QFT — Introduction to Feynman Diagrams: assignment 4

1. **cross section**

Show that the cross section for two-to-two particle scattering in the centre of mass frame is given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}|\mathcal{M}|^2.$$

The magnitudes of the three-momenta in the initial and final states are taken to be p_i and p_f respectively. You may use the formula in terms of *dLips* to obtain this. Here $s = (p_1 + p_2)^2$.

2. **decay rate**

Show that the decay rate for the process $A \to 12$ is given by

$$\frac{d\Gamma}{d\Omega} = \frac{p_f}{32\pi^2 m_A^2}|\mathcal{M}|^2$$

in the rest frame of particle A. You may use the formula in terms of *dLips* to obtain this.

3. **two-point function**

Use Wick's theorem to evaluate

$$\frac{\langle 0|T[\phi_x\phi_y \mathrm{e}^{+i\int d^4z\mathcal{L}}]|0\rangle}{\langle 0|T[\mathrm{e}^{+i\int d^4z\mathcal{L}}]|0\rangle}$$

to order λ^2 in ϕ^4 theory. You may leave you answer in terms of spatial integrals over Feynman propagators. Verify explicitly that bubbles cancel in the ratio. Correctly obtain all symmetry factors.

4. **two-point function (ii)**

(i) Use the Feynman rules to obtain all the diagrams contributing to the two-point function to order λ^2 in ϕ^3 theory. Obtain all symmetry factors.

(ii) Write expressions for your Feynman diagrams in momentum space.

(iii) Think about evaluating your expressions. How do they behave in the ultraviolet limit? (put an upper cutoff of Λ on your integrals and determine how the integral behaves as a function of Λ when Λ is large). One of your diagrams should be a tadpole; show that the tadpole can be completely absorbed in a redefinition of the 'phion' mass. Tadpoles are normally ignored due to this.

5. **phion decay**

(i) Use the Feynman rules to draw all diagrams of any type (disconnected, unamputated) that contribute to $\varphi \to \phi\phi$ decay in the model,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}M^2\varphi^2 + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{g}{2!}\varphi\phi^2$$

to order g^3.

(ii) Which of these diagrams is AmpCon?

(iii) Determine the symmetry factors of all of the AmpCon diagram(s).