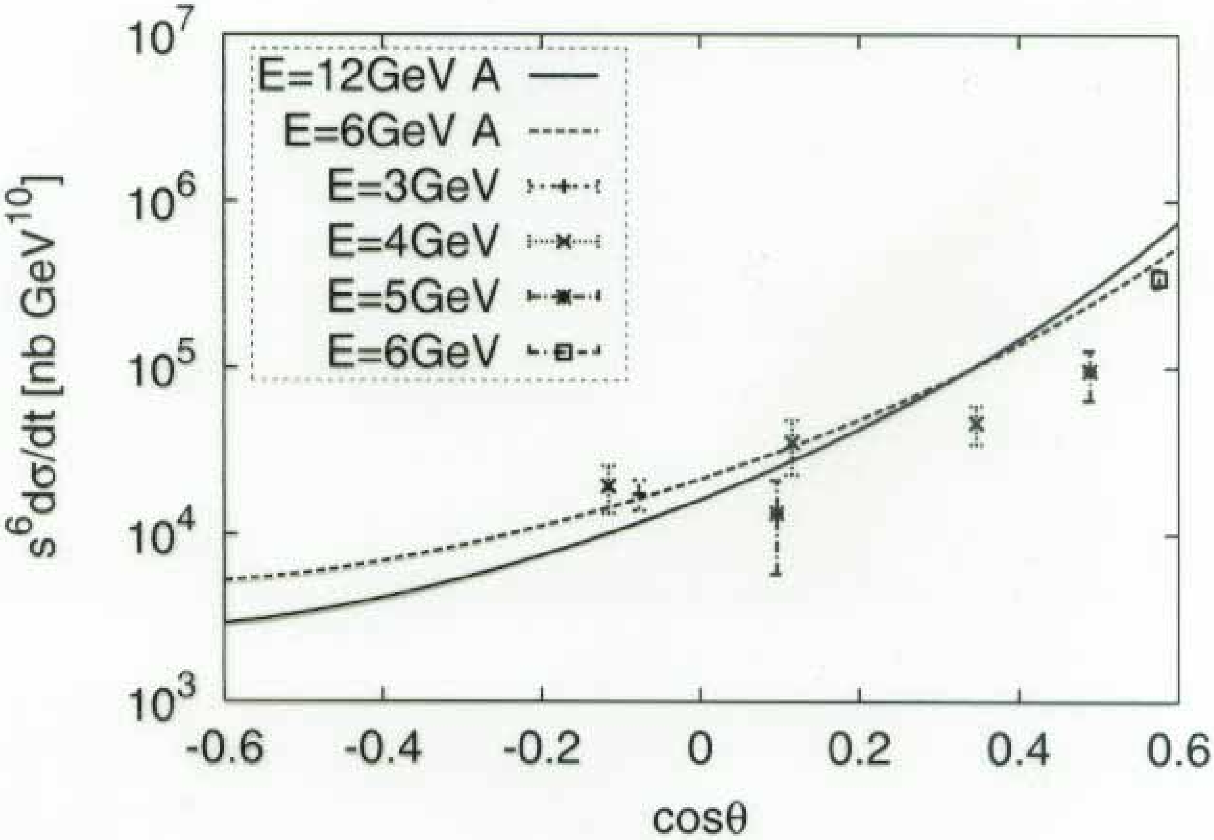
(see also Ralston et al (2000))

