The Search for Unity: Notes for a History of Quantum Field Theory
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Introduction

QUANTUM FIELD THEORY is the theory of matter and its interactions, which grew out of the fusion of quantum mechanics and special relativity in the late 1920s. Its reputation among physicists suffered frequent fluctuations in the following years, at times dropping so low that quantum field theory came close to be abandoned altogether. But now, partly as a result of a series of striking successes over the last decade, quantum field theory has become the most widely accepted conceptual and mathematical framework for attacks on the fundamental problems of physics. If something like a set of ultimate laws of nature were to be discovered in the next few years (an eventuality by no means expected), these laws would probably have to be expressed in the language of quantum field theory.

To the best of my knowledge, there does not exist anything like a full history of the past fifty years of quantum field theory. Existing histories of modern physics cover special and general relativity pretty thoroughly, and they take us through the early years of quantum mechanics, but their treatment of quantum mechanics generally ends with the triumph of the statistical interpretation around 1927. This is a pity, not only because of the fundamental nature of quantum field theory, but also because its history offers an interesting insight to the nature of scientific advance.

It is widely supposed that progress in science occurs in large or small revolutions. In this view, the successes of previous revolutions tend to fasten upon the scientist's mind a language, a mind-set, a body of doctrine, from which he must break free in order to advance further. There is great debate about the degree to which these revolutions are brought about by the individual scientific genius, able to transcend the fixed ideas of his times, or by the accumulation of discrepancies between existing theory and experiment. However, there seems to be general agreement that the essential element of scientific progress is a decision to break with the past.

I would not quarrel with this view, as applied to many of the major advances in the history of science. It certainly seems to apply to the great revolutions in physics in this century: the development of special relativity and of quantum mechanics. However, the development of quantum field theory since 1930 provides a curious counterexample, in which the essential element of progress has been the realization, again and again, that a revolution is unnecessary. If quantum mechanics and relativity were revolutions in the sense of the French Revolution of 1789 or the Russian Revolution of 1917, then quantum field theory is more of the order of the Glorious Revolution of 1688: things changed only just enough so that they could stay the same.

I will try to tell this story here without using mathematics. To some extent, I will let the history do the job of explication—I will go into some of the historical developments more fully than others, because they help to introduce ideas which are needed later. Even so, this is not an easy task—there is no branch of natural science which is so abstract, so far removed from everyday notions of how nature behaves, as quantum field theory.

This cannot be a story of physics in one country. As we shall see, quantum field theory had its birth in Europe, especially in Germany and Britain, and was revived after World War II by a new generation of theoretical physicists in Japan and the United States. The United States has been somewhat the center of the intense activity of the last decade, but physicists from many countries in Europe and Asia have made essential contributions. And although there are discernible national styles in physics, they have played only a minor role in this history. It is not the national or the social or the cultural setting that has determined the direction of research in physics, but rather the logic of the subject itself, the need to understand nature as it really is.

This article is not a history of quantum field theory, but only "notes for a history." A great deal of work needs to be done by professional historians of science in uncovering the story of the last half-century of theoretical physics. I would be delighted if this article were to spur someone to take on this overdue task.

Prehistory: Field Theory and Quantum Theory

Before quantum field theory came field theory and quantum theory. The history of these earlier disciplines has been told again and again by able historians of science, so I will do no more here than remind the reader of the essential points.

The first successful classical field theory was based on Newton's theory of gravitation. Newton himself did not speak of fields—for him, gravitation was a force which acts between every pair of material particles in the universe, "according to the quantity of solid matter which they contain and propagates on all sides to immense distances, decreasing always as the inverse square of the distances."[1] It was the mathematical physicists of the eighteenth century who found it convenient to replace this mutual action at a distance with a gravitational field, a numerical quantity (strictly speaking, a vector) which is defined at every point in space, which determines the gravitational force acting on any particle at that point, and which receives contributions from all the material particles at every other point. However, this was a mere mathematical *façon de parler*—it really made no difference in Newton's theory of gravitation whether one said that the earth attracts the moon, or that the earth contributes to the general gravitational field, and that it is this field at the location of the moon which acts on the moon and holds it in orbit.

Fields really began to take on an existence of their own with the development in the nineteenth century of the theory of electromagnetism. Indeed, the word "field" was introduced into physics by Michael Faraday in 1849. It was still possible for Coulomb and Ampère to consider electromagnetic forces as acting directly between pairs of electric charges or electric currents, but it became very much more natural to introduce an electric field and a magnetic field as conditions of space, produced by all the charges and currents in the universe, and acting in turn on every charge and current. This interpretation became almost unavoidable after James Clerk Maxwell demonstrated that electromagentic waves travel at a finite speed, the speed of light. The force which acts on electrons in the retina of my eye at this moment is not produced by the electric currents in atoms on the sun at the same moment, but by an electromagnetic wave, a light wave, which was produced by these currents about eight minutes ago, and which has only now reached my eye. (As we shall see, an attempt was made in 1945 by Richard Feynman and John Wheeler to account for this retardation of electromagnetic forces in an action-at-a-distance framework, but the idea did not catch on, and they both went on to more promising work).

Maxwell himself did not yet adopt the modern idea of a field as an independent inhabitant of our universe, with as much reality as the particles on which it acts. Instead (at least at first) he pictured electric and magnetic fields as disturbances in an underlying medium—the aether—like tension in a rubber membrane.[2] This would have had one obvious experimental implication—the observed speed of electromagnetic waves would depend on the speed of the observer with respect to the aether, just as the observed speed of elastic waves in a rubber membrane depends on the speed of the observer relative to the membrane. Maxwell himself thought that his own field equations were in fact valid in only one special frame, at rest with respect to the aether.[3] The notion of an aether that underlies the phenomena of electromagnetism persisted well into the twentieth century, despite the repeated failure of experimentalists to discover any effects of the motion through the aether of the earth as it revolves about the sun.

