

QFT assignment 6: ϕ^3 Theory at one-loop

1. Obtain the Callan-Symanzik beta function for ϕ^3 theory at one loop. Use

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{3!}g\phi^3$$

- (i) Rewrite the lagrangian using the BHP method.
- (ii) Relate the bare and physical (renormalised) parameters. Give the finite renormalisation group equations.
- (iii) Identify the classes of divergent diagrams. In how many dimensions is the theory renormalisable? Use DimReg in the following, taking $d = d_{renorm} - \epsilon$.
- (iv) Obtain the one-loop counterterms using the \overline{MS} scheme.
- (v) Determine the imaginary part of the renormalised pion self-energy. Is your result sensible?
- (vi) While we are at it, discuss the renormalisation of the self-energy in the MOM scheme. Specifically, note that two conditions on Σ imply that Σ' must be finite. Argue that this must be true in general.
- (vii) Obtain the beta function. What is the fixed point of the theory at this order?