Take R_T from Jlab Hall A experiment

$$\kappa_T = \frac{\sqrt{-t}}{2m} \frac{R_T}{R_V} \simeq 0.37$$

in general $\kappa_T(t)$

The Compton cross section



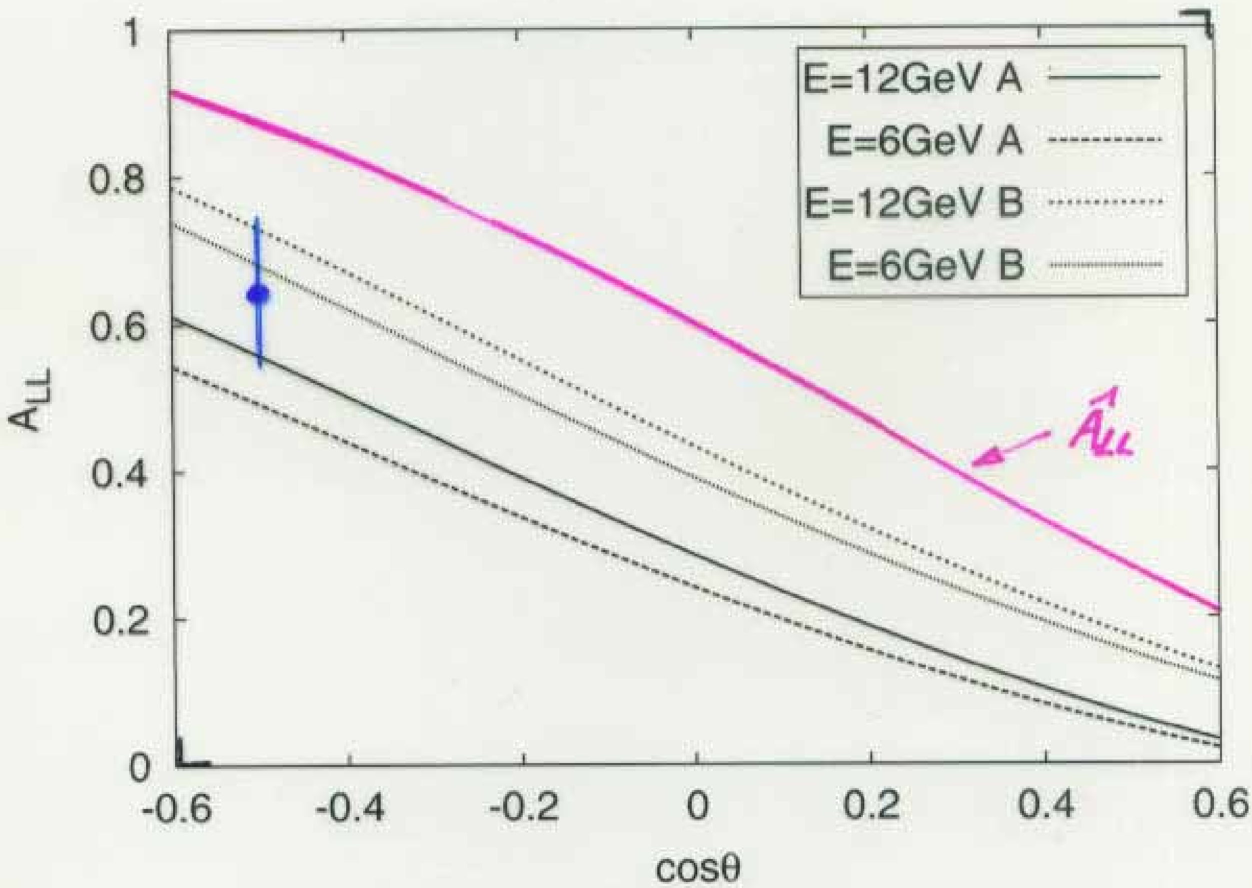
$$\frac{d\sigma}{dt} = \frac{d\hat{\sigma}}{dt}\Big\{\frac{1}{2}\left[R_V^2(1+\kappa_T^2)+R_A^2\right] - \frac{us}{s^2+u^2}\left[R_V^2(1+\kappa_T^2)-R_A^2\right]\Big\} + \mathcal{O}(\alpha_s)$$

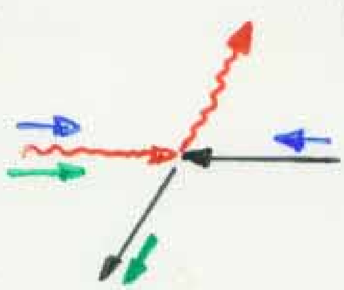
$\frac{d\hat{\sigma}}{dt}$ Klein-Nishina cross section

if $R_i \sim 1/t^2 \quad \Longrightarrow \quad d\sigma/dt(\theta = \text{fixed}) \sim 1/s^6$

data: Shupe et al (79)