But even with the problem of the aether unresolved, the idea of a field as an entity in its own right grew stronger in physicists' minds. Indeed, it became popular to suppose that matter itself is ultimately a manifestation of electric and magnetic fields, a theme explored in the theories of the electron developed in 1900–1905 by Joseph John Thomson, Wilhelm Wien,[4] M. Abraham,[5] Joseph Larmor, Hendrik Antoon Lorentz,[6] and Jules Henri Poincaré.[7] Finally, in 1905 the essential element needed to free electromagnetic theory from the need for an aether was supplied by the special theory of relativity of Albert Einstein.[8] New rules were given for the way that the observed space and time coordinates of an event change with changes in the velocity of the observer. These new rules were specifically designed so that the observed speed of a light wave would be just that speed calculated in Maxwell's theory, whatever the velocity of the observer. Einstein's theory removed any hope of detecting the effects of motion through the aether, and although the aether lingered on in theorists' minds for a while, it eventually died away, leaving electromagnetic fields as things in themselves—the tension in the membrane, but without the membrane.

As it happens, it was the study of electromagnetic phenomena that gave rise to the quantum theory as well as to special relativity. By the end of the nineteenth century, it was clear that the classical theories of electromagnetism and statistical mechanics were incapable of describing the energy of electromagnetic radiation at various wavelengths emitted by a heated opaque body. The trouble was that classical ideas predicted too much energy at very high frequencies, so much energy in fact that the total energy per second emitted at all wavelengths would turn out to be infinite! In a paper read to the German Physical Society on December 15, 1900, a resolution of the problem was proposed by Max Karl Ernst Ludwig Planck.[9] It will be worthwhile for us to concentrate on Planck's proposal for a moment, not only because it led to modern quantum mechanics, but also because an understanding of this idea is needed in order to understand what quantum field theory is about.

Planck supposed that the electrons in a heated body are capable of oscillating back and forth at all possible frequencies, like a violin with a huge number of strings of all possible lengths. Emission or absorption of radiation at a given frequency occurs when the electron oscillations at that frequency give up energy to or receive energy from the electromagnetic field. The amount of energy being radiated per second by an opaque body at any frequency therefore depends on the average amount of energy in electron oscillations at that particular frequency.

It was in calculating this average energy that Planck made his revolutionary suggestion. He proposed that the energy of any mode of oscillation is *quantized*—that is, that it is not possible to set an oscillation going with any desired energy, as in classical mechanics, but only with certain distinct allowed values of the energy. More specifically, Planck assumed that the difference between any two successive allowed values of the energy is always the same for a given mode of oscillation, and is equal to the frequency of the mode times a new constant of nature which has come to be called *Planck's constant*. It follows that the allowed states of the modes of oscillation of very high frequency are widely separated in energy, so that it takes a great deal of energy to excite such a mode at all. But the rules of statistical mechanics tell us that the probability of finding a great deal of energy in any one mode of oscillation falls off rapidly with increasing energy; hence the average energy in oscillations of very high frequency must fall off rapidly with the frequency, and the energy radiated by a heated body must also fall off rapidly with the frequency of the radiation, thus avoiding the catastrophe of an infinite total rate of radiation.

Planck was not ready to apply the idea of energy quantization to radiation itself. (George Gamow[10] has described Planck's view as follows: "Radiation is like butter, which can be bought or returned to the grocery store only in quarter-pound packages, although the butter as such can exist in any desired amount.") It was Einstein[11] who in 1905 proposed that radiation comes in bundles of energy, later called *photons*, each with an energy proportional to the frequency.

In 1913 the ideas of Planck and Einstein were brought together by Niels Bohr in his theory of atomic spectra.[12] Like the hypothetical modes of oscillation in Planck's work, atoms in Bohr's theory are supposed to exist in distinct states with certain definite energies, but not generally equally spaced. When an excited atom drops to a state of lower energy, it emits a photon with a definite energy, equal to the difference of the energies of the initial and final atomic states. Each definite photon energy corresponds to a definite frequency, and it is these frequencies that we see vividly displayed when we look at the bright lines crossing the spectrum of a fluorescent lamp or a star.

This early quantum theory, from Planck to Bohr and for a decade after, was inspired guesswork, ad hoc mathematical manipulation, justified by its brilliant success in explaining the behavior of atoms and radiation. Quantum theory became the coherent scientific discipline known as *quantum mechanics*, through the work of Louis de Broglie, Werner Heisenberg, Max Born, Pascual Jordan, Wolfgang Pauli, Paul Adrian Maurice Dirac, and Erwin Schroedinger in 1925–1926.[13] Armed with this formalism, theorists were able to go back to the problem of determining the allowed energy levels of material systems, and to reproduce the successful results first found by Bohr. But despite its origins in the theory of thermal radiation, quantum mechanics still dealt in a coherent way only with material particles—electrons in atoms—and not with radiation itself.

The Birth of Quantum Field Theory

The first application of the new quantum mechanics of 1925–1926 to fields rather than particles came in one of the founding papers of quantum mechanics itself. In 1926, Born, Heisenberg, and Jordan[14] turned their attention to the electromagnetic field in empty space, in the absence of any electric charges or currents. Their work can best be understood by an analogy with Planck's 1900 theory of thermal radiation.

