QFT — renormalisation preamble: assignment 5

1. **two-component Klein-Gordon fields**

(i) A second order differential equation can be reduced to coupled first order differential equations. Do this for the free complex KG field by setting

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_- \end{pmatrix}$$

with $\phi_\pm = \frac{1}{\sqrt{2m}}(\pm i\partial_t + m)\phi$. If

$$i\frac{\partial\phi}{\partial t} = H\phi$$

determine the 2x2 matrix H.

(ii) Set

$$\phi^{(\pm)} = \begin{pmatrix} \chi \\ \eta \end{pmatrix} \mathrm{e}^{i(\vec{k}\cdot\vec{x}\mp Et}$$

and determine $\phi_k^{(+)}(x)$ and $\phi_{-k}^{(-)}(x)$ (up to a normalisation).

(iii) Consider charge conjugation in this approach. Let $\phi \to \phi^C = \mathcal{C}\phi^*$ where $\mathcal{C}$ is a 2x2 matrix to be determined. Find $\mathcal{C}$ such that ϕ^C obeys the same equation as ϕ and show that $\phi_{-k}^{(-)C} = \phi_k^{(+)}$. Interpret this equation.

2. **LSZ**

Discuss as fully as possible (extensive equations are not required) the application of the LSZ theorem to the computation of $\varphi\varphi \to \phi\phi$ in φ^4 theory. Here ϕ is a bound state consisting of at least two phions.

3. **DimReg**

Compute

$$\int \frac{d^4k}{(2\pi)^4} \frac{4k^\mu k^\nu - k^2 g^{\mu\nu}}{(k^2-\Delta)^3}$$

using dimensional regularisation. Express your answer in terms of $\epsilon = 4 - d$. Comment on your result.