Helicity correlation A_{LL}



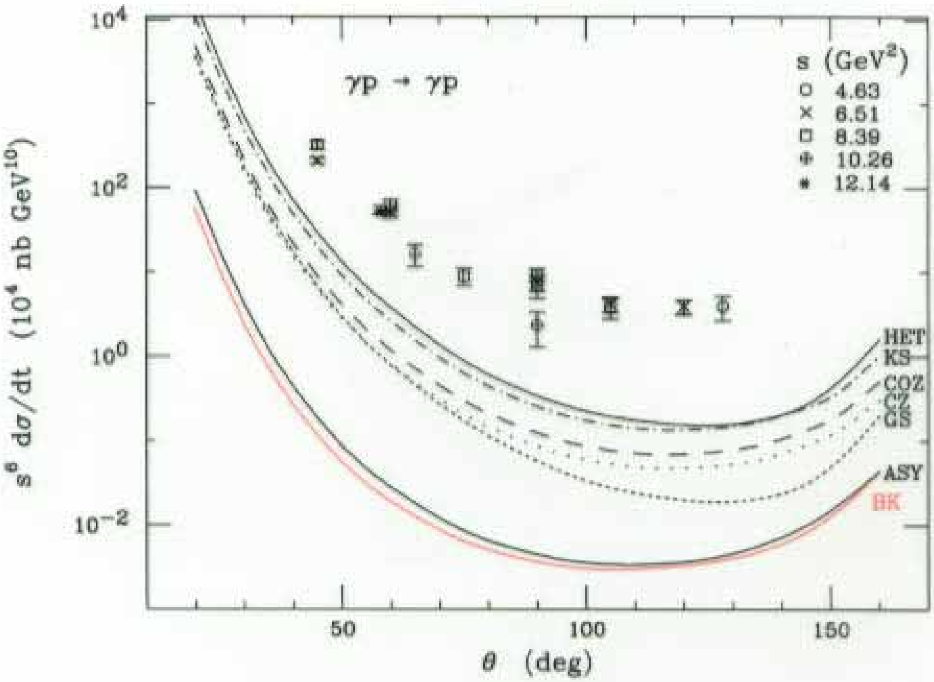


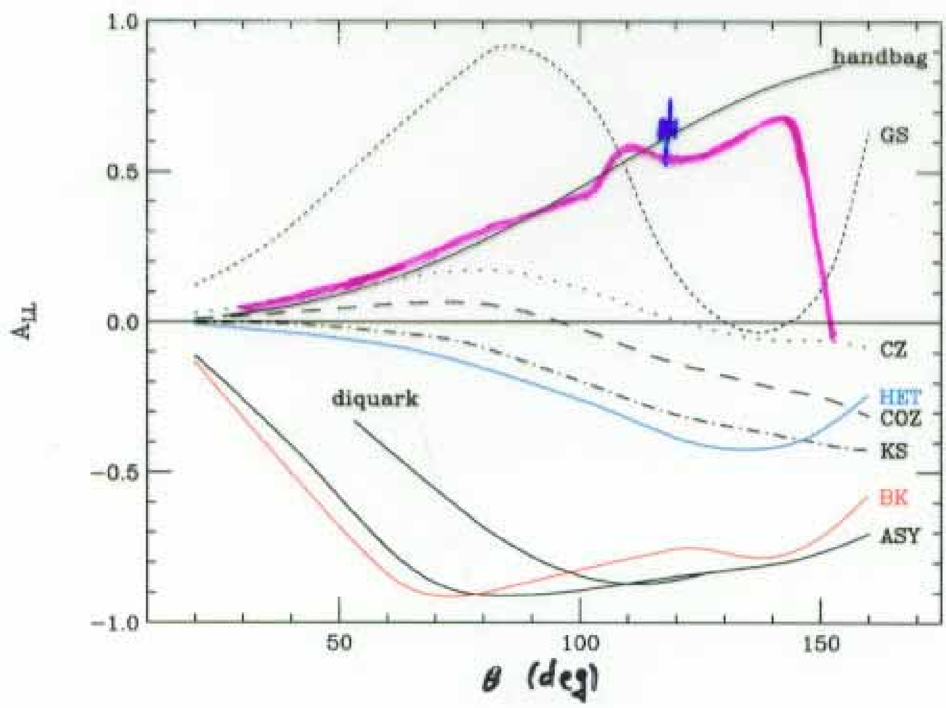
$$\hat{A}_{LL} = \frac{s^2-u^2}{s^2+u^2}$$

$$A_{LL} = K_{LL}$$

$$\simeq \hat{A}_{LL}\,\frac{R_A}{R_V} + \mathcal{O}(\alpha_s, \kappa_T, \beta)$$

Leading-twist results





taken from Brooks and Dixon, hep-ph/0004143

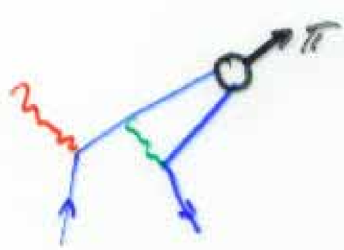
Photo- and electroproduction of π^0

handbag contribution

$$\mathcal{M}^{\pi}_{0+\mu+} = \frac{e}{2}\left\{\mathcal{H}^{\pi}_{0+\mu+}\,[R^{\pi}_V + R^{\pi}_A] + \mathcal{H}^{\pi}_{0-\mu-}\,[R^{\pi}_V - R^{\pi}_A]\right\}$$
$$\mathcal{M}^{\pi}_{0+\mu-} = \frac{e}{2}\left\{\mathcal{H}^{\pi}_{0+\mu+} + \mathcal{H}^{\pi}_{0-\mu-}\right\} R^{\pi}_T$$