Planck, it will be recalled, had treated the motion of the electrons in a heated body in terms of an idealized picture, in which the electrons were replaced with an unlimited number of modes of simple oscillation, like a violin with a huge number of strings of all possible lengths. He had further proposed that the allowed energies of any one mode of oscillation were separated by a definite quantity, equal to the frequency of the mode times Planck's constant. One of the products of the new quantum mechanics of 1925–1926 was a confirmation of Planck's proposal: it was proved that the energies of a simple oscillator, like a violin string, were indeed quantized, in just the way that Planck had guessed. The essential feature of the dynamics of simple oscillations, used in obtaining this result, is that the energy required to produce any given displacement in the oscillator is proportional to the square of the displacement—as we pull a violin string farther and farther from its equilibrium position, it becomes harder and harder to produce any further displacement.

But essentially the same is true of an electromagnetic field—the energy in any one mode of oscillation of the field is proportional to the square of the field strength—in a sense, to the square of its "displacement" from the normal state of field-free empty space. Thus, by applying to the electromagnetic field the same mathematical methods that they had used for material oscillators, Born et al. were able to show that the energy of each mode of oscillation of an electromagnetic field is quantized—the allowed values are separated by a basic unit of energy, given by the frequency of the mode times Planck's constant. The physical interpretation of this result was immediate. The state of lowest energy is radiation-free empty space, and can be assigned an energy equal to zero. The next lowest state must then have an energy equal to the frequency times Planck's constant, and can be interpreted as the state of a single photon with that energy. The next state would have an energy twice as great, and therefore would be interpreted as containing two photons of the same energy. And so on. Thus, the application of quantum mechanics to the electromagnetic field had at last put Einstein's idea of the photon on a firm mathematical foundation.

Born, Heisenberg, and Jordan had dealt only with the electromagnetic field in empty space, so although their work was illuminating, it did not lead to any important quantitative predictions. The first "practical" use of quantum field theory was made in a 1927 paper of Paul Adrian Maurice Dirac.[15] Dirac was grappling with an old problem: how to calculate the rate at which atoms in excited states would emit electromagnetic radiation and drop into states of lower energy. The difficulty was not so much in deriving an answer—the correct formula had already been derived in an ad hoc sort of way by Born and Jordan[16] and by Dirac[17] himself. The problem was to understand this guessed-at formula as a mathematical consequence of quantum mechanics. This problem was of crucial importance, because the process of spontaneous emission of radiation is one in which "particles" are actually created. Before the event, the system consists of an excited atom, whereas after the event, it consists of an atom in a state of lower energy, plus one photon. If quantum mechanics could not deal with processes of creation and destruction, it could not be an all-embracing physical theory.

The quantum-mechanical theory of such processes can best be understood by returning to the analogy between fields and oscillators. In the absence of any interaction with atoms, the electromagnetic field is like an ensemble of completely isolated violin strings; whatever energy is given to any mode of oscillation, or equivalently, whatever the number of photons of a particular frequency, it will stay the same forever. Similarly, if an atom did not interact with radiation, it would remain indefinitely in whatever state it was placed. But atoms do interact with radiation, because electrons carry an electric charge. So the true analogy is with a set of violin strings that are weakly coupled together, as by the violin soundboard. Every musician knows what happens when one oscillator is set going—it will gradually feed energy into the other modes of oscillation until they are all excited. In quantum mechanics this cannot happen gradually because the energies are quantized, so instead the probability gradually increases that energy which was originally stored in the atom will be found in the electromagnetic field—in other words, that a photon will have been created.

Dirac's successful treatment of the spontaneous emission of radiation confirmed the universal character of quantum mechanics. However, the world was still conceived to be composed of two very different ingredients—particles and fields—which were both to be described in terms of quantum mechanics, but in very different ways. Material particles like electrons and protons were conceived to be eternal; to describe the physical state of a system, one had to describe the probabilities for finding each particle in any given region of space or range of velocities. On the other hand, photons were supposed to be merely a manifestation of an underlying entity, the quantized electromagnetic field, and could be freely created and destroyed.

It was not long before a way was found out of this distasteful dualism, toward a truly unified view of nature. The essential steps were taken in a 1928 paper of Jordan and Eugene Wigner,[18] and then in a pair of long papers in 1929–1930 by Heisenberg and Pauli.[19] (A somewhat different approach was also developed in 1929 by Enrico Fermi.[20]) They showed that material particles could be understood as the quanta of various fields, in just the same way that the photon is the quantum of the electromagnetic field. There was supposed to be one field for each type of elementary particle. Thus, the inhabitants of the universe were conceived to be a set of fields—an electron field, a proton field, an electromagnetic field—and particles were reduced in status to mere epiphenomena. In its essentials, this point of view has survived to the present day, and forms the central dogma of quantum field theory: the essential reality is a set of fields, subject to the rules of special relativity and quantum mechanics; all else is derived as a consequence of the quantum dynamics of these fields.

This field-theoretic approach to matter had an immediate implication: given enough energy, it ought to be possible to create material particles, just as photons are created when an atom loses energy. In 1932 Fermi[21] used this aspect of quantum field theory to formulate a theory of the process of nuclear beta decay. Ever since Becquerel discovered in 1896 that a crystal containing uranium salts would fog a photographic plate, it was known that nuclei were subject to various kinds of radioactive decay. In one of these modes of decay, known as beta decay, the nucleus emits an electron, and changes its own chemical properties. Throughout the 1920s it was believed that the nuclei are composed of protons and electrons, so there was no great paradox in supposing that every once in a while one of the electrons gets out. However, in 1931 Paul Ehrenfest and Julius Robert Oppenheimer[22] presented a compelling though indirect argument that nuclei do not in fact contain electrons, and in 1932 Heisenberg[23] proposed instead that nuclei consist of protons and the newly discovered neutral particles, the neutrons. The mystery was, where did the electron come from when a nucleus suffered a beta decay? Fermi's answer was that the electron comes from much the same place as the photon in the radiative decay of an excited atom—it is *created* in the act of decay, through an interaction of the field of the electron with the fields of the proton, the neutron, and a hypothesized particle, the neutrino.

One problem remained to be solved after 1930, in order for quantum field theory to take its modern form. In formulating the pre-field-theoretic theory of individual electrons, Dirac[24] in 1928 had discovered that his equations had solutions corresponding to electron states of negative energy, that is, with energy less than the zero energy of empty space. In order to explain why ordinary electrons do not fall down into these negative-energy states, he was led in 1930 to propose[25] that almost all these states are already filled. The unfilled states, or "holes" in the sea of negative energy electrons would behave like particles of positive energy, just like ordinary electrons but with opposite electrical charge: plus instead of minus. Dirac thought at first that these "antiparticles" were the protons, but their true nature as a new kind of particle was revealed with the discovery[26] of the positron in cosmic rays in 1932.

Dirac's theory of antimatter allowed for a kind of creation and annihilation of particles even without introducing the ideas of quantum field theory. Given enough energy, a negative-energy electron can be lifted up into a positive-energy state, corresponding to the creation of a positron (the hole in the negative-energy sea) and an ordinary electron. And of course the reverse annihilation process can also occur. Dirac himself had always resisted the idea that quantum field theory is needed to describe any sort of particle but photons. However, in 1934 a pair of papers by Wendell Furry and Oppenheimer[27] and by Pauli and Victor Weisskopf[28] showed how quantum field theory naturally incorporates the idea of antimatter, without introducing unobserved particles of negative energy, and satisfactorily describes the creation and annihilation of particles and antiparticles. For most theorists, this settled the matter, and particles and antiparticles are now seen as coequal quanta of the various quantum fields.

It is important to understand that quantum field theory gave rise to a new view not only of particles but also of the forces among them. We can think of two charged particles interacting at a distance not by creating classical electromagnetic fields which act on one another, but by exchanging photons, which continually pass from one particle to the other. Similarly, other kinds of force can be produced by exchanging other kinds of particle. These exchanged particles are called *virtual particles*, and are not directly observable while they are being exchanged, because their creation as real particles (e.g., a free electron turning into a photon and an electron) would violate the law of conservation of energy. However, the quantum-mechanical uncertainty principle dictates that the energy of a system that survives for only a short time must be correspondingly highly uncertain, so these virtual particles can be created in intermediate states of physical processes, but must be reabsorbed again very quickly.

From this line of reasoning, one can infer that the force produced by the exchange of a given type of particle has a range (the distance beyond which it falls off very rapidly) inversely proportional to the mass of the exchanged particle. Thus, the photon, which has zero mass, gives rise to a force of infinite range, the familiar inverse-square force of Coulomb. The force between protons and neutrons in an atomic nucleus was known to have a range of a little less than a million millionth of a centimeter, so Hidekei Yukawa[29] was able in 1936 to predict the existence of an entirely new kind of particle, the meson, with a mass a few hundred times that of the electron. In calculating these forces, one assumes that the energy density at a point is not just a sum of squares of fields, as for uncoupled simple mechanical oscillators, but also contains products of the values of the different fields (and their rates of change) at that point. These multifield interactions are the unknowns that have to be sought by our theoretical and experimental efforts. From the viewpoint of quantum field theory, all questions about the particles of which matter is composed and the forces that act among them are only means to an end—the real problem is to determine what are the fundamental quantum fields, and what are the interactions among them.

The Problem of Infinities

I have described the early days of quantum field theory as if it were a grand progress from triumph to triumph. This has been a somewhat distorted picture, for almost from the beginning the theory was thought to be subject to a grave internal inconsistency.

The problem first appeared in a 1930 paper of Oppenheimer,[30] who was trying to calculate the effect on the energy of an atomic electron produced by its interaction with the quantum electromagnetic field. Just as the exchange of virtual photons between two electrons produces an energy of interaction between them, so also in quantum field theory the emission of virtual photons and their reabsorption by the same electron produces a self-energy, which might depend on the atomic orbit occupied by the electron, and which might show up as an observable shift in atomic energy levels. Unfortunately, Oppenheimer discovered that the energy shift predicted by the quantum theory of the electromagnetic field was infinite!

The infinity here arises because when an electron in an atom turns briefly into a photon and an electron, these two particles can share the momentum of the original electron in an infinite variety of ways. The self-energy of the electron involves a sum over all the ways that the momentum can be shared out, and because there is no limit to how large the momenta can be, this sum turns out to be infinite. It was not obvious that this would have to happen; after all, there are many examples of mathematical series in which one adds up an infinite number of terms and gets a finite result. (For example, $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$). However, Oppenheimer found that the self-energy of the electron behaved more like the series $1 + 1 + 1 + 1 + \ldots$, and could hardly be interpreted as a finite quantity.

The problem was ameliorated a bit a few years later, when Weisskopf[31] included the effects of processes in which a virtual electron, positron, and photon are created out of empty space, with the positron and photon then being annihilated along with the original electron, leaving the new electron over as a real particle in the final state. This contribution to the self-energy cancelled the worst part of the original infinity found by Oppenheimer, but the self-energy was left in the form of a sum over virtual momenta which behaved like the series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots$, and which still could not be interpreted as a finite quantity.