For each flavor: $R_i^{\pi q} = R_i^{\gamma q}$ universality

leading twist formation of meson (as in DVEM)



$\mathcal{H}^{\pi}_{0+++} \simeq \frac{\sqrt{-2t}}{-u}$; $\mathcal{H}^{\pi}_{0+-+} \simeq \frac{\sqrt{-2t}}{s}$

$\Rightarrow \hat{A}^{\pi}_{LL} = \frac{s^2-u^2}{s^2+u^2}$

as in Compton scattering

A^{π}_{LL} diluted by form factors

(analog to Compton scattering)

difficulty: photoproduction cross section turns out to be too small

reasons:
- handbag correct but leading-twist mechanism underestimates meson production (see π form factor)
- soft VDM contribution still dominant

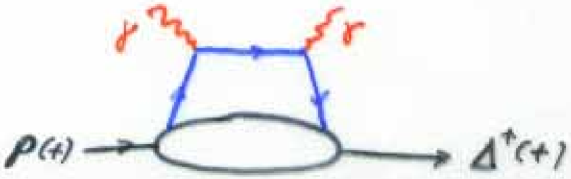
experimental check: do data show handbag characteristics?

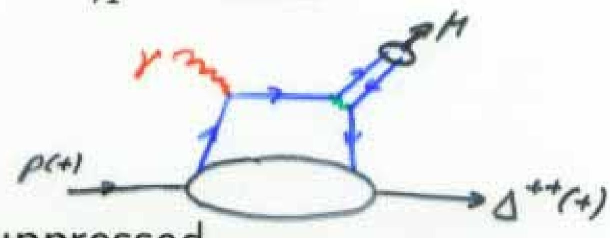
Huang-K,, hep-ph/0005318

Excited nucleons in wide-angle scattering

in principle possible, analogously to proton case

e.g. $\gamma p \to \gamma \Delta$ $\qquad\qquad$ $\gamma p \to M \Delta$





$\Delta(3/2)$ requires $L_q \neq 0$ suppressed

$\rho^-(\mp 1)\Delta^{++}(1/2 \pm 1)$ $\quad$ helicity correlations

new GPDs: $\quad H^i_{p\Delta}, \widetilde{H}^i_{p\Delta} \qquad i = 1,2,3$

new form factors: $G^i_{p\Delta}(t) = \int d\bar{x} H^i_{p\Delta}(\bar{x}, \xi, t)$

$$R^i_{p\Delta} = \int d\bar{x}/\bar{x}\, H^i_{p\Delta}$$

and for $\widetilde{H}$ analogously

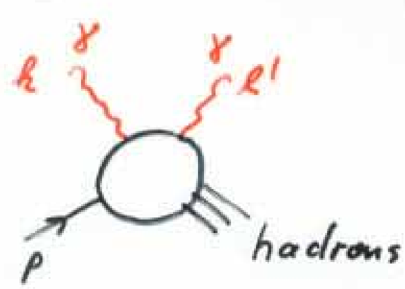
overlap model possible

but less restrictive since forward limit unknown

Frankfurt et al (99): large N_c limit provides relations

e.g. $\qquad \widetilde{H}^1_{p\Delta} = \sqrt{3}(\widetilde{H}_u - \widetilde{H}_d)$

sum over several final states Bjorken-Paschos (69)



inelastic Compton scattering *(at wide angles)*

$s = 2kp \qquad \nu = (k - k')p/m$

$t = 2kk' \qquad u = -s - t$

$$\frac{d\sigma}{dt d\nu} = \frac{d\hat{\sigma}^{KN}}{dt} \frac{2m}{-t} \sum_a e_a^4 q_a\left(x = \frac{-t}{2m}\right)$$

How many active partons

participate in hard exclusive reactions

at momentum transfer of order 10 GeV^2

?

0: only hadronic degrees of freedom
(e.g. Regge poles)

1: handbag

2: cat's ears, diquark correlations

3: leading twist

or more: from higher Fock states

Experiments tell us:

the number of active partons is small

1 seems to be favoured

but 0 is not excluded