Similar infinities were found in other problems, such as the "polarization of the vacuum" by applied electric fields,[32] and the scattering of electrons by the electric fields of atoms.[33] (One of the few bright spots in this otherwise discouraging picture came in the treatment of infinities associated with photons of very low momenta; it was shown in 1937 by Felix Bloch and Arnold Nordsieck[34] that these infrared infinities all cancel in the total rates for collisions.) Of course, if one uses a theory to calculate an observable quantity, and finds that the answer is infinite, one concludes either that a mathematical mistake has been made, or that the original theory was no good. Throughout the 1930s, the accepted wisdom was that quantum field theory was in fact no good, that it might be useful here and there as a stopgap, but that something radically new would have to be added in order for it to make sense.

The problem of infinities in fact has provided the single greatest impetus toward a radical revision of quantum field theory. Some of the ideas which were tried out in the 1930s and 1940s are listed below:

1. In 1938 Heisenberg[35] proposed that there is a fundamental unit of energy, and that quantum field theory works only at scales of energy that are small compared with this energy unit. The analogy was with the other fundamental constants, Planck's constant and the speed of light. Quantum mechanics comes

into play when ratios of energies and frequencies approaches values as small as Planck's constant, whereas special relativity is needed when velocities approach values as large as the speed of light. In the same way, Heisenberg supposed, some entirely new physical theory might be needed when energies exceed the fundamental unit, and some mechanism in this theory might wipe out the contributions of virtual particles with such high energies, thus avoiding the problem of infinities. (Heisenberg's idea drew support in the 1930s from the observation that the showers of charged particles produced by high energy cosmic rays did not behave as expected in quantum electrodynamics. This discrepancy was later realized to be due to the production of new particles, the mesons, and not to a failure of quantum field theory.)

2. John Archibald Wheeler[36] in 1937 and Heisenberg in 1943[37] independently proposed a positivistic approach to physics, which would have replaced quantum field theory with a different sort of theory, sometimes called an "S-matrix theory," which would involve only directly measurable quantities. They reasoned that experiments do not actually allow us to follow what happens to electrons in atoms or in collision processes. Instead, it is only really possible to measure the energies and a few other properties of bound systems like atoms, and the probabilities for various collision processes. These quantities obey certain very general principles, such as reality, conservation of probabilities, smooth energy dependence, conservation laws, etc., and it was these general principles that were supposed to replace the assumptions of quantum field theory.

3. Dirac[38] in 1942 suggested that quantum mechanics ought to be expanded to include states of negative probability, which could not appear as the initial or final state of any physical process, but which would have to be included among the intermediate states in these processes. In this manner, minus signs might be introduced into the sums over the ways that the intermediate states share out the momentum of the system, so that a finite answer would be obtained, just as $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ is a finite quantity (the natural logarithm of 2) whereas $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots$ is infinite.

4. As already mentioned, Richard Feynman and John Wheeler[39] in 1945 considered the possibility of abandoning field theory altogether, replacing the field-mediated interaction among particles with a direct action at a distance.

Some of these ideas have survived and are now part of the regular equipment of theoretical physics. In particular, the idea of a pure S-matrix theory has flourished in the development of so-called "dispersion relations" which involve only observable quantities.[40] Also, states of negative probability are now a handy mathematical device, useful especially for dealing with the polarization of virtual photons.[41] However, none of these ideas proved to be the key to the problem of infinities.

The solution turned out to be far less revolutionary than most theorists had expected. Recall that the energy of an electron had been found by Oppenheimer, Waller, and Weisskopf to receive an infinite contribution from the emission and reabsorption of "virtual" photons. An infinite self-energy of this sort appears not only when the electron is moving in orbit in an atom, but also when it is at rest in empty space. But special relativity tells us that the energy of a particle at rest is related to its mass by the famous formula $E = mc^2$. Thus the electron mass found in tables of physical data could not be just the "bare" mass, the quantity appearing in our equations for the electron field, but would have to be identified with the bare mass *plus* the infinite "self mass," produced by the interaction of the electron with its own virtual photon cloud. This suggests that the bare mass might itself be infinite, with an infinity which just cancels the infinity in the self mass, leaving a finite total mass to be identified with the mass that is actually observed. Of course, it goes against the grain to suppose that a quantity like the bare mass, which appears in our fundamental field equations could be infinite; but after all, we can never turn off the electron's virtual photon cloud to measure the bare mass, so no paradox arises. Similar remarks apply to other physical parameters, like the charge of the electron. Is it then possible that the infinities in the bare masses and bare charges cancel the infinities found in quantum field theory, not only when we calculate the total masses and charges, but in all other calculations as well? This method, of eliminating infinities by absorbing them into a redefinition of physical parameters, has come to be called *renormalization*.

The renormalization idea was suggested by Weisskopf in 1936,[42] and again by Kramers[43] in the mid 1940s. However, it was far from obvious that the idea would work. In order to eliminate infinities by absorbing them into redefinitions of physical parameters, the infinities must appear in only a special way, as corrections to the observed values of these parameters. For instance, in order to absorb the infinite shift in atomic energy levels found by Oppenheimer into a redefinition of the electron mass, the infinite part of the self-energy would have to be the same for all atomic energy levels. The mathematical methods available in the 1930s and early 1940s were simply inadequate for the task of sorting out all the infinities that might appear in all possible calculations to see if they could all be eliminated by renormalization. Perhaps even more important, there was no compelling reason to do so—there were no experimental data which forced theorists to come to grips with these problems. And of course, physicists had other things on their minds from 1939 to 1945.

Revival at Shelter Island

On June 1, 1947, a four-day conference on the foundations of quantum mechanics opened at Shelter Island, a small island near the end of Long Island in New York State. The conference brought together young American physicists from the new generation who had started their scientific work during the war at Los Alamos and the MIT Radiation Laboratory, as well as older physicists who had been active in the 1930s. Among the younger participants was Willis Lamb, an experimentalist then working in the remarkable group of physicists founded at Columbia University by I. I. Rabi. Lamb announced the results of a beautiful experiment, in which he and a student, R. C. Retherford, had for the first time measured an effect of the self-energy of the electron in the hydrogen atom.[44]

The existing theory of the hydrogen atom had been first advanced by Niels Bohr in 1913,[12] then put on a sound mathematical foundation by the quantum mechanics of 1925–1926,[13] and finally corrected to include effects of relativity and the spin of the electron by Heisenberg and Jordan,[45] C. G. Darwin,[46] and

Dirac.[24] In the final version of this theory, in particular as formulated by Dirac, certain pairs of excited states of the atom were expected to have exactly equal energy. (These pairs of states correspond to the two different ways that the spin of the electron and the angular momentum associated with its revolution around the nucleus could combine to give a definite total angular momentum.) But this theory ignored all effects of the interaction of the electron with its own electromagnetic field—the effects that Oppenheimer[30] had tried to calculate when he discovered the infinities. If such effects were real they would presumably shift the energies of these pairs of states, so that they would no longer be exactly equal.

This is what Lamb and Retherford found. By using the new techniques of handling microwave radiation that had come out of wartime work in radar, they were able to show that the energies of the first two excited states of hydrogen (the $2s_{1/2}$ and $2p_{1/2}$ states), which were supposed to be equal according to the 1928 Dirac theory, actually differed by about 0.4 parts per million. This is now known as the Lamb shift.

Stimulated in part by Lamb's results, the participants at Shelter Island entered into an intense discussion of the underlying theory. I was not at Shelter Island (having just entered high school) and I cannot trace the historical development of the different reformulations of quantum field theory that developed around that time. It would be most valuable for a historian of science to gather the recollections of the participants at Shelter Island and succeeding conferences, read the papers that were written at that time, and put together a coherent account. I will sketch here only a few of the products of this period.

The Lamb shift itself was first calculated by Hans Bethe,[47] I believe on the train ride back from Shelter Island. Using mass renormalization to eliminate infinities, he obtained a result in reasonable agreement with the value announced by Lamb. However, as acknowledged by Bethe himself, this was a rough calculation, involving approximations that were not fully consistent with the Special Theory of Relativity.

There were at least three general reformulations of quantum field theory worked out in the late 1940s that were thoroughly relativistic and that were sufficiently simple and elegant to allow a systematic treatment of the infinities. One of these approaches had actually been developed well before the Shelter Island Conference, by Sin-Itiro Tomonaga[48] and his colleagues in Japan, but I believe that their work had not yet become known in the United States in the summer of 1947. The two other approaches were contributed by participants at Shelter Island, Julian Schwinger[49] and Richard Feynman.[50]

Feynman's work led to a set of pictorial rules which allowed one to associate a definite numerical quantity to each picture of how the momentum and energy could flow through the intermediate states of any collision process: the probability for the process is given by the square of the sum of these individual quantities. The Feynman rules were very much more than a handy calculational algorithm, because they incorporated an essential feature of quantum field theory—the symmetry between particles and antiparticles. Each line in a Feynman diagram can represent either a particle created at one end of the line and destroyed at the other, or an antiparticle going the other way. It is this equal treatment of particles and antiparticles that ensures that the quantities calculated by Feynman diagrams are independent of the velocity of the observer, as required by the Special Theory of Relativity, at every stage of the calculation. As had been shown long before by Weisskopf,[31] intermediate states involving antiparticles play a crucial role in cutting down the degree of infinity, from disasters like $1 + 2 + 3 + \ldots$, to something more manageable like $1 + \frac{1}{2} + \frac{1}{3} \ldots$. The Feynman rules automatically ensured the cancellations of the worst infinities, leaving over the more manageable infinities, which could be eliminated by renormalization.

Before the end of 1947 Schwinger[51] had used his approach to carry out what I believe was the first calculation of another effect of the electron's cloud of virtual photons, the *anomalous magnetic moment* of the electron. One of the triumphs of Dirac's 1928 theory[24] was its prediction of the magnetic moment of the electron, a number which characterizes the strength of the electron's interaction with magnetic fields, and the strength of its own magnetic field. However, experiments[52] at Columbia in 1947 had revealed that the magnetic moment of the electron is actually a little larger than the Dirac value, by 1.15 to 1.21 parts per thousand. By absorbing the infinite effects of virtual photons into a renormalization of the charge of the electron, Schwinger was able to calculate a finite magnetic moment that was larger than the Dirac value by just 1.16 parts per thousand!

Of course, both the experimental and the theoretical determinations of effects like the Lamb shift and the anomalous magnetic moment of the electron have been enormously improved since 1947.[53] For instance, right now the experimental value of the magnetic moment of the electron is larger than the Dirac value by 1.15965241 parts per thousand, whereas the theory gives this anomalous magnetic moment as 1.15965234 parts per thousand, with uncertainties of about 0.00000020 and 0.00000031 parts per thousand, respectively. The precision of the agreement between theory and experiment here can only be called spectacular.

Finally, Freeman Dyson[54] in 1949 showed that the formalisms of Schwinger and Tomonaga would yield the same graphical rules that had been found by Feynman. Dyson also carried out an analysis of the infinities in general Feynman diagrams, and sketched out a general proof that these infinities are always of precisely the sort which could be removed by renormalization.[55] As a graduate student in the mid-1950s, I learned the new approach to quantum field theory by reading Dyson's marvelously lucid papers.

I should emphasize that the theory of Schwinger, Tomonaga, Feynman, and Dyson was not really a new physical theory. It was simply the old quantum field theory of Heisenberg, Pauli, Fermi, Oppenheimer, Furry, and Weisskopf, but cast in a form far more convenient for calculation, and equipped with a more realistic definition of physical parameters like masses and charges. The continued vitality of the old quantum field theory after fifteen years of attempts to find a substitute is truly impressive.

This raises an interesting historical question. All the effects that were calculated in the great days of 1947–1949 could have been estimated if not actually calculated at any time after 1934. True, without the renormalization idea, the answers would have been formally infinite, but at least it would have been possible to guess the order of magnitude of quantities like the Lamb shift and the correction to the magnetic moment of the electron. (An infinite series like $1 + 1/2 + 1/3 + 1/4 + \ldots$ grows very slowly; after a million terms, it is still less than 14.4.) Not only was this not done—most theorists seemed to have believed that these quantities were zero! Indeed, some evidence for what was later called the Lamb shift had actually been discovered[56] in 1938, but to the best of my knowledge, no theorist checked to see whether the order of magnitude of this reported energy splitting was more or less what would be expected in a quantum field theory.

Why was quantum field theory not taken more seriously? One reason is the tremendous prestige of the Dirac theory[24] of 1928, which had worked so well in accounting for the fine structure of the hydrogen spectrum without including self-energy effects. Even more important, the appearance of infinities discredited quantum field theory altogether in many physicists' minds. But I think that the deepest reason is a psychological difficulty, that may not have been sufficiently appreciated by historians of science.[57] There is a huge apparent distance between the equations that theorists play with at their desks, and the practical reality of atomic spectra and collision processes. It takes a certain courage to bridge this gap, and to realize that the products of thought and mathematics may actually have something to do with the real world. Of course, when a branch of science is well under way, there is continual give and take between theory and experiment, and one gets used to the idea that the theory is about something real. Without the pressure of experimental data, the realization comes harder. The great thing accomplished by the discovery of the Lamb shift was not so much that it forced us to change our physical theories, as that it forced us to take them seriously.

Weak and Strong Interactions

For a few years after 1949, enthusiasm for quantum field theory was at a high level. Many theorists expected that it would soon lead to an understanding of all microscopic phenomena, not only the dynamics of photons, electrons, and positrons. However, it was not long before there was another collapse in confidence—shares in quantum field theory tumbled on the physics bourse, and there began a second depression, which was to last for almost twenty years.

Part of the problem arose from the limited applicability of the renormalization idea. In order for all infinities to be eliminated by a renormalization of physical parameters like masses and charges, it is necessary for these infinities to arise in only a limited number of ways, as corrections to masses, charges, etc., but not otherwise. Dyson's work[54] showed that this would be the case for only a small class of quantum field theories, which are called *renormalizable* theories. The simplest theory of photons, electrons, and positrons (known as *quantum electrodynamics*) is renormalizable in this sense, but most theories are not.

Unfortunately, there was one important class of physical phenomena which apparently could not be described by a renormalizable field theory. These were the *weak interactions*, which cause the radioactive beta decay of nuclei mentioned above in the section on the Birth of Quantum Field Theory. Fermi[21] had invented a theory of weak interactions in 1932 which, with a few modifications, adequately described all weak interaction phenomena in the lowest order of approximation, that is, including only a single simple Feynman diagram in calculations of transition rates. However, as soon as this theory was pushed to the next order of approximation, it exhibited infinities which could not be removed by a redefinition of physical quantities.

The other major problem had to do with the limited validity of the approximation techniques used in 1947–1949. Any physical process is represented by an infinite sum of Feynman diagrams, each one representing a particular sequence of intermediate states consisting of definite numbers of particles of various types. To each diagram we associate a numerical quantity; the rate of the process is the square of the sum of these quantities. Now, in quantum electrodynamics the quantities associated with complicated diagrams are very small; for each additional photon line there is one additional factor of a small number known as the *fine-structure constant*, roughly 1/137. In the Fermi theory of weak interactions, the corresponding factor is even smaller—at the typical energies of elementary particle physics, it is 10^{-5} to 10^{-7}. It is the rapid decrease of the contributions associated with complicated Feynman diagrams (together with the renormalizability of the theory) that makes it possible to carry calculations in quantum electrodynamics to such a fantastic degree of accuracy.

However, in addition to electromagnetic and weak interactions, there is one other class of interaction in elementary particle physics, known as the *strong interactions.* It is the strong interactions that hold atomic nuclei together against the electrostatic repulsion of the protons they contain. For strong interactions, the factor corresponding to the fine structure constant is roughly of the order of one instead of 1/137, so complicated Feynman diagrams are just as important as simple ones. (This of course is why these interactions are strong.) Thus, although attempts have been made time and time again to use quantum field theory in calculations of the nuclear force, it has never really worked in a convincingly quantitative way. New kinds of strongly interacting particles called mesons and hyperons were being discovered from 1947 on, first in cosmic rays and then in accelerator laboratories, and quantum field theory was at first enthusiastically used to study their strong interactions, but again with little quantitative success. It was not that there was any difficulty in thinking of renormalizable quantum field theories that *might* account for the strong interactions—it was just that having thought of such a theory, there was no way to use it to derive reliable quantitative predictions, and to test if it were true.

The nonrenormalizability of the field theory of weak interactions and the uselessness of the field theory of strong interactions led in the early 1950s to a widespread disenchantment with quantum field theory. Some theorists turned to the study of symmetry principles and conservation laws, which can be applied to physical phenomena without detailed dynamical calculations. Others picked up the old S-matrix theory of Wheeler and Heisenberg, and worked to develop principles of strong interaction physics that would involve only observable quantities. In both lines of work, quantum field theory was used heuristically, as a guide to general principles, but not as a basis for quantitative calculation.

Now, as a result of work by many physicists over the last decade, quantum field theory has again become what it was in the late 1940s, the chief tool for a detailed understanding of elementary particle processes. There are quantum field theories of the weak and strong interactions, called *gauge theories*, which are not subject to the old problems of nonrenormalizability and incalculability, and which are to some extent even true! We are still in the midst of this revival, and I will not try to outline its history, but will only summarize how the old problems of quantum field theory are surmounted in the new theories.[58]

The essence of the new theories is that the weak and the strong interactions are described in a way that is almost identical to the successful older quantum field theory of electromagnetic interactions. Just as electromagnetic interactions among charged particles are produced by the exchange of photons, so the weak interactions are produced by the exchange of particles called *intermediate vector bosons* and the strong interactions by the exchange of other particles called *gluons*. All these particles—photons, intermediate vector bosons, and gluons—have equal spin, and have interactions governed by certain powerful symmetry principles known as *gauge symmetries*. (A gauge symmetry principle states that the fundamental equations do not change their form when the fields are subjected to certain transformations, whose effect varies with position and time.) Because these theories are so similar to quantum electrodynamics, they share its fundamental property, of being renormalizable. Indeed, the relation between weak and electromagnetic interactions is not merely one of analogy—the theory unifies the two, and treats the fields of the photon and the intermediate vector bosons as members of a single family of fields.

The intermediate vector bosons are not massless, like the photon, but instead are believed to have perhaps 70 to 80 times the mass of a proton or neutron. This huge mass is not due to any essential dissimilarity between the photon and the intermediate vector boson fields, but instead arises from the way that the symmetry of the underlying field theory breaks down when the field equations are solved. The family of intermediate vector bosons, of which the photon is a member, is believed to contain one heavy charged particle and its antiparticle, called the W^+ and W^-, and one even heavier neutral particle, called the Z^0. Exchange of the W produces the familiar weak interactions, like nuclear beta decay, whereas exchange of the Z^0 would produce a new kind of weak interaction, in which the participating particles do not change their charge. Such *neutral current processes* were discovered in 1973, and are found to have just about the properties expected in these theories. All of the intermediate vector bosons are much too heavy to have been produced with existing accelerator facilities, but there are great hopes of producing them with colliding beams of protons and antiprotons before too long.

In contrast, the gluons which mediate the strong interactions may well have zero mass. Such theories with massless gluons have a remarkable property known as *asymptotic freedom*—at very high energy or very short distances, the strength of the gluon interactions gradually decreases. In consequence, it is now possible to use quantum field theory to carry out detailed calculations of strong interaction processes at sufficiently high energy. In particular, it has been possible to account for some of the features observed in a process such as high energy electron-nucleon scattering.

Just as the gluon interactions become weak at high energy and short distances, they also become strong at low energy and long distances. For this reason, it is widely believed (though not yet proved) that particles which carry the quantity called *color* with which gluons interact (in the same sense that photons interact with electric charge) cannot be produced as separate free particles. The colored particles include the gluons themselves, which is presumably why gluons have never been observed as real particles. The colored particles are also believed to include the quarks discussed in this double issue of *Daedalus* in the article by Sidney Drell. The observed strongly interacting particles such as neutrons, protons, and mesons are believed to be compound states, consisting of quarks, antiquarks, and gluons, but with no net color. This picture represents a nearly complete triumph of the field over the particle view of matter: the fundamental entities are the quark and gluon fields, which do not correspond to any particles that can be observed even in principle, whereas the observed strongly interacting particles are not elementary at all, but are mere consequences of an underlying quantum field theory.

There are hopes of a unified gauge theory of weak, electromagnetic, and strong interactions. The photon, intermediate vector bosons, and gluons would then form part of a single family of fields. However, in order for this to be possible, there would have to be other fields in this family, corresponding to particles of extraordinarily high mass. According to one estimate, the expected mass of these new particles would be 10^{17} (a hundred thousand million million) proton masses. These masses are so high that it is no longer possible to ignore the gravitational fields of these particles, as done almost everywhere else in particle physics.

Unfortunately, despite strenuous efforts which continue to the present, there still has not been found a satisfactory (e.g., renormalizable) quantum field theory of gravitation. It is ironic that gravitation, which provided the first classical field theory, has so far resisted incorporation into the general framework of quantum field theory.

Throughout this history I have put great emphasis on the condition of renormalizability, the requirement that it should be possible to eliminate all infinities in a quantum field theory by a redefinition of a small number of physical parameters. Many physicists would disagree with this emphasis, and indeed, it may eventually be found that all quantum field theories, renormalizable or not, are equally satisfactory. However, it has always seemed to me that the requirement of renormalizability has just the kind of restrictiveness that we need in a fundamental physical theory. There are very few renormalizable quantum field theories. For instance, it is possible to construct quantum field theories of electromagnetism in which the electron has any magnetic moment we like, but only one of these theories, corresponding to a magnetic moment of 1.0011596523 . . . times the Dirac value, is renormalizable. Also, as we have seen, it took a long time before it was found that there are any renormalizable theories at all of the weak interactions. We very much need a guiding principle like renormalizability to help us to pick the quantum field theory of the real world out of the infinite variety of conceivable quantum field theories. Thus, if renormalizability is ultimately to be replaced with some other condition, I would hope that it will be one that is equally or even more restrictive. After all, we do not want merely to describe the world as we find it, but to explain to the greatest possible extent why it has to be the way it is